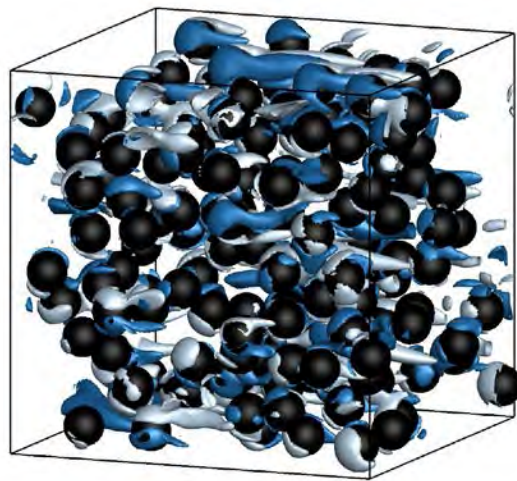


# CCMT

CENTER FOR COMPRESSIBLE  
MULTIPHASE TURBULENCE

## Multiphase Physics Deep-dive Workshop October 6-7, 2016





# CENTER FOR COMPRESSIBLE MULTIPHASE TURBULENCE

## Multiphase Physics Deep-dive Workshop

### October 6-7, 2016 Attendee List

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NNSA



# CENTER FOR COMPRESSIBLE MULTIPHASE TURBULENCE

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Philip Smith  
Sean Smith  
Prashanth Sridharan  
Jeff St. Clair  
Cameron Stewart  
Jeremy Thornock  
Gretar Tryggvason  
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ASC AST  
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## **Agenda Multiphase Physics Deep-Dive, October 6-7, 2016 St. Petersburg Marriott Clearwater,**

12600 Roosevelt Blvd, North St. Petersburg, FL 33716, Phone: 727-572-7800

### Thursday Oct 6<sup>th</sup>, 2016

7:30 – 8:30 Breakfast (Provided)

8:30 – 8:45 Welcome (S. Balachandar)

8:45 – 10:15 Overviews

Gretar Tryggvason

Challenges and opportunities in fully resolved simulations of multi fluid flows

Ali Mani

Overview of computational modeling at Stanford PSAAP: particle-laden flows subject to radiative heating

Kambiz Salari

Research activities for energetic dispersal of particles

10:15 – 10:30 Coffee

10:30 – 12:00 Overviews

Alex Brown

Multiphase Methods for Modeling Fire Environments

Phil Smith

Multi-phase flow modeling at Utah PSAAP – predictivity in application

S. Balachandar

Overview of multiphase flow computational strategy at UF PSAAP

12:00 – 12:15 Further Discussion

12:15 – 1:30 Lunch (Provided)

1:30 – 3:00 Macroscale

Duan Zhang:

Equations and Closures for Deformation and Flow of Continuous and Disperse Materials

Allen Khul

3-Phase Model of Explosion Fields

Marco Arienti

Multiphase Flow Simulation Strategies at the CRF



3:00 – 3:15 Coffee

3:15 – 5:15 Microscale

Jeremy Horwitz	Point-particle modeling for two-way-coupled problems: Challenges, verification, and physics-based improvements
Georges Akiki	Extended point particle model
Jesse Capecehatro	Recent insights on turbulence modeling of strongly-coupled particle-laden flows
Tom Jackson	Microscale simulations of shock particle interaction

6:30 – 9:00 Dinner (Provided – all attendees)

Friday Oct 7<sup>th</sup>, 2016

7:30 – 8:30 Breakfast (Provided)

8:30 – 10:30 Modeling & numerical methods

Sean Smith	Particle dynamics: coal-specific modeling
A. Subramaniam	Microscale modeling based on Generalized Faxen theorem
Alan Harrison	Modeling of Ejecta Particles in the FLAG Continuum Mechanics Code
Markus Uhlmann	Large scale microscale simulations and modeling opportunities

10:30 – 10:45 Coffee

10:45 – 12:15 Experiments and simulations

Ankur Bordoloi	Experimental measurements of drag on shocked particles
Laura Villafañe	Including real experimental effects in validation of numerical models for confined particle-laden flows
Fady Najjar	Meso-scale Simulations of Shock-Particle Interactions

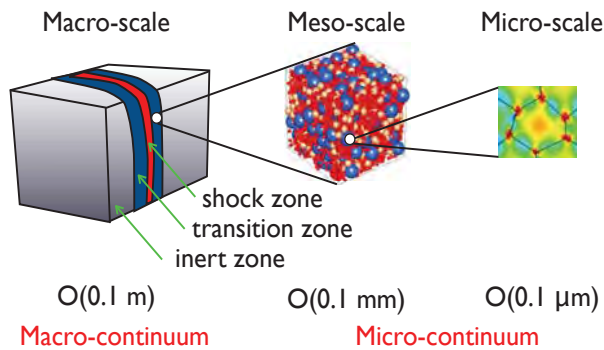
12:15 – 1:30 Lunch (Provided)

1:30 – 3:00 Meso/macroscale

Mahdi Esmaily	A systematic study of turbophoresis by four-way-coupled simulation of Stokesian particles in channel flow
Balu Nadiga	Bayesian Analysis of Inter-Phase Momentum Transfer in the Dispersed Eulerian Formulation of Multiphase Flow
John Parra-Álvarez	Eulerian Models and Polydispersity Treatment for Dilute Gas-Particle Flows

3:00 – 4:00 Discussion & Closing remarks

## PSAAP II: Center for Shock Wave-processing of Advanced Reactive Materials (C-SWARM)



**The C-SWARM framework represents a transformative engineering and science achievement, the ability to synthesize new materials,** that exemplifies what Exascale computing can deliver.

Develop simulations tools that allow prediction of solid-solid transformation, using continuum framework, including chemo-thermo-mechanical behavior



## Challenges and Opportunities in Fully Resolved Simulations of Multi Fluid Flows

Grétar Tryggvason,  
University of Notre Dame

Multiphase Physics Deep-Dive  
October 6-7, 2016 St. Petersburg Marriott Clearwater

Work supported by NSF & DOE (CASL & PSAAP-II)



### CFD of Multiphase Flows—one slide history

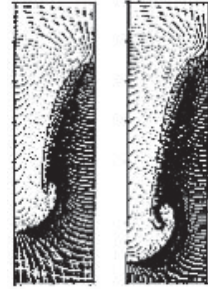
BC: Birkhoff and boundary integral methods  
for the Rayleigh-Taylor Instability

65' Harlow and colleagues at Los Alamos:  
The MAC method

75' Boundary integral methods for Stokes flow and  
potential flow

85' Alternative approaches (body fitted, unstructured,  
etc.)

95' Beginning of DNS of multiphase flow. Return of the  
“one-fluid” approach and development of other  
techniques



From: B. Daly (1969)



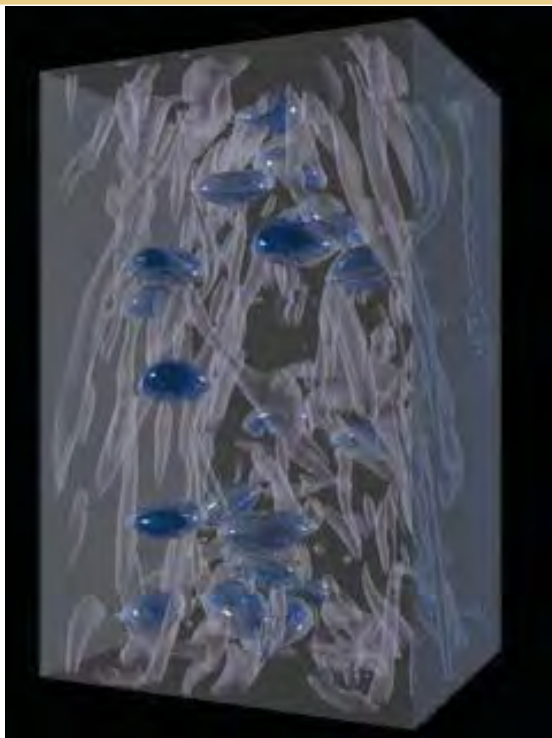
A number of methods have now been developed for  
incompressible multiphase flows. Many of the difficulties  
encountered by early methods have been more or less  
overcome. Those include:

- Artificial fluid motion at high surface tension (parasitic currents), induced by inconsistent numerical approximations.
- Consistent momentum advection near the interface, particularly for large density differences;
- Accurate and robust solution of the pressure equation for large density differences;
- Mass conservation, particularly in the level set method
- Accurate treatment of the viscous stresses.

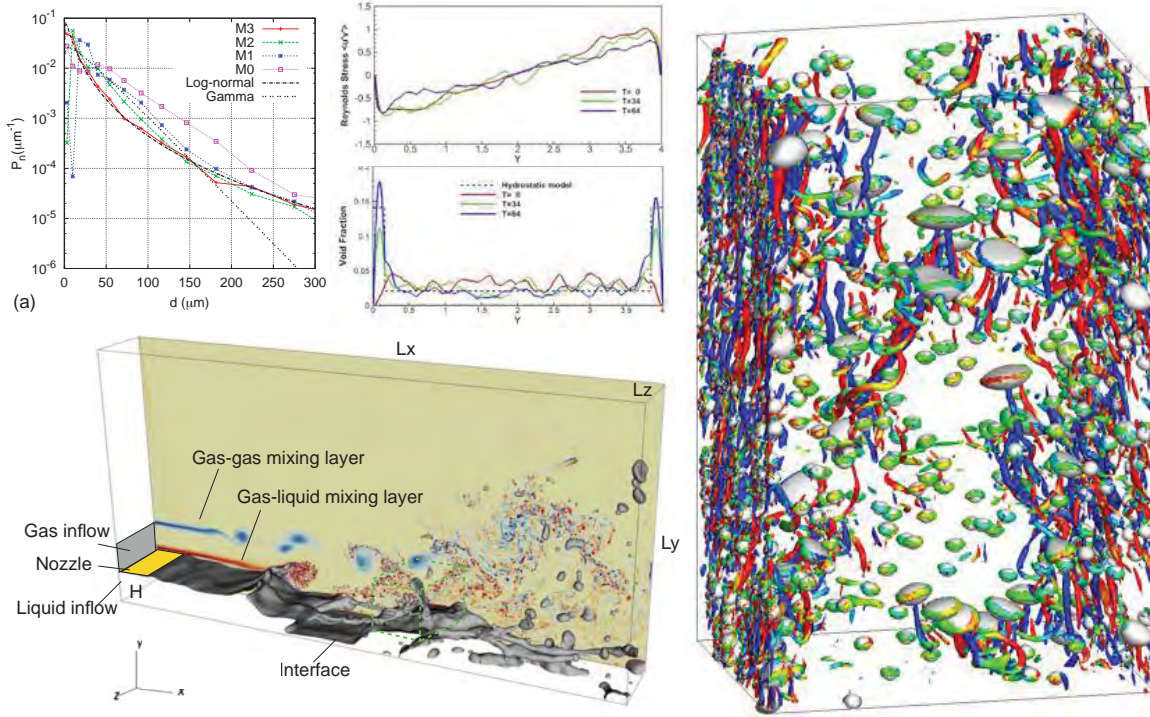




# “Simple” Problems





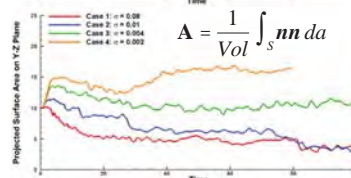
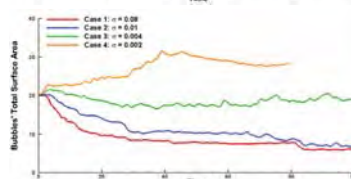
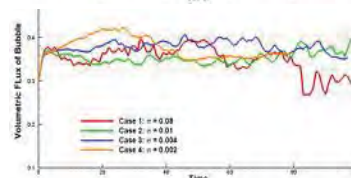
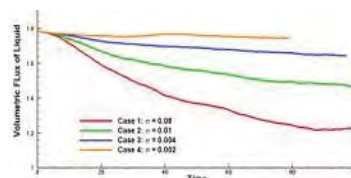
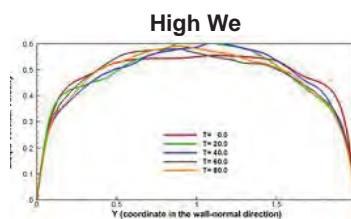
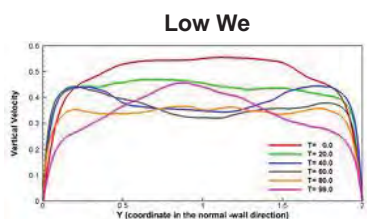
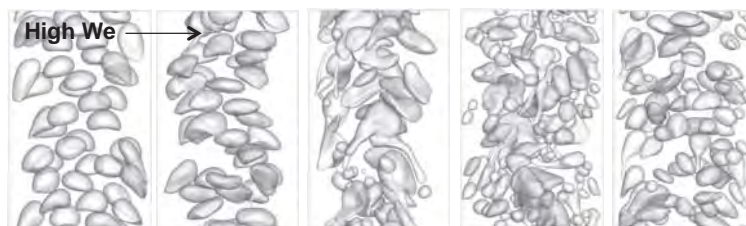
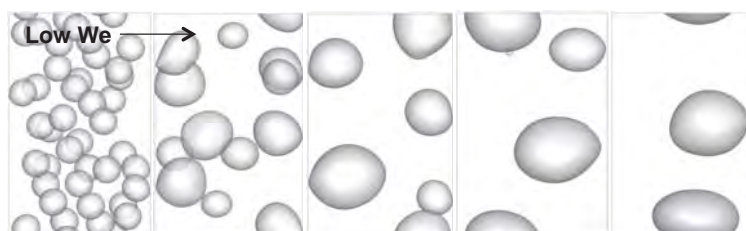


For incompressible two-fluid problems with a clean interface a number of methods now exist that are capable of simulating fairly complex situations, such as hundreds of bubbles in turbulent flows. Thus, current tasks include:

- Extensions to more complex physics. For simple problems this has been done but new physics often includes new scales that are difficult to resolve and the number of studies of very large systems are modest
- Development of multiscale strategies to account for very small scales that appear spontaneously or are imposed by additional physical
- Development of advanced strategies to process the results, including using simulations data to help building reduced order models.



# “Complex” Physics



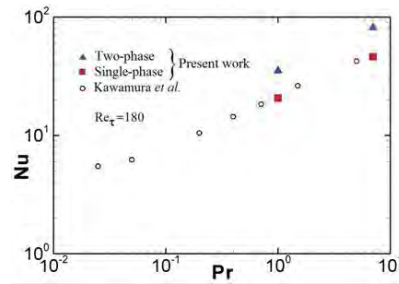
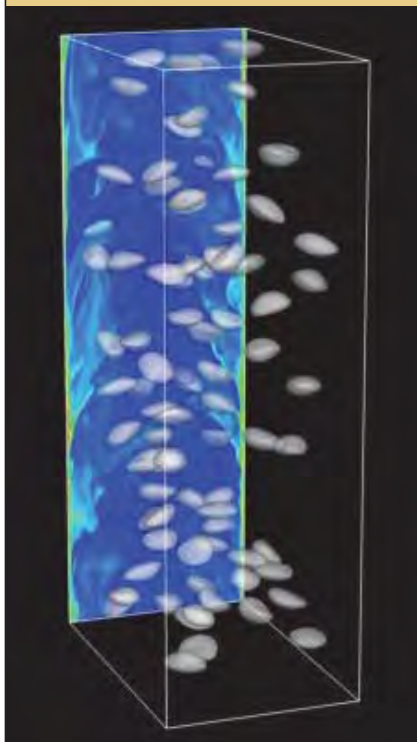
$$A = \frac{1}{Vol} \int_s n n da$$





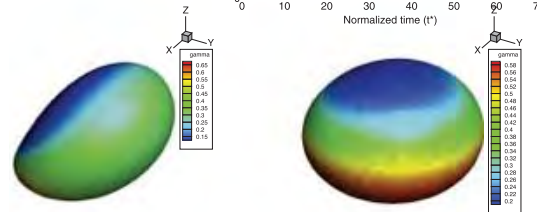
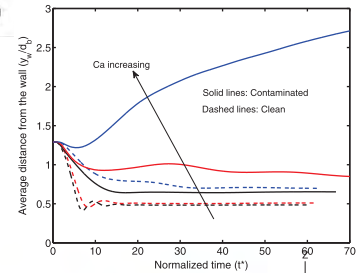
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## DNS of Multiphase Flows Heat/Mass Transfer & Surfactants



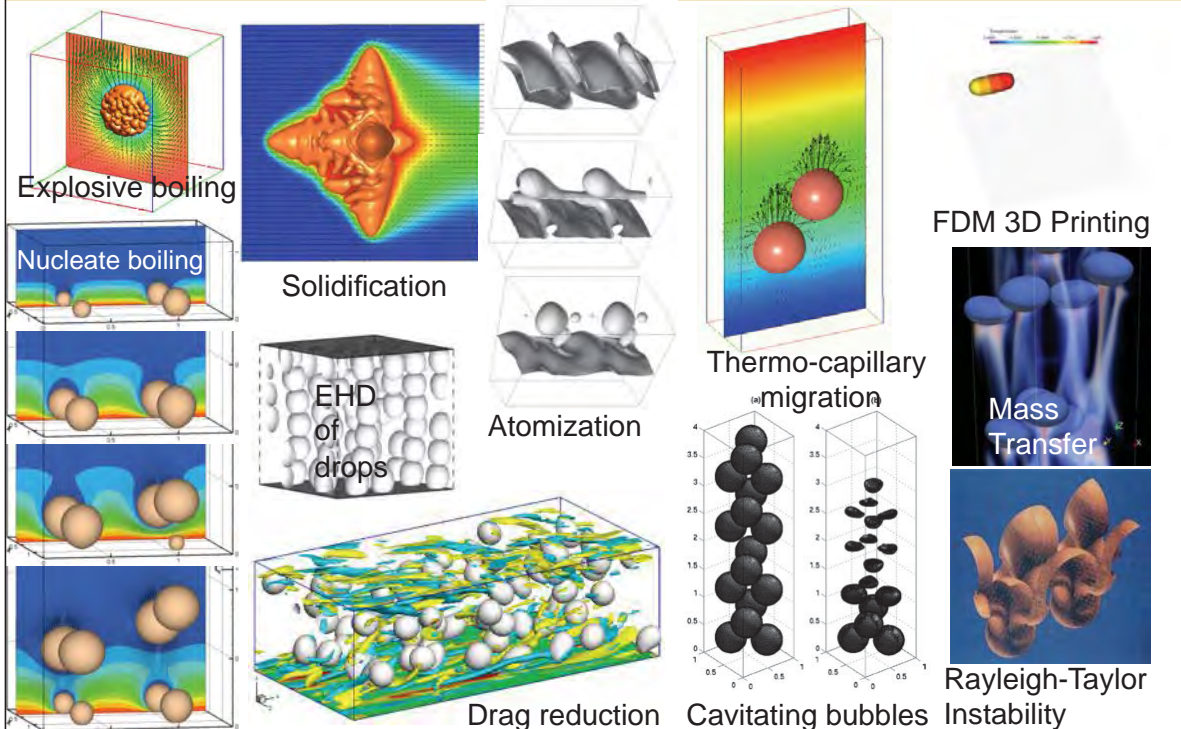
M. Muradoglu and G. Tryggvason. Simulations of Soluble Surfactants in 3D Multiphase Flow. *Journal of Computational Physics*, 274 (2014), 737-757.

S. Dabiri and G. Tryggvason. Heat transfer in turbulent bubbly flow in vertical channels. *Chemical Engineering Science*. 122 (2015), 106-113.



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## DNS of Multiphase Flows Examples





# Multiscale Strategies

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Capturing isolated small-scale motion in simulations where the focus is on the larger scales can be done in many ways, such as be various grid refinement techniques (unstructured grids, AMR for Cartesian grids, wavelets, etc.) or reduced order models

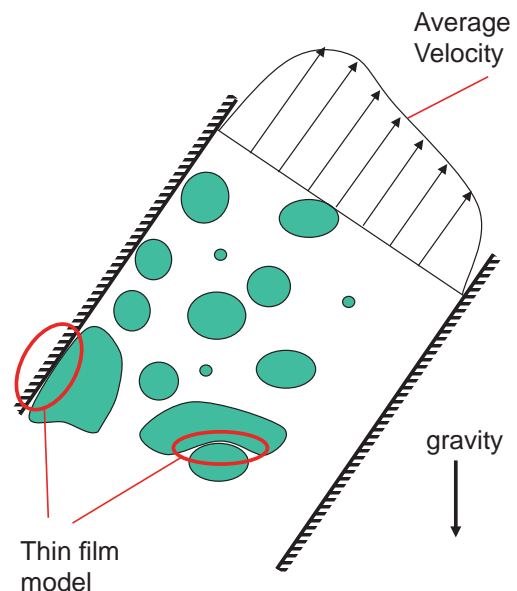
**However:**

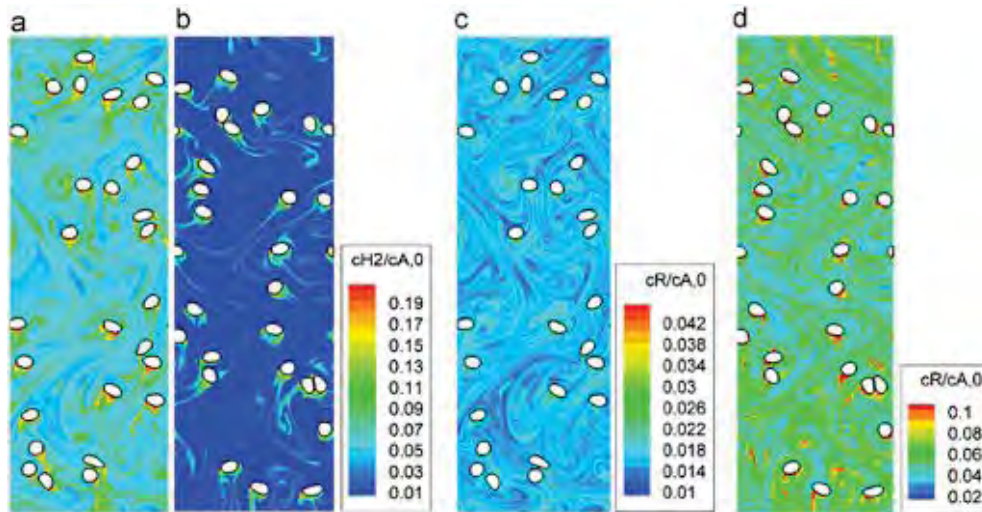
At small scales, the effect of surface tension is strong so interface geometries are simple

At small scales the effect of viscosity is strong so the flow is simple

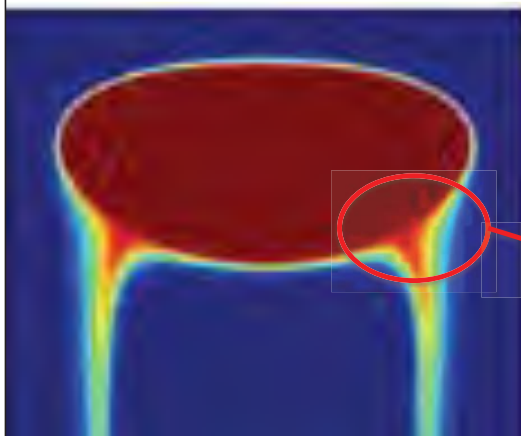
Those are exactly the situation that can be—and have been—handled analytically

Buoyant bubbles in an inclined channel flow

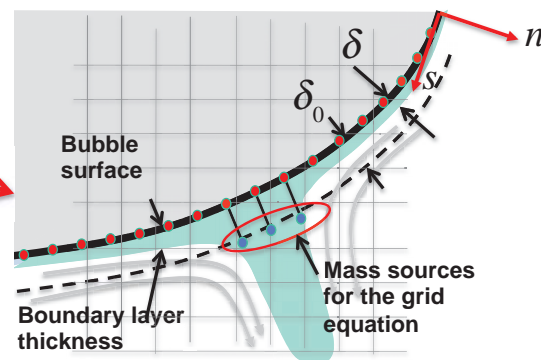




Results from simulations of the catalytic hydrogenation of nitroarenes. The hydrogen (frames a and b) and hydroxylamine (frames c and d) concentration profiles are shown for one time for two simulations. In frames a and c the reaction rates are relatively slow, compared with the mass transfer, but in frames b and d the reaction is relatively fast. From Radl, Koynov, Tryggvason, and Khinast (2008).



Capturing the mass boundary layer



$$\frac{\partial f}{\partial t} = \sigma n \frac{\partial f}{\partial n} + D \frac{\partial^2 f}{\partial n^2}$$

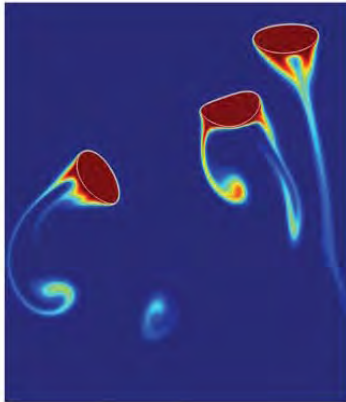
$$M_0 = \int_0^\infty f \, dn$$

$$M_1 = \int_0^\infty n f \, dn$$

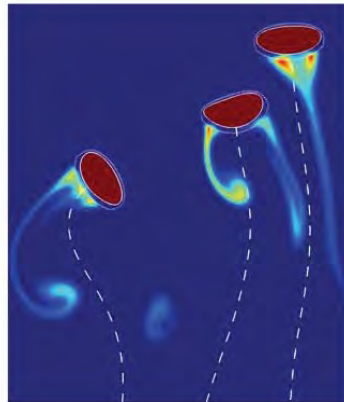
$$\frac{dM_0}{dt} = -\sigma (M_0 - \delta_o f_\delta) - D \left. \frac{\partial f}{\partial n} \right|_0$$

$$\frac{dM_1}{dt} = -\sigma (2M_1 - \delta_o^2 f_\delta) + D (f_o - f_\delta)$$

B. Aboulhasanzadeh, S. Thomas, M. Taeibi-Rahni, and G. Tryggvason. *Chemical Engineering Science* 75 (2012) 456–467.



Full



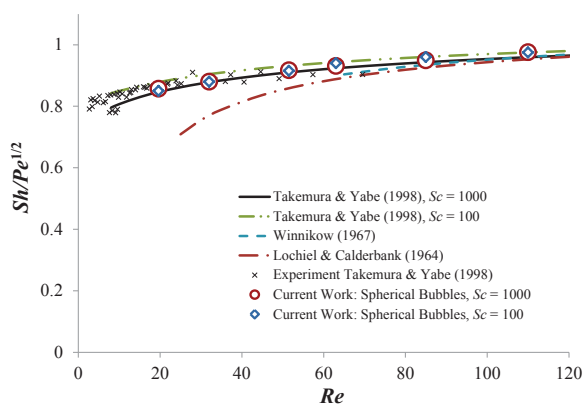
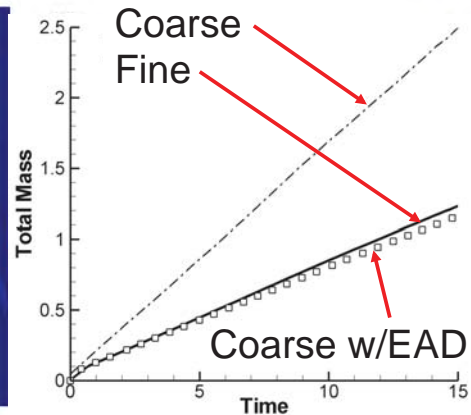
Using Model

Re = 60, Sc = 15

Using one moment and a one-parameter profile

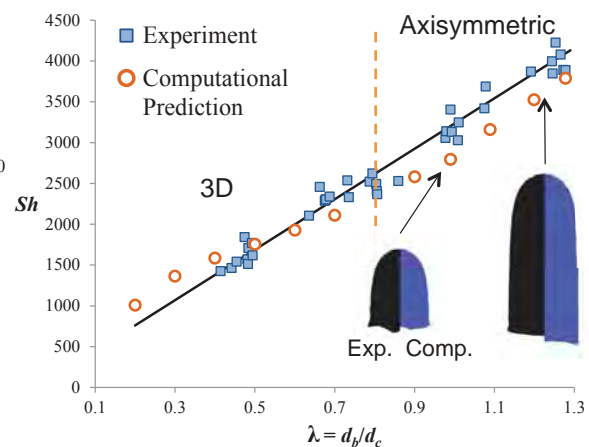
$$\frac{dM_0}{dt} = -\sigma(M_0 - \delta_0 f_\delta) - D \left. \frac{\partial f}{\partial n} \right|_0;$$

$$f(n) = \begin{cases} f_0 \left( 1 - 2 \left( \frac{n}{\delta} \right) + \left( \frac{n}{\delta} \right)^2 \right); & n \leq \delta_0 \\ 0; & n > \delta \end{cases}$$



The mass transfer versus Re,  
for a single bubble in a large  
domain, along with the  
predictions of experimental  
correlations

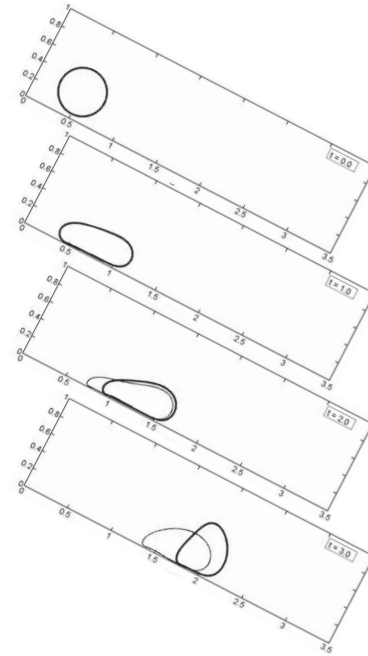
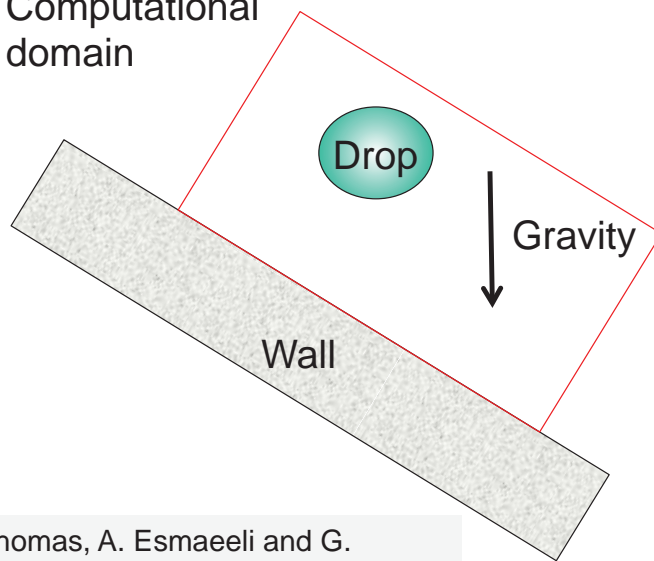
Comparison with experimental  
results from A. Tomiyama:  
Eo = 24.7  
Mo = 10<sup>-7.78</sup> and  
Sc = 8260







Computational  
domain



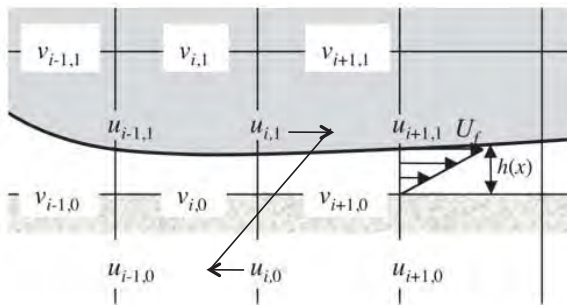
S. Thomas, A. Esmaeeli and G. Tryggvason. "Multiscale computations of thin films in multiphase flows." *Int'l J. Multiphase Flow* 36 (2010), 71-77.



Film model—linear velocity profile

$$\frac{\partial h}{\partial t} + \frac{1}{2} \frac{\partial}{\partial x} (h U_f) = 0;$$

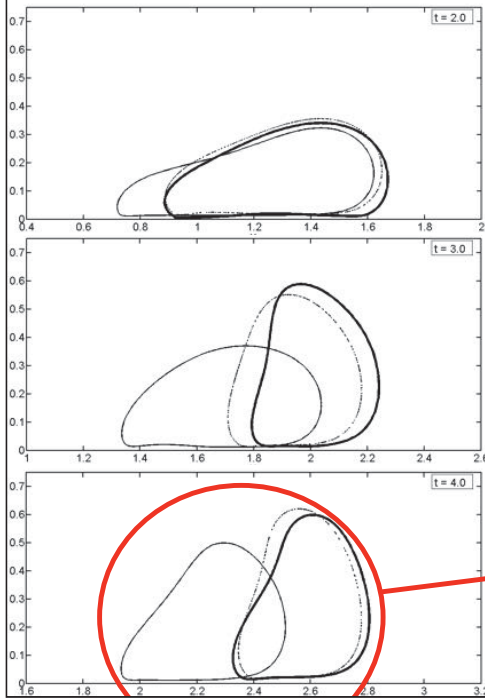
$$\frac{\partial}{\partial t} (h U_f) + \frac{2}{3} \frac{\partial}{\partial x} (h U_f^2) = - \frac{2h}{\rho_o} \left( \frac{dp}{dx} \right)_f,$$



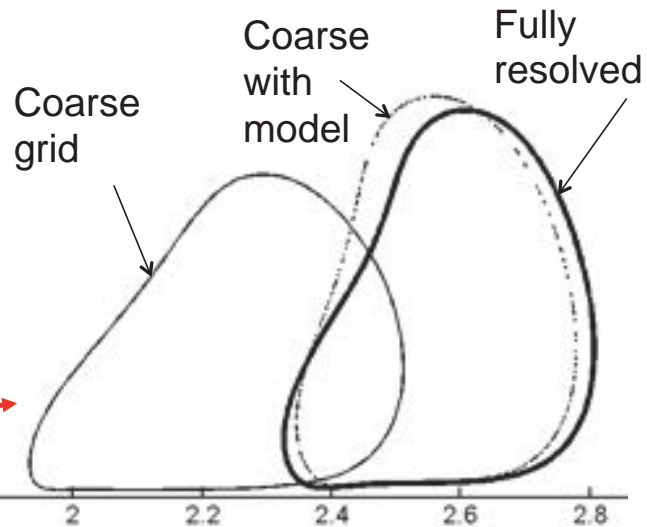
Wall shear and ghost velocity:

$$\tau_f = \mu_o \frac{U_f}{h} \Rightarrow u_{i,0} = u_{i,1} - \frac{\tau_f \Delta y}{\mu_d}$$

1. Identify whether a grid points at a wall belong to a film or not.
2. For film wall-points, given  $h$  and  $U_f$ , find the wall-shear and set the ghost velocities. For points outside the film, use the no-slip boundary condition.
3. Solve the Navier-Stokes equations for the velocity and pressure at the next time step, using the ghost velocities set above.
4. Integrate the thin film equations, using the pressure at the wall as computed by solving the Navier-Stokes equations (step 3).
5. Go back to (1)



Drop motion on a sloping wall.  
Impact of fully resolving the film  
between the drop and the wall



### “Almost” DNS

- For two-fluid and multi-phase flows the range of scales is sometimes simply too large for everything to be fully resolved, either because of spontaneous generation of small films, ligaments or drops due to collision of fluid masses or topology changes or because of scales introduced by additional physical processes.
- The other reason is that as we consider more complex physics, we are often working with equations that do not fully describe the physics. For surfactants we may be using simplified equations of state and for reactions we may be using a reduced set of equations for the chemistry. So even if the solution is accurate, the physics may not be fully captured.





# What to do with the Results

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## **Modeling Challenges and Opportunities:**

The enormous amount of data generated by DNS—and increasingly by experiments—will allow reduced order models that involve large number of variables and complex relationships between the resolved and unresolved variables and are applicable to complex flows

Determining complex nonlinear relationships from massive data involving a range of physical scales using modern statistical learning is becoming easier

Modeling challenges will therefore shift to the development of more sophisticated and comprehensive models, the identification of the appropriate variables, and the incorporation and propagation of physical and model uncertainties

The inclusion of limiting cases, such as where the relationships are known, or the scaling is understood, in fitting is currently difficult but is likely to become increasingly important



A simple description of the average flow is derived by integrating the vertical momentum equation and taking the density and viscosity of the gas is zero

Void fraction and phase averaged velocity

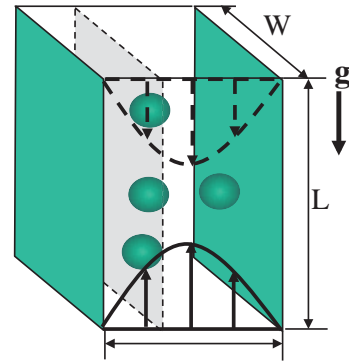
$$\alpha_i = \frac{1}{A_{zy}} \int \chi_i da \quad \langle v \rangle = \frac{1}{\alpha_i A_{zy}} \int \chi v da$$

Horizontal flux of bubbles

$$\frac{\partial \alpha_l}{\partial t} + \frac{\partial F_l}{\partial x} = 0, \quad F_i = \frac{1}{\alpha A_{zy}} \int \chi_i u_i da$$

Averaged vertical momentum of the liquid:

$$\begin{aligned} \frac{\partial}{\partial t} \alpha_l \langle v \rangle_l + \frac{\partial}{\partial x} F_l \langle v \rangle_l = \\ -\frac{1}{\rho_l} \frac{dp_o}{dy} - g_y \alpha_l + \nu_l \frac{\partial}{\partial x} \left( \alpha_l \frac{\partial \langle v \rangle_l}{\partial x} \right) \\ - \frac{\nu_l}{L} \frac{\partial}{\partial x} \sum_{int} (v' n_x + u' n_y) - \frac{\partial}{\partial x} \alpha_l \langle u' v' \rangle_l + \frac{1}{\rho_l} (f_\sigma)_y \end{aligned}$$



$$\begin{aligned} \alpha_b + \alpha_l &= 1 \\ \alpha_b \langle u \rangle_b + \alpha_l \langle u \rangle_l &= 0 \\ \alpha_l \langle u \rangle_l \langle v \rangle_l &= F_l \langle v \rangle_l \\ F_l &= \alpha_l \langle u \rangle_l \end{aligned}$$



“Closure” variables  
needed for models  
of the average flow

Resolved average  
variables

Quantities  
summarizing the state  
of the unresolved flow

$F_g$	$\langle u'v' \rangle$	$f_\sigma$	$\alpha_g$	$\frac{\partial \alpha_g}{\partial x}$	$\frac{\partial \langle v \rangle_l}{\partial x}$	$d_w$	$k_t$	$\varepsilon_t$	$a$	$a_{ij}$
Data obtained by averaging the DNS results										

Not  
include  
yet

By averaging the DNS results over planes parallel to the walls, we construct the Table above with quantities that are known and unknown in the averaged equations. Using Neural Networks, we fit the data, resulting in:

$$F_b = f_1(\mathbf{x}); \quad \langle u'v' \rangle = f_2(\mathbf{x}); \quad F_\sigma = f_3(\mathbf{x}); \quad \mathbf{x} = \left( \alpha, \frac{\partial \alpha}{\partial x}, \frac{\partial \langle v \rangle}{\partial x}, d_w \right)$$

These relationships are used when solving the average equations for the void fraction and the vertical liquid velocity

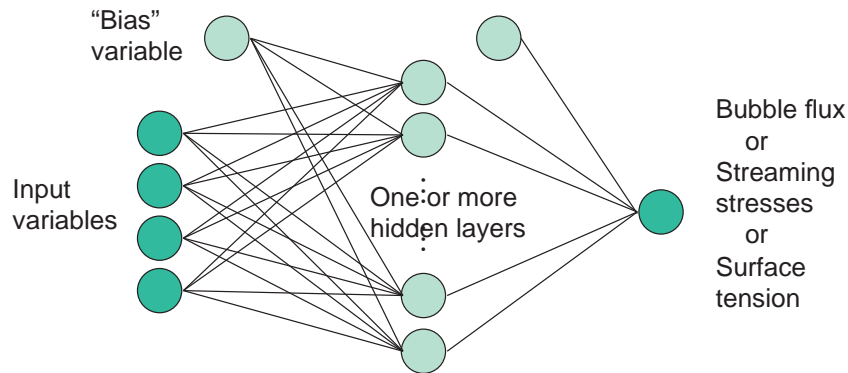


Most recently our emphasize has been neural network as they seem to be giving the most promising results.

$$F_b = f_1(\mathbf{x}); \quad \langle u'v' \rangle = f_2(\mathbf{x}); \quad F_\sigma = f_3(\mathbf{x}); \quad \mathbf{x} = \left( \alpha, \frac{\partial \alpha}{\partial x}, \frac{\partial \langle v \rangle}{\partial x}, d_w \right)$$

A neural network with one hidden layers

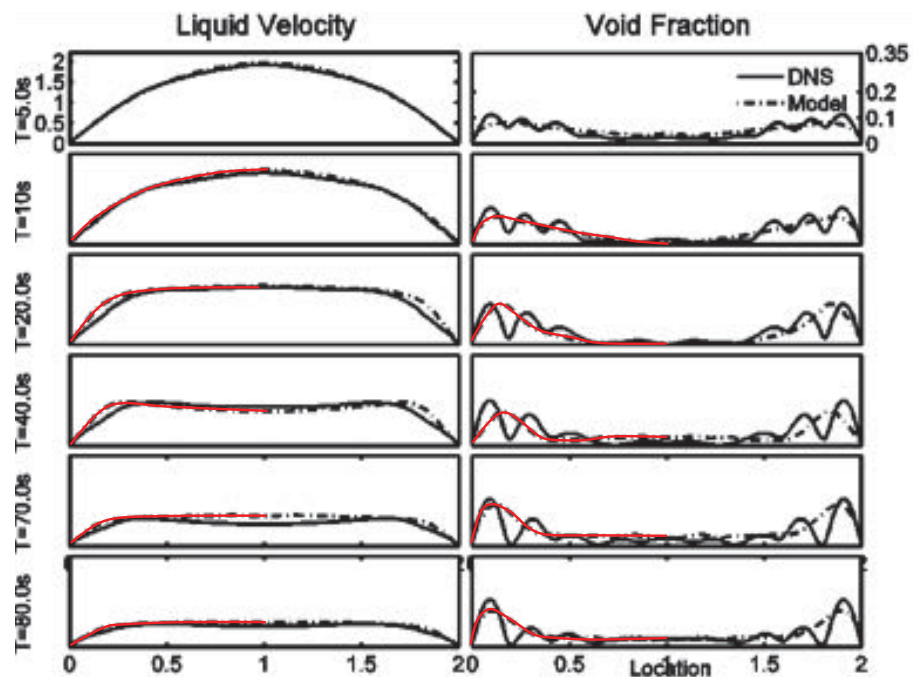
m: input variables  
n: hidden variables



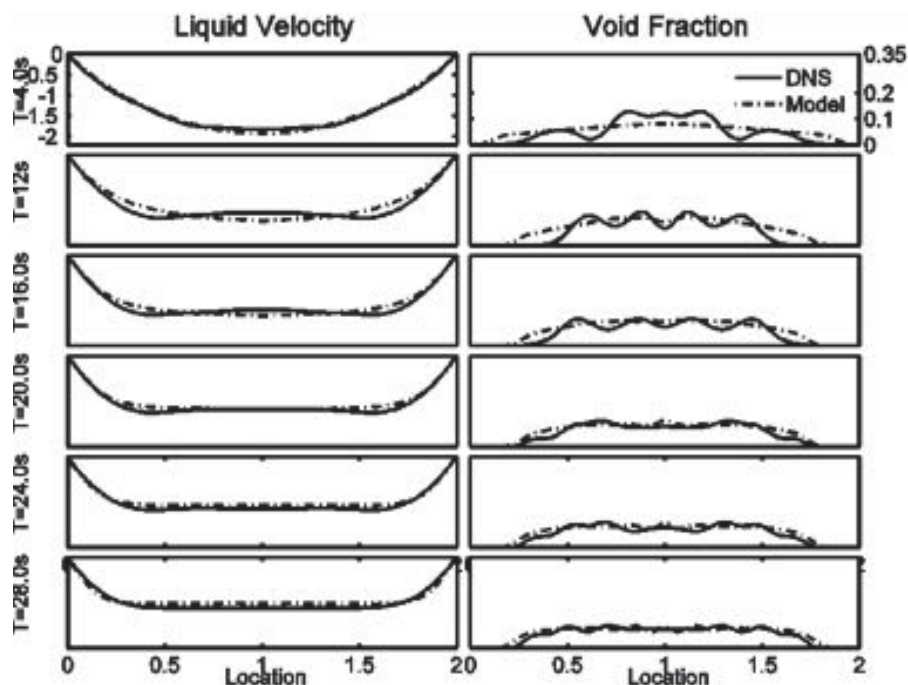
$$f(\mathbf{x}) = b_0 + \sum_{i=1}^n b_i h_i(\mathbf{x}); \quad h_i(\mathbf{x}) = s \left( a_{0i} + \sum_{j=1}^m x_j a_{j,i} \right); \quad s(u) = \frac{2}{1 + e^{-2u}} - 1$$



Averaged DNS results and Model predictions using the ANN closure terms at several different times



The closure relations derived from the upflow cases applied to downflow



DNS of multifluid turbulent flows have been developed to the point that they should be able to help produce new models for “industrial” simulations

DNS data is putting new demands on the modeling of complex multiphase flows. Currently, such modeling relies of fairly basic ideas, first put forward many years ago. DNS should make much more comprehensive models possible

DNS needs to be extended to handle flows with more complex topology and those undergoing flow regime transitions

Complex isothermal flows and flows with phase change and other additional physics, such as mass transfer, need multiscale modeling that must be developed further and put on a rigorous theoretical basis.

One of the biggest obstacle for more rapid increase in the use of DNS is the high “entry barrier” for new investigators. Many “things” to learn!





# Education

31



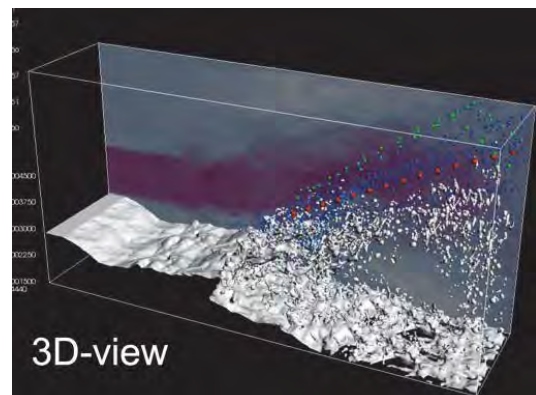
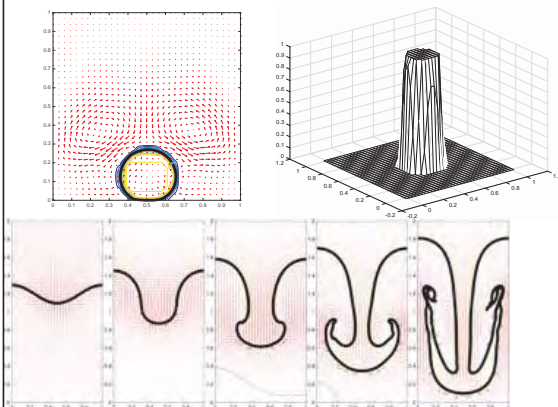
There is a need for software for a variety of purposes. Those include:

- Codes for education and/or testing of ideas

<http://www3.nd.edu/~gtryggva/MultiphaseDNS/index.html>

- Large scale “somewhat” general purpose codes that represent close to the state-of-the-art and often can be used as “black-boxes:

<http://www.ida.upmc.fr/~zaleski/paris/index.html>



[illegible]





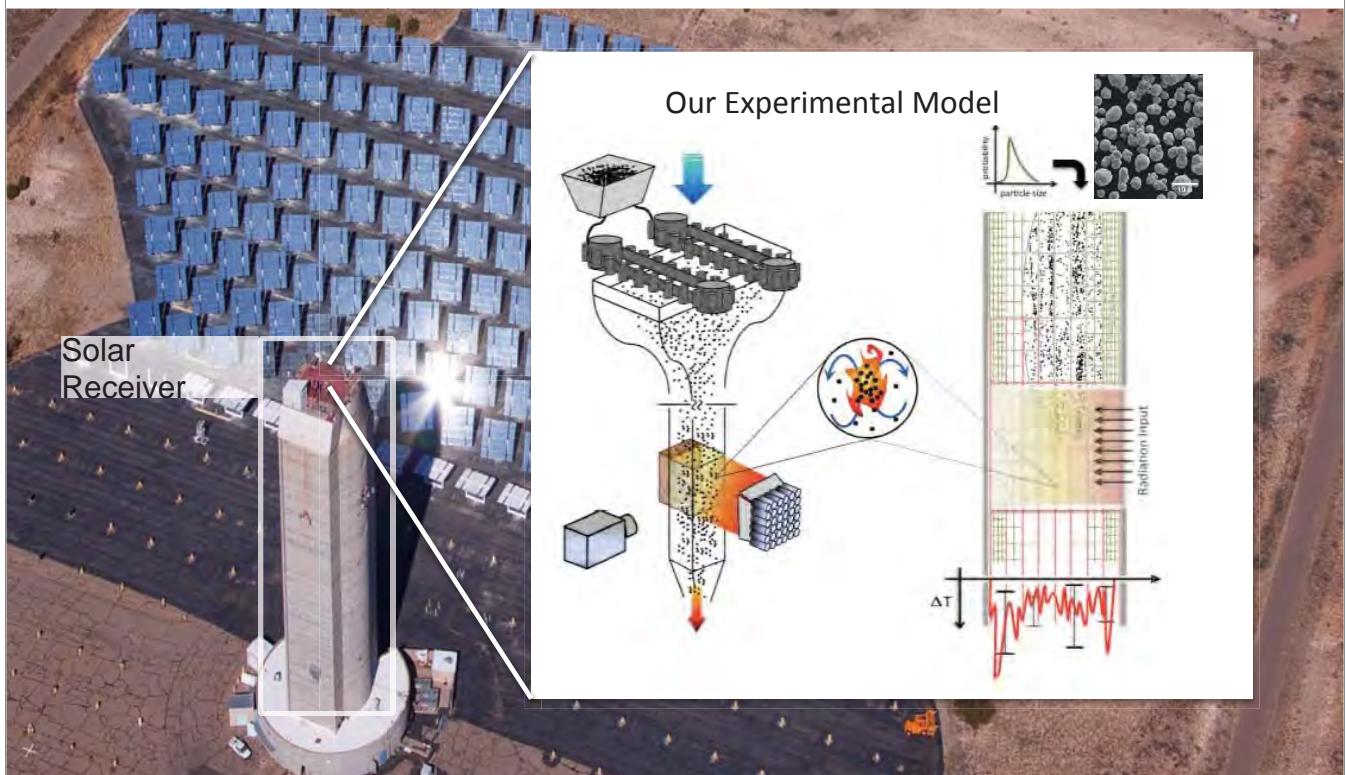
# Overview of computational modeling at Stanford PSAAP: Particle-laden flows subject to radiative heating

Ali Mani

Multiphase Physics Deep Dive 2016



## Particle-Based Solar Receiver



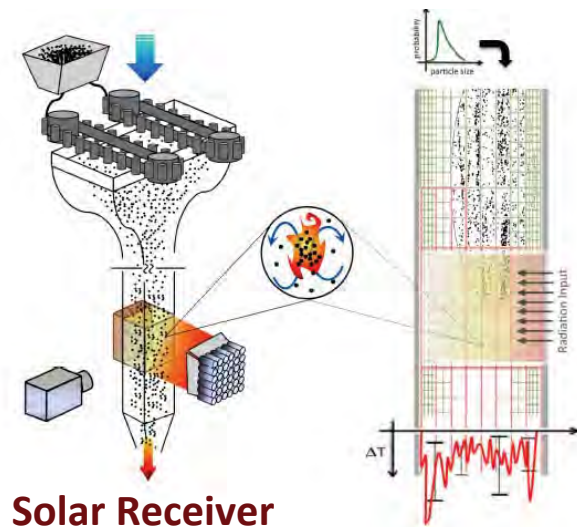


# Scientific Challenge

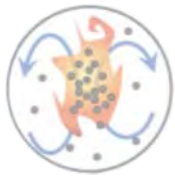
Multiphysics problem involving turbulence, particle transport, and radiation in a tightly coupled environment.

Experiment

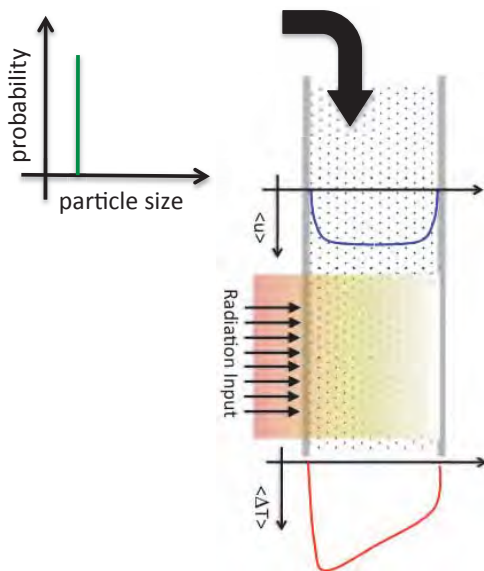
Simulations



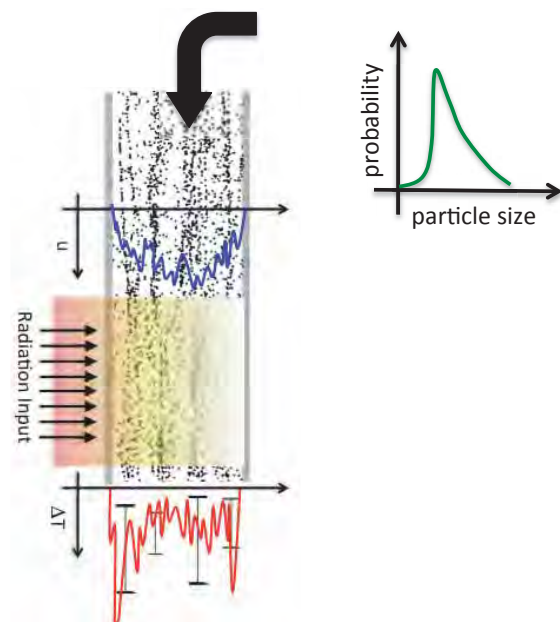
- Cold gas-particle mixture enters
- Sunlight is absorbed by particles
- Particle temperature increases
- Air convects energy from particles
- Hot gas-particle mixture exits
- Particles are separated out, hot gas used for energy extraction



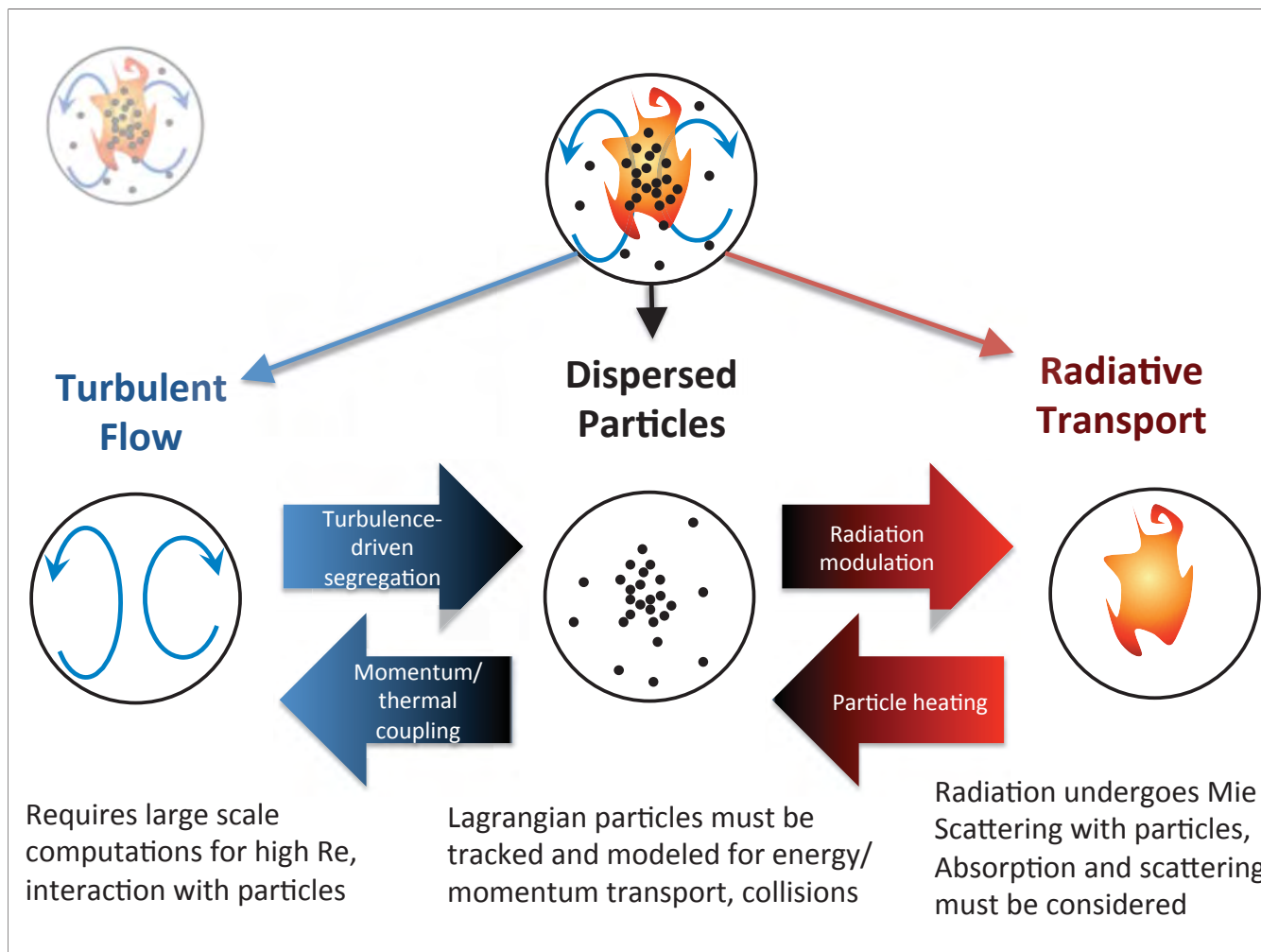
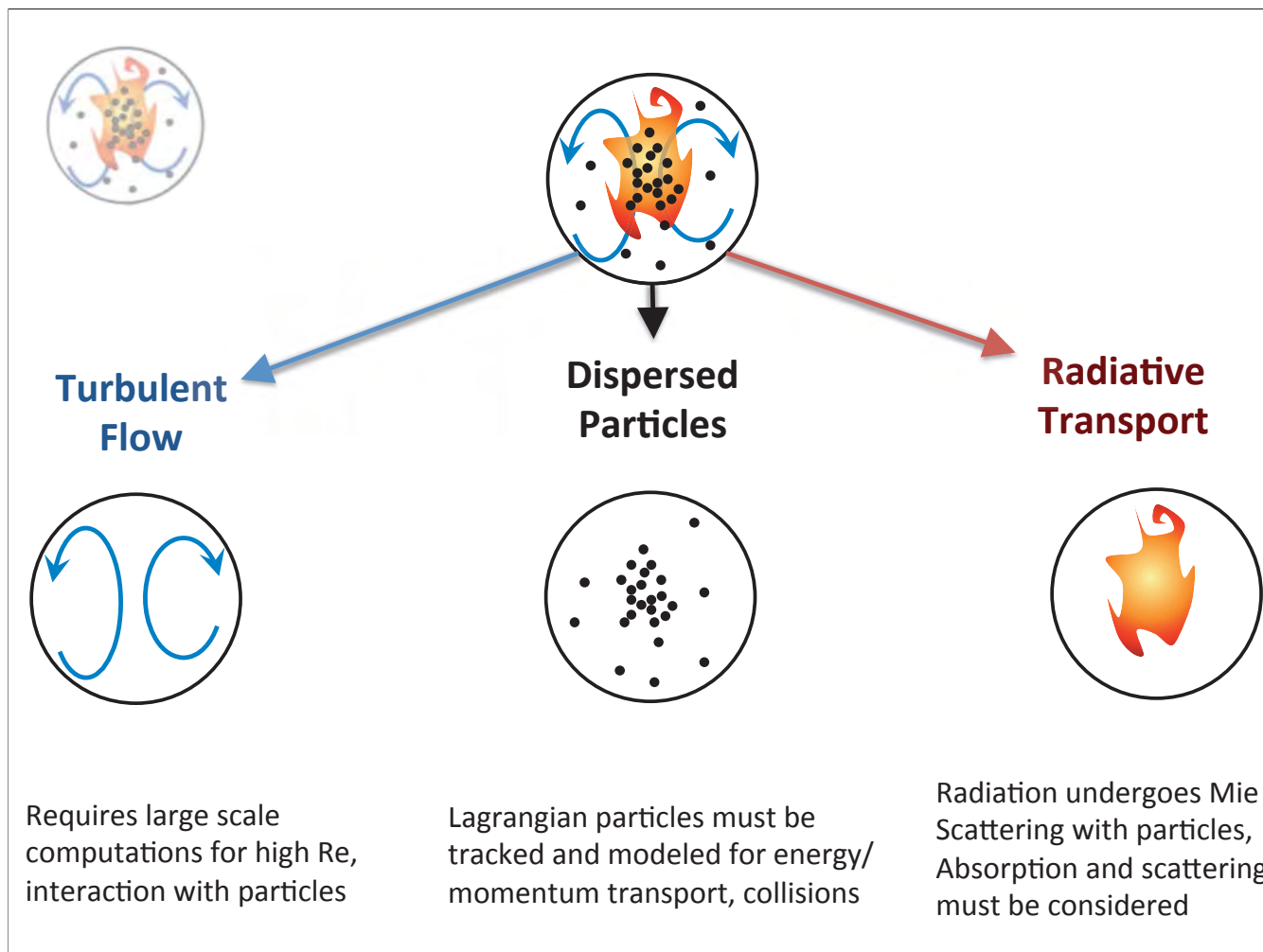
# Scientific Challenge



Idealized Scenario  
Weak Coupling



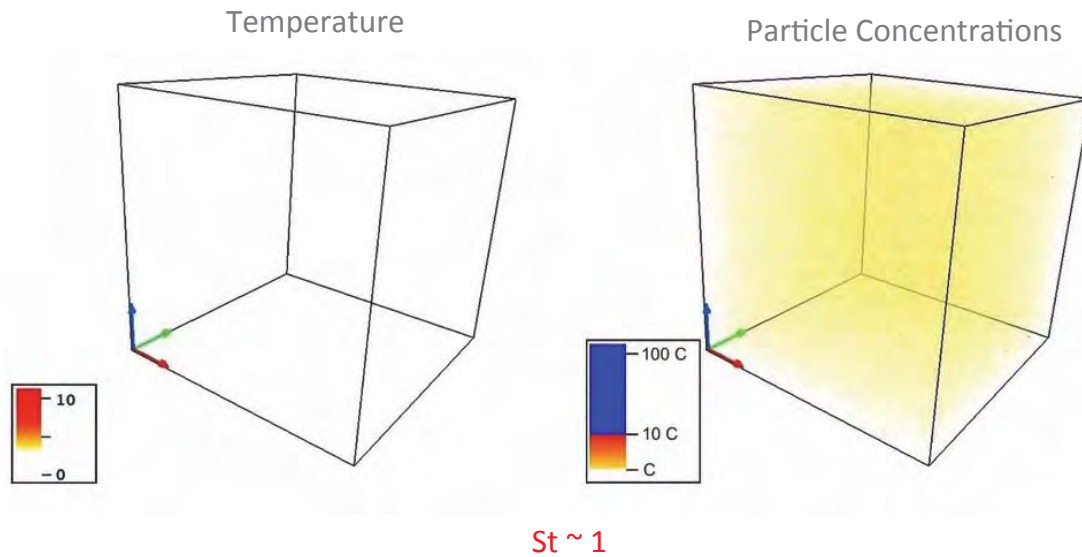
Real-world Scenario  
Strong Coupling





# Early Demonstrations

Detailed coupling of particle-laden turbulence and radiation



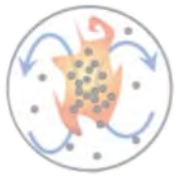
Ref: Zamansky et al. PoF 26, 071701 (2014)



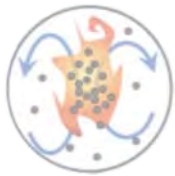
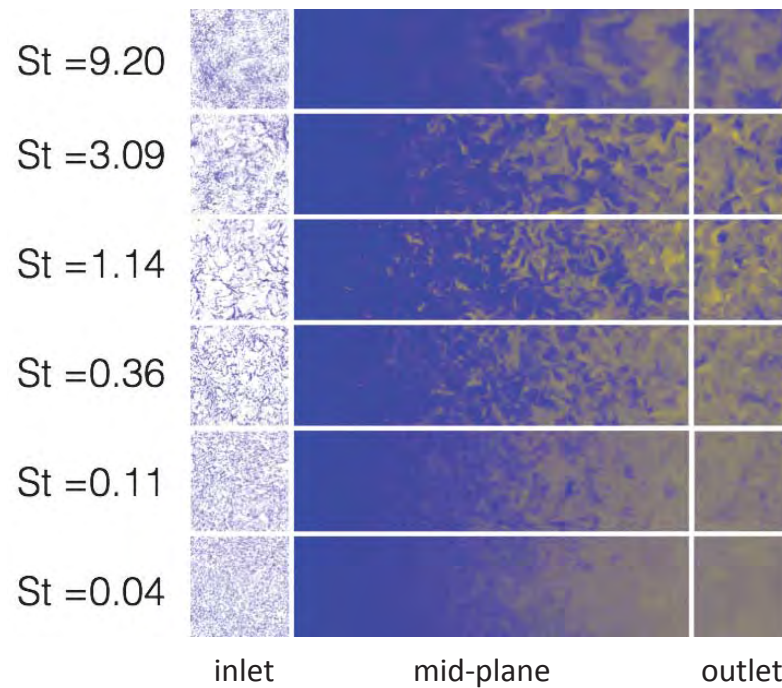
## Stanford PSAAP Regimes and Parameters

- 12 dimensionless parameters describe the full problem
- Typical parameters
  - $Re \sim 10^4$
  - $St \sim 10$
  - $d_p/\eta \sim 0.1$



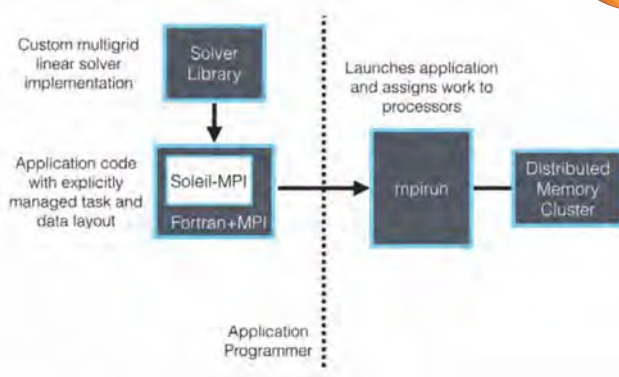


# Effect of Preferential Concentration on Heat Transfer



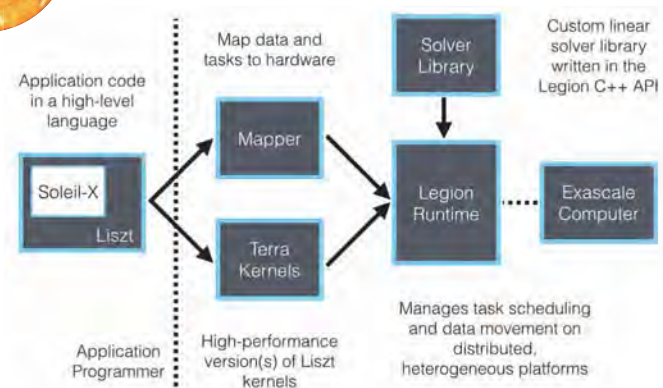
# Simulation Framework

## SOLEIL-MPI



State-of-the-art MPI implementation targeting 100K core clusters

## SOLEIL-X Legion/Liszt/Terra/Lua



Implementation using data & task based parallelism targeting heterogeneous architectures – includes development of new programming framework

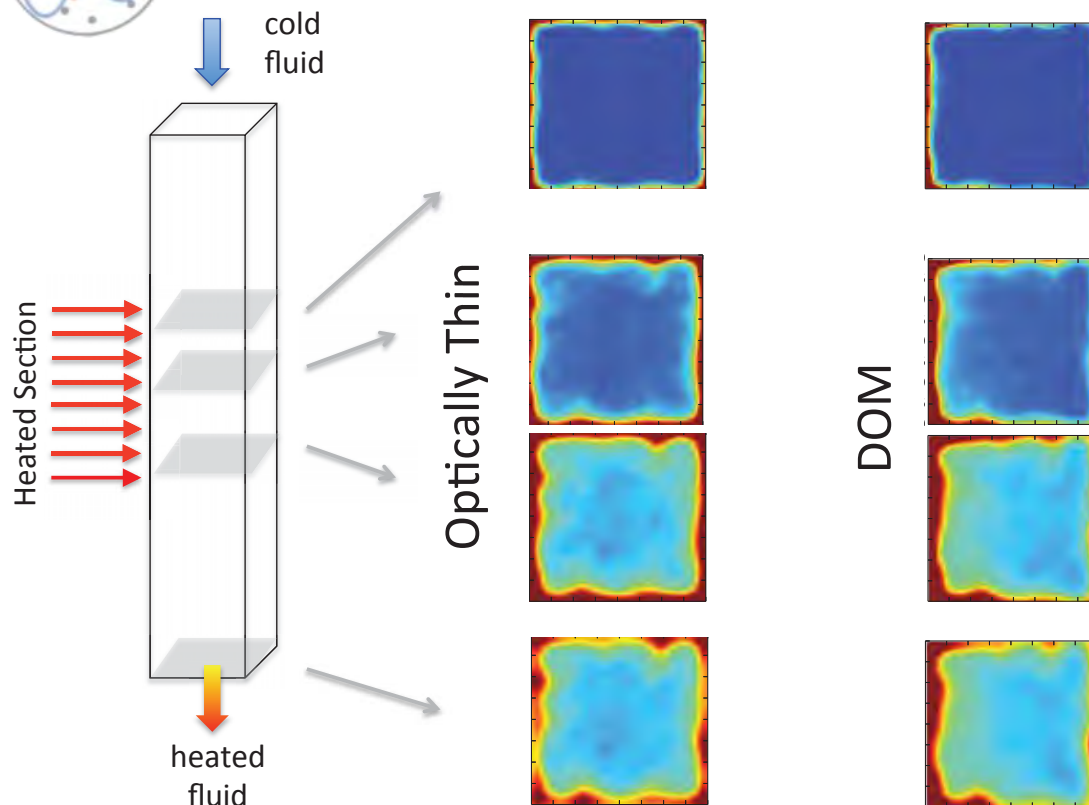


## ...where are we?

- **Physics Models:** Demonstrate ability to simulate fully coupled (flow, particle, radiation, heat transfer) system in nominal experimental scenario
- **Exascale:** Integrate complete software stack (code, compiler, scheduler, libraries)
- **Validation:** First comparison between experiments and computation
- **UQ:** Evaluate uncertainties and their effects on key output of interest



## Example Simulation

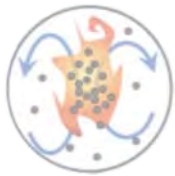






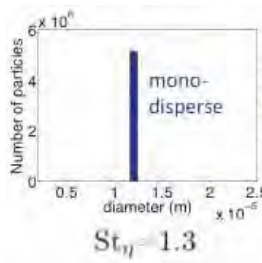
## Challenges (multiphase)

- Point particle models
  - Verification → Afternoon talk by J. Horwitz
  - Validation → Experiment/Particle Resolved DNS
- Turbophoresis → Tomorrow talk by M. Moghadam
  - Impact of particle collision model
  - Impact of near wall models
- Poly-dispersity
  - Impact on turbophoresis
  - Impact on heat transfer
- Experimental validation → Tomorrow talk by L. Villafañe
  - Particle concentration
  - Cluster analysis



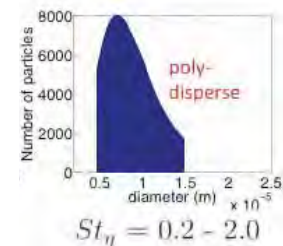
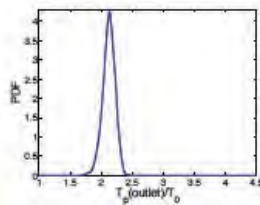
## Challenges (multiphase)

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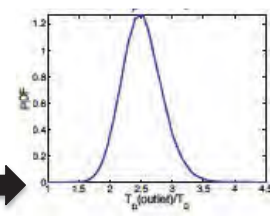
Particle  
Inlet Size

Particle  
Exit Temperature



Particle  
Inlet Size

Particle  
Exit Temperature



## Challenges (heat transfer)

- Lagrangian  $\leftrightarrow$  Eulerian heat exchange
  - Verification issues
  - Validation
- Radiative heat transfer
  - Particle resolved: expensive but verifiable
  - Eulerian (using homogenization): no verifiable model available



# Broader Science Challenges

- LES models
  - Lagrangian particle models
  - Subgrid Heat transfer
- Develop Understanding
  - What is effect of preferential concentration on
    - heat transfer?
    - radiative transfer?
  - Reduced-order models suitable for analysis and design



# Publications

## Physics:

- “Radiation induces turbulence in particle-laden fluids,”  
Zamansky et al. PoF, 26, 071701 (2014)
- “Settling of heated particles in homogeneous turbulence,”  
Frankel, et al. JFM, 792, 869-893, (2016)
- “Spatially-localized wavelet-based spectral analysis of preferential concentration in particle-laden turbulence,” Bassenne, et al. CTR Annual Research Briefs (2015)
- “Polydisperse particles in an irradiated turbulent gas-particle mixture,”  
Rahmani et al. CTR Annual Research Briefs (2015)

## Modeling:

- “Particle-laden flows forced by the disperse phase: comparison between Lagrangian and Eulerian simulations ,” Vie et al. IJMF, 79, 144-158 (2016)
- “Accurate calculation of Stokes drag for point-particle tracking in two-way coupled flows,” Horwitz et al. JCP, 318, 85-109, (2016)
- “A dynamic subgrid-scale model based on differential filters for LES of particle-laden turbulent flows,” Park et al. CTR Annual Research Briefs (2015)
- “Parallel variable-density particle-laden turbulence simulation,”  
Pouransari et al. CTR Annual Research Briefs (2015)

# Research activities for energetic dispersal of particles

Multiphase physics deep-dive workshop, St. Petersburg, FL

Kambiz Salari

October 6-7, 2016

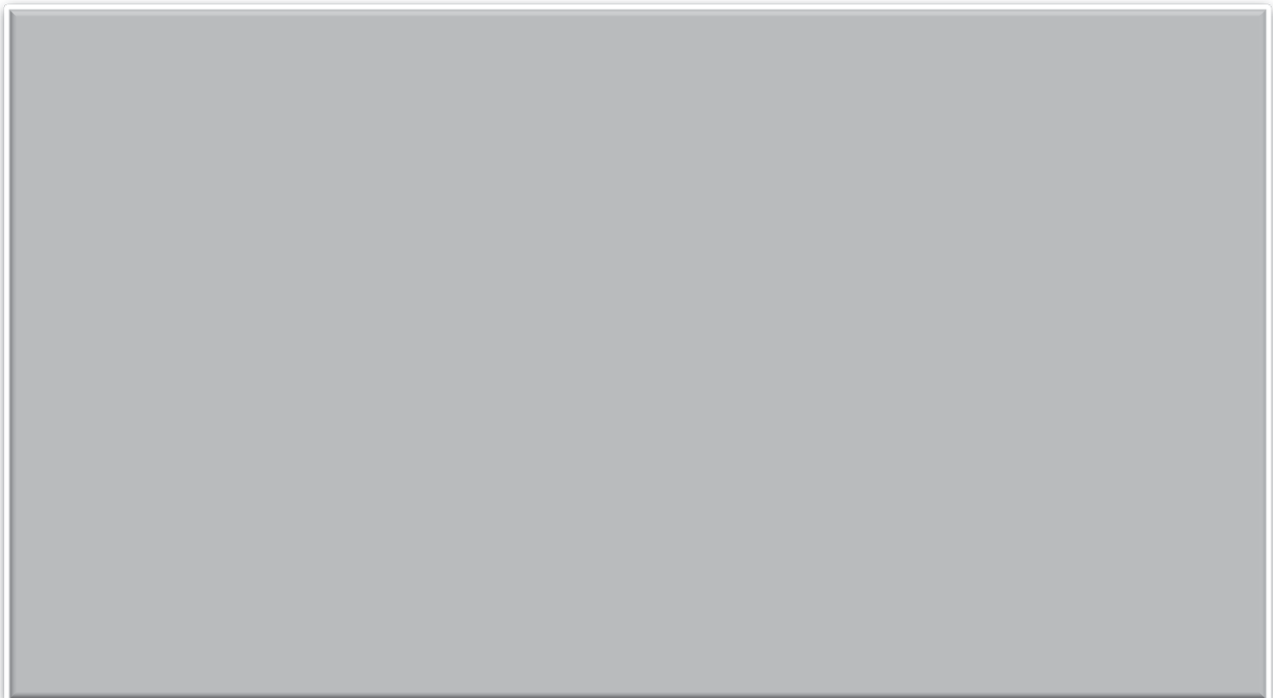


LLNL-PRES-704180

This work was performed under the auspices of the U.S. Department of Energy by Lawrence Livermore National Laboratory under contract DE-AC52-07NA27344. Lawrence Livermore National Security, LLC

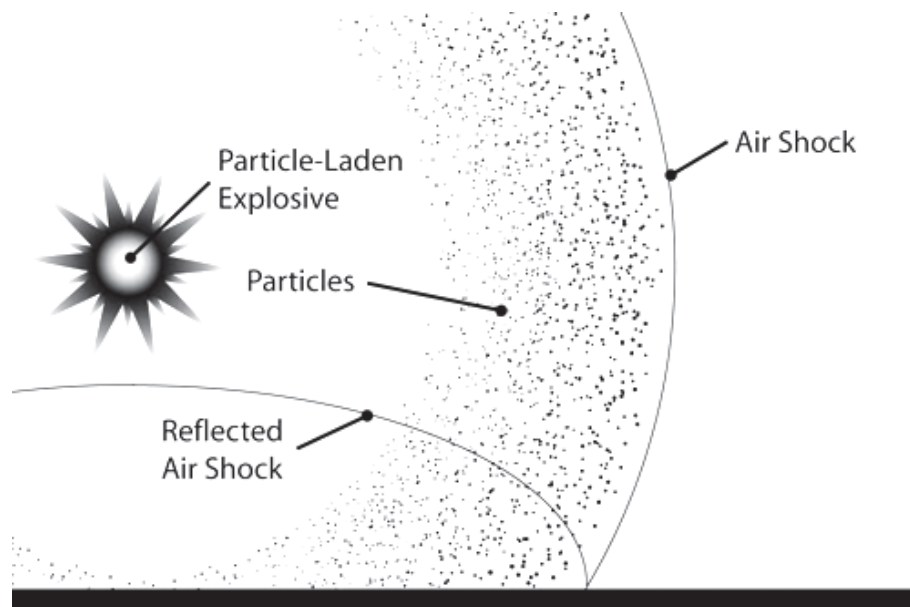
 Lawrence Livermore  
National Laboratory

## We rely on high explosives to energetically disperse particles





# Presence of particles enhances momentum and energy transport

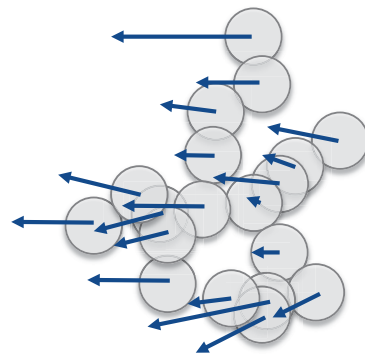


High velocity particles pose a challenge to modeling and simulation

## Particle laden flows in shock dominated environment are highly complex

Forces acting on a particle are due to many physical drivers

- Drag, Volumetric, and Pressure gradient
- Shape and surface roughness
- Inviscid and viscous
- Steady and unsteady
- Rotation
- Buoyancy/gravity
- Contact

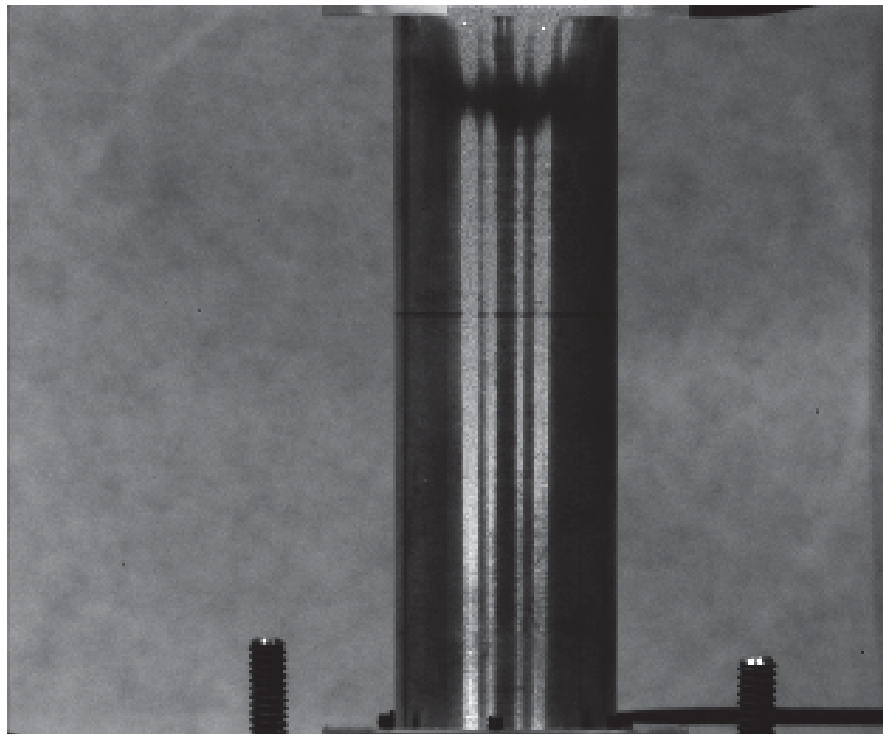


# Multiphase particle dispersal process is a multi-physics and multi-scale problem

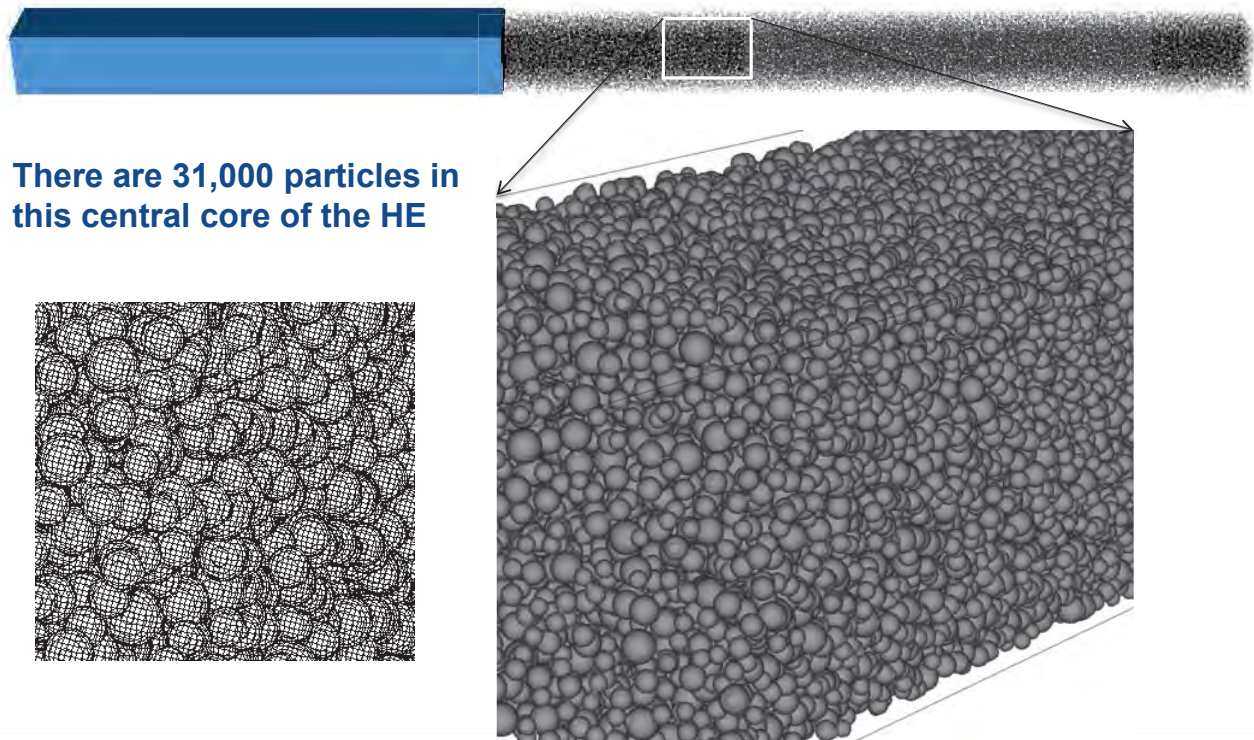
- Detonics
  - Non-ideal high explosive, particles mixed with HE
  - Inter-particle collision/contact
  - Inter-phase interactions, mass, momentum, and energy
  - Reactive flow
  - Particle surface chemistry
- Transport
  - Particulate plume surface instabilities, particle clustering and jetting
  - Particle drag for dense, dense-dilute transition, and dilute regime
  - Fireball combustion
  - Turbulent mixing
  - Particle surface chemistry
- Characterization of material properties
- Insufficient number of high fidelity experiments for validation and phenomenological study

## Standard cylinder test is needed to measure energy release from a non-ideal high explosive

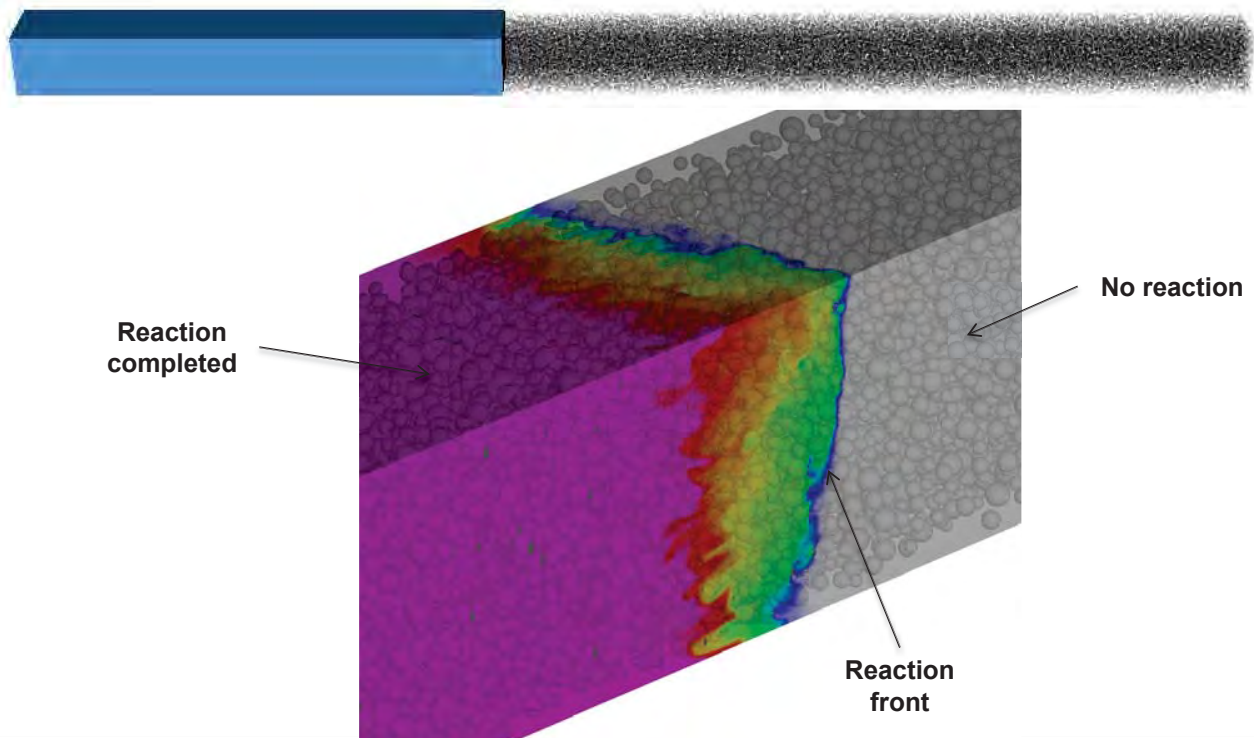
Shimadzu HPV-2 camera,  
1 million frame per second



## We rely on Mesoscale simulations to gain insight into particle impact on detonation process

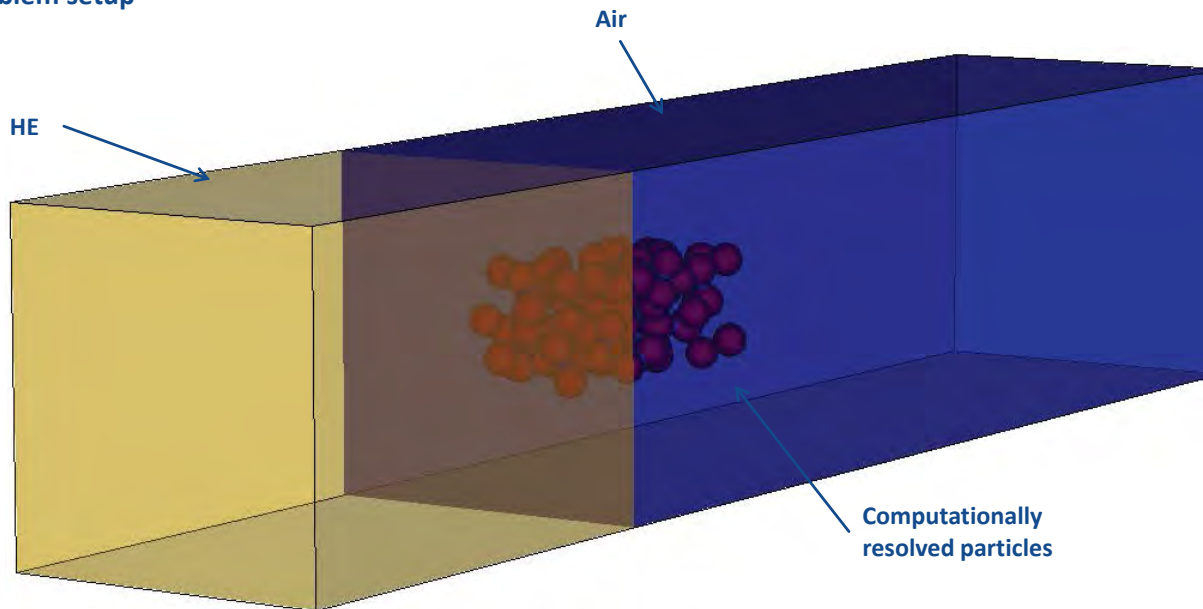


## The reaction zone in the presence of particles is wider compared to the parent explosive

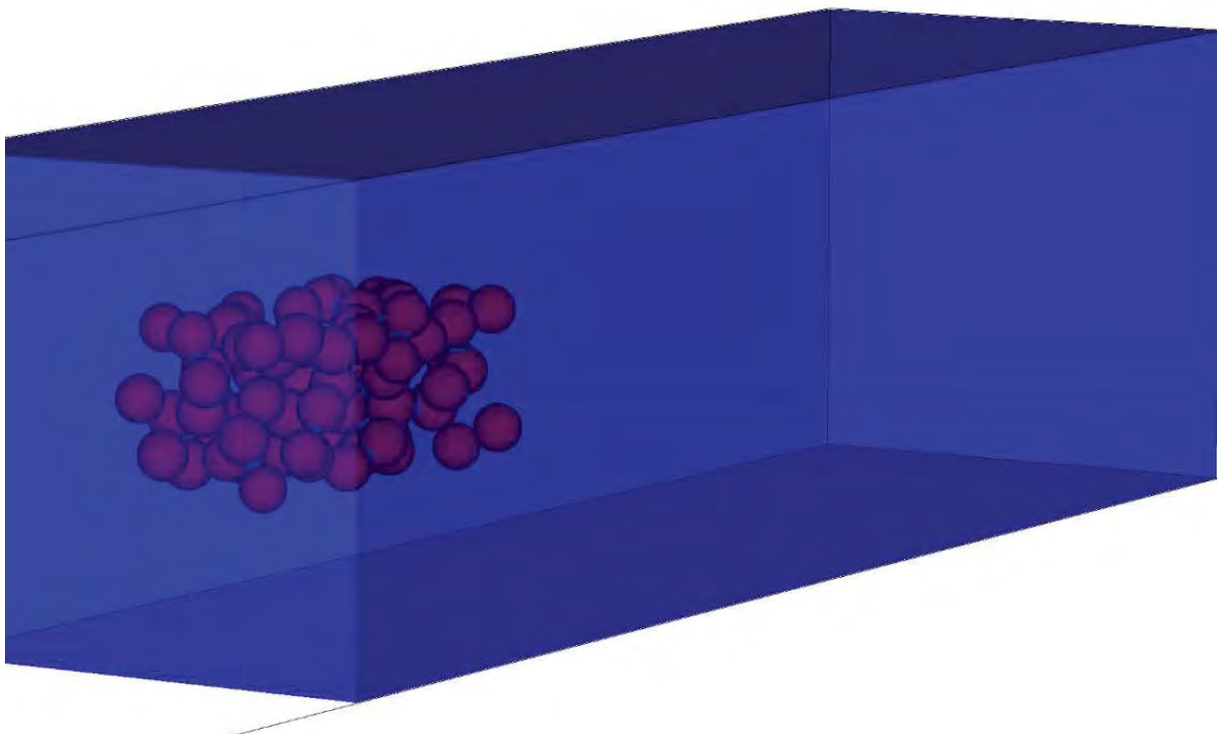


## Mesoscale simulations with embedded grid can provide insight into the physics of particle-particle interaction

Problem setup

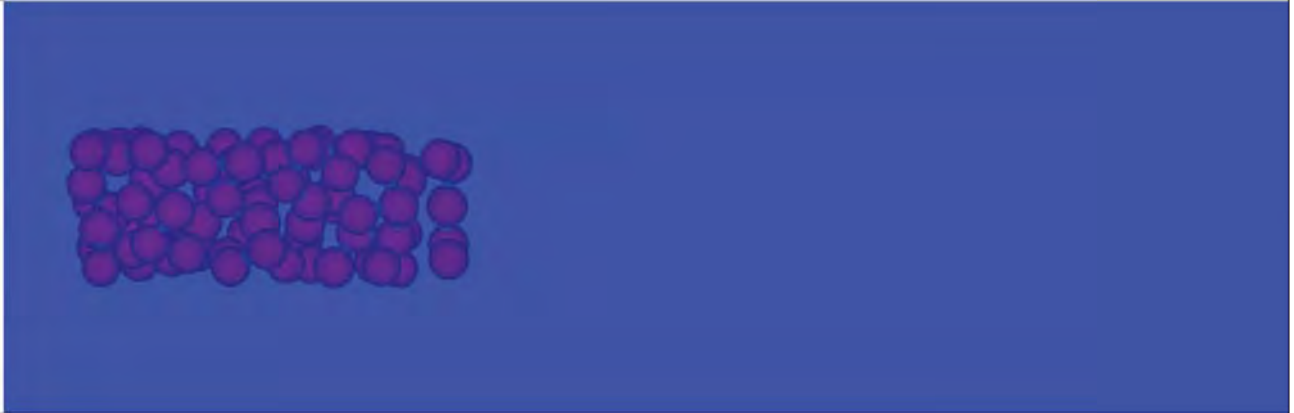


## Mesoscale simulations could provide details of particle interactions and transport

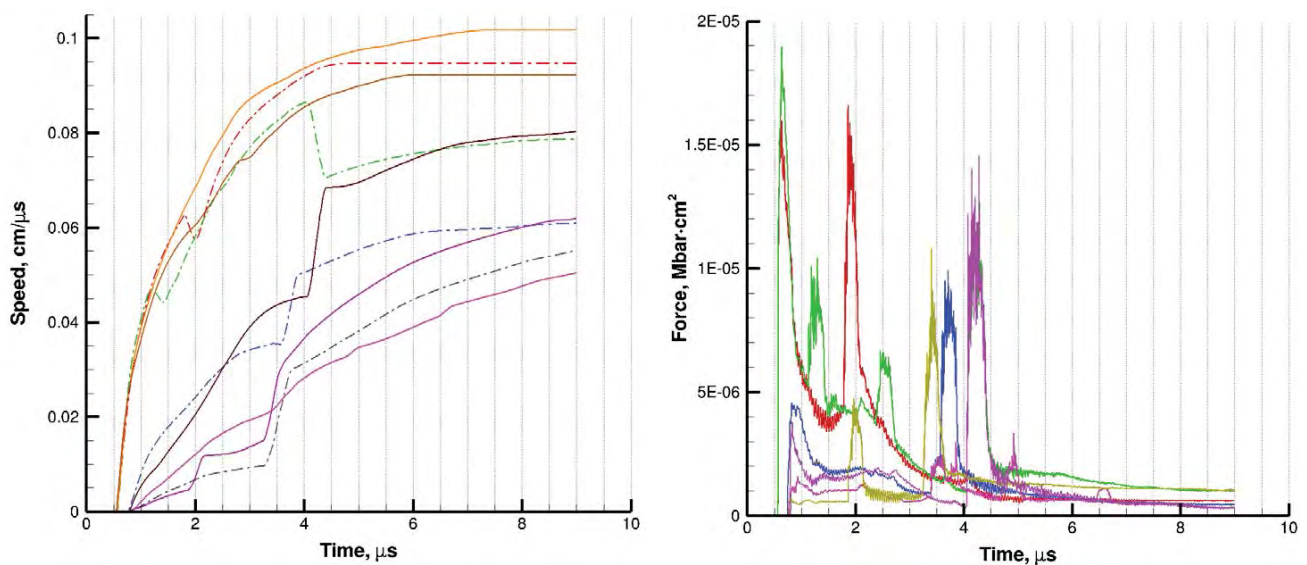




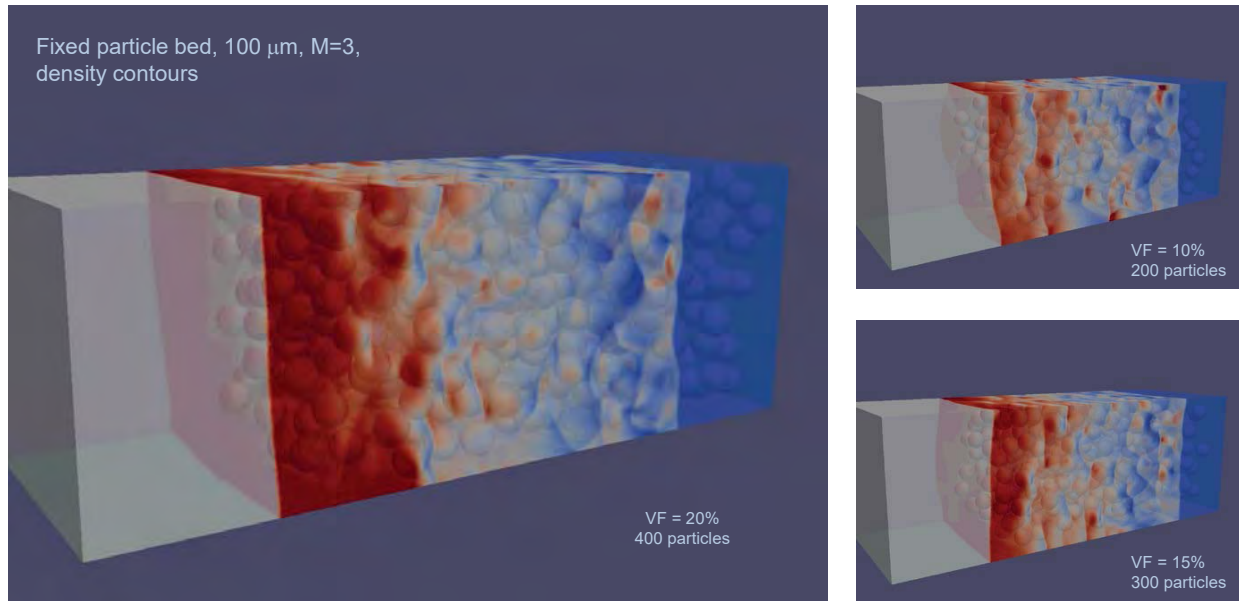
# Process of compaction is visible in this simple Mesoscale simulation



# Soft and hard contacts are observed in particle dispersal

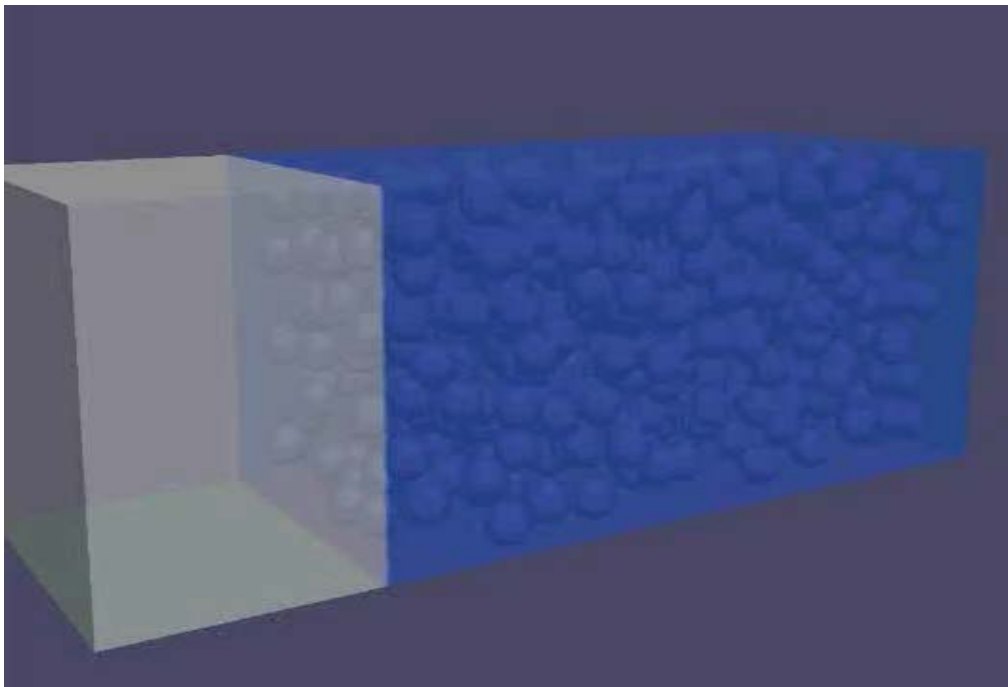


# Primary shock slows down as it goes through a particle field

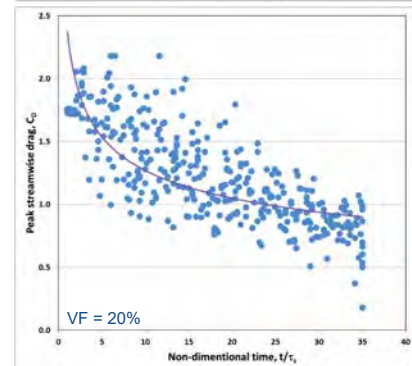
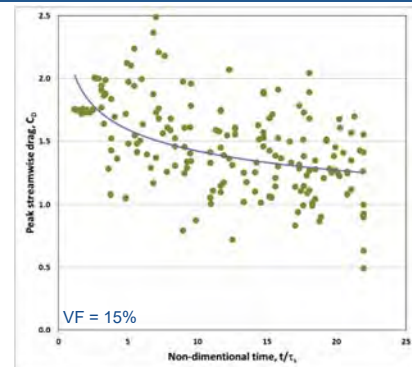
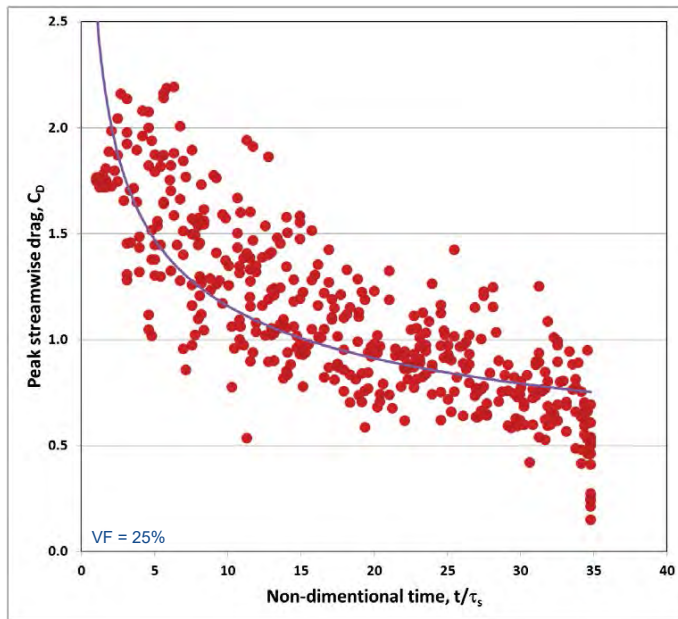


Student Chris Neal from University of Florida – shock particle interaction study using ROCFLU

# Reflected shock is more pronounced at higher particle volume fraction – ROCFLU

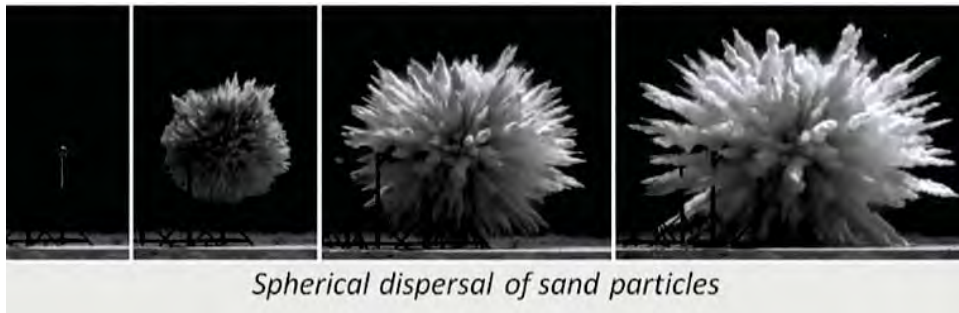


# Streamwise particle drag shows noticeable reduction within the particle field

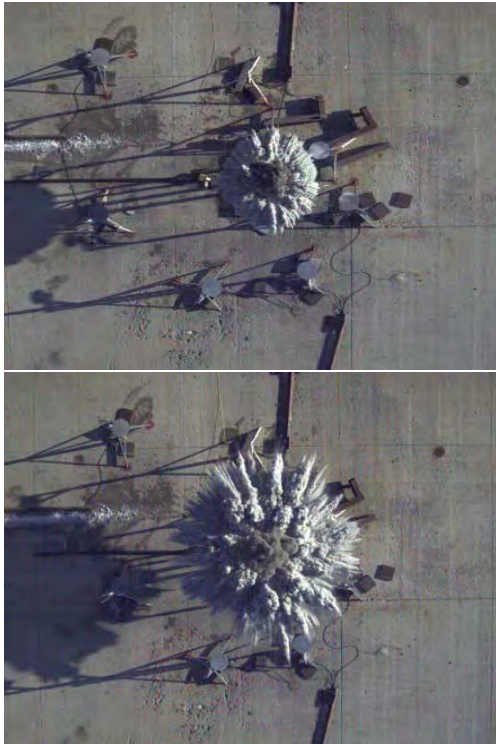


# Particle clustering/jetting is a common feature of energetic dispersal

- Particle jets can influence:
  - Turbulent mixing and burning
  - Blast wave propagation
- Particle clustering/jetting can have many physical drivers including instability mechanisms

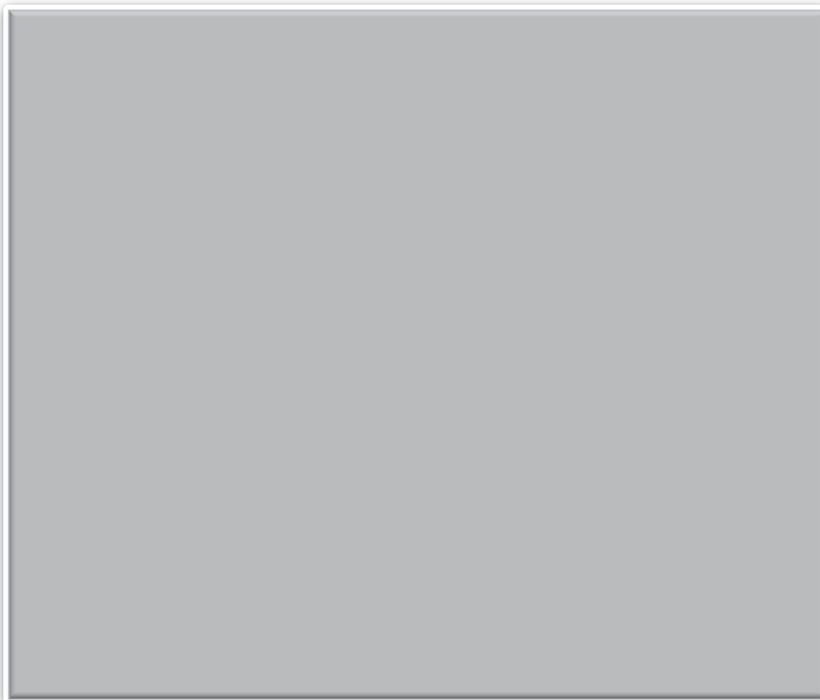


## Fan Zhang demonstrated the presence of particle jets and their correlation with booster case fragments



Plenary Talk; 12th Hypervelocity Impact Symposium, Baltimore Sept 16 – 20, 2012

## Particulate plume surface instabilities can produce fine clustering/jetting

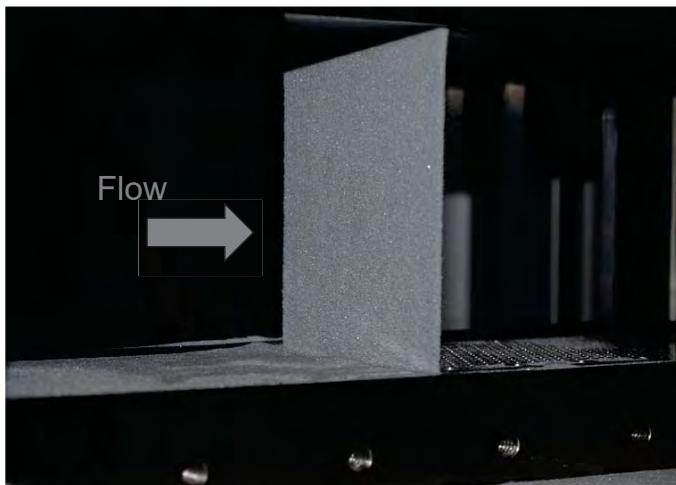




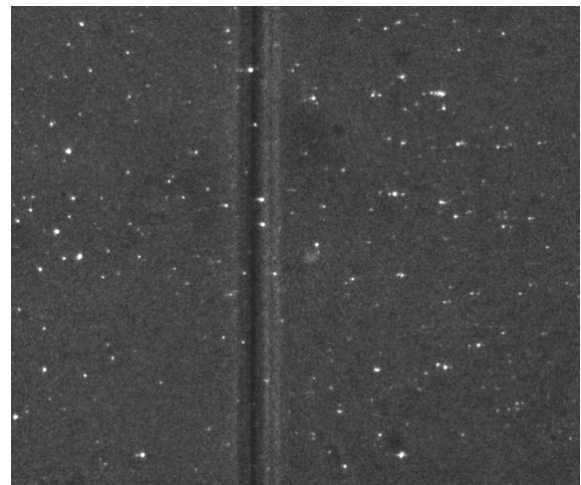
## LLNL activities in energetic release of particles

- LLNL has a long history in designing and testing of charges with particles
- Internally funded experimental campaign to conduct experiments with complex diagnostic to measure gas and particle behavior during detonation and transport, **headed by Ed Kokko**
- 3-Phase model of explosion fields with detailed fireball combustion, **headed by Allen Khul**

## SNL and LANL shock tube particle experiments are valuable in evaluation of particle drag in shock dominated conditions



**Justin Wagner, SNL**



**Kathy Prestridge, LANL**

# Multiphase model can be developed based on the hypothesis of phase separation

- A basic premise of *two-phase mixture theory is that the mixture can be separated into discrete components*. Each of these components behaves as a single material except when interacting with another component.
- This hypothesis holds if *the internal state of one phase is not related to the internal state of another phase and the thermodynamic state of a particular phase depends only on the properties of that phase (mass, momentum, energy, and EOS)*
- Each phase mass, momentum, and energy equation contains terms not present in one-phase models that are known as *phase interaction terms*. These terms model the transfer of mass, momentum, and energy from one phase to another

Akhatov & Vainshtein, 1984, Baer & Nunziato, 1986, Saurel & Abgrall, 1998, Chinnayya, Daniel, and Saurel, 2004, Balakrishnan, Nance, Menon, 2010

## We are using Multiphase Discrete Equations Method (DEM)

- Eulerian-Eulerian (DEM-EE): all phases are represented as a continuum
- Eulerian-Lagrangian (DEM-EL): mixed representation. We assume solid particles are represented as a Lagrangian phase

### Benefits of DEM approach:

- Each phase has it's own velocity, density, pressure, energy, and equation of State (EOS)
- All phases can be compressible
- Interface quantities can be calculated via a Riemann solve
- DEM allows massively parallel coupling to multi-physics and chemistry

# Multiphase theory governing equations

Volume fraction

$$\frac{\partial \alpha_k}{\partial t} + u_i^* \frac{\partial \alpha_k}{\partial x} = \mu(p_1 - p_2)$$

Mass

$$\frac{\partial \alpha_k \rho_k}{\partial t} + \frac{\partial \alpha_k \rho_k u_{k,j}}{\partial x_j} = \dot{m}$$

Mom.

$$\frac{\partial \alpha_k \rho_k u_{k,i}}{\partial t} + \frac{\partial (\alpha_k \rho_k u_{k,i} u_{k,j} + \alpha_k p_k \delta_{ij})}{\partial x_j} = p^* \frac{\partial \alpha_k}{\partial x_j} \delta_{ij} + \dot{m} u_i^* + F_i + \mu p^* (p_1 - p_2)$$

energy

$$\frac{\partial \alpha_k \rho_k E_k}{\partial t} + \frac{\partial \alpha_k (\rho_k E_k + p_k) u_{k,j}}{\partial x_j} = p^* u_j^* \frac{\partial \alpha_k}{\partial x_j} + \dot{m} E_k + F_i u_i^* + Q$$

Equation of state

$$(f(E, p, \rho))_K = 0$$

Saturation constraints

$$\sum_{k=1,n} \alpha_k = 1$$

\* Indicates interface quantity

Equations for phase K

Phase exchange terms

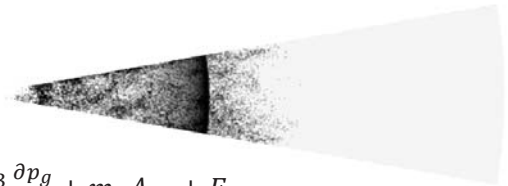
Pressure relaxation terms

Saurel & Abgrall, 1998

## Modification to governing equations for free lagrangian particles

- Particles are free lagrangian objects

$$\begin{aligned} - \frac{dm_p}{dt} &= \dot{m} \\ - \frac{dx_{p,i}}{dt} &= u_{p,i} \\ - m_p \frac{du_{p,i}}{dt} &= \frac{\pi}{2} r_p^2 C_D \rho_g |\vec{u}_g - \vec{u}_p| (u_{g,i} - u_{p,i}) - \frac{4}{3} \pi r_p^3 \frac{\partial p_g}{\partial x_i} + m_p A_{c,i} + F_{body} \\ - m_p C_p \frac{dT_p}{dt} &= 2 \pi r_p \kappa_g Nu (T_g - T_p) \end{aligned}$$



- DEM-EL volume fractions entirely determined by location of lagrangian particles
  - $\alpha_g = 1 - \alpha_p$
- Interface terms handled identically to DEM-EE
  - Interface quantities  $p^*$  and  $u^*$  determined by one-sided Riemann Solve and particle velocities respectively
- No pressure relaxation terms
- Phase exchange terms are calculated as sums of mass, momentum, and energy transfer over all particles

# DEM-EL computational simulation captures relevant flow features of multiphase blast



Kambiz Salari  
[Salari1@llnl.gov](mailto:Salari1@llnl.gov)

## Questions?



## CCMT Multiphase Deep Dive

October 6-7, 2016, St. Petersburg, FL, USA

### Multiphase Methods for Modeling Fire Environments

**Alexander L. Brown;** [albrown@sandia.gov](mailto:albrown@sandia.gov); (505)844-1008  
Fire Science and Technology Department

SAND2016-10001 C



Sandia National Laboratories is a multi-program laboratory managed and operated by Sandia Corporation, a wholly owned subsidiary of Lockheed Martin Corporation, for the U.S. Department of Energy's National Nuclear Security Administration under contract DE-AC04-94AL85000.

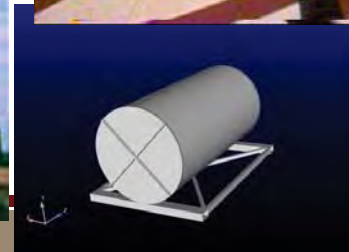
## Outline



- Part 1: Drop Break-up and Dense Spray Simulations
  - Why we care?
  - Description of L-J methods
  - Issues with the dense region
  - Test conditions for validating the model
- Part 2: Liquid Pool Fire Modeling
  - Motivating experiment description
  - Modeling approach
  - Preliminary results (verification)
- Summary

# Water Slug Impact Experiment

- Tests performed in 2002 provided data for validating liquid spread dynamics for an aluminum tank impacting a concrete slab
- Liquid deposition, particle sizing, and video data
- This is representative of some fireball initiation problems



## Water Slug Simulations

A coupled capability using SIERRA solid mechanics and fluid mechanics in tandem can simulate fluid behavior beyond just the impact

**Water Slug Test  
and  
Simulation  
Sandia National Labs**

# Fireball Simulations

- Follow-on work has demonstrated that predictions can be made with impacting fuels, resulting in fireball formation

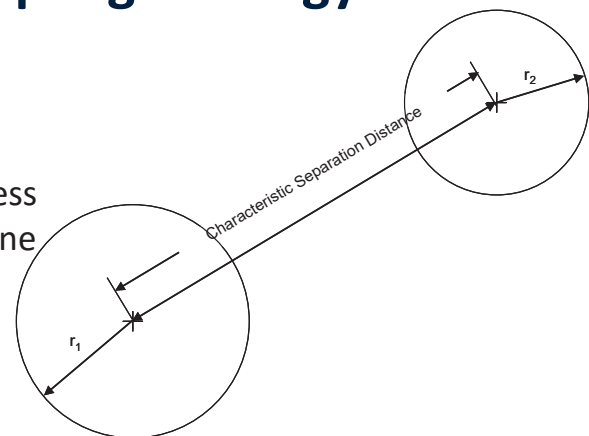


## Presto/Fuego Coupling Strategy

- SPH mass/momentum conserved in a transfer between the two codes
- Mass is transferred according to an algorithm that uses a critical dimensionless particle separation distance ( $B_{crit}$ ) to define transfer times:

$$B = \frac{\text{Characteristic Separation Distance}}{\text{Characteristic Drop Length}}$$

- All liquid mass is assumed to be spherical drops until transfer to CFD code
- Drops subsequently are predicted to distort and break-up according to the Taylor Analogy Break-up (TAB) model
- Lagrangian/Eulerian coupling employed for evaporating drops



$B_i$  assessed at each time for each particle

$$B_i = \min_{j=1 \text{ to } N} \left\{ \frac{\sqrt{(x_i - x_j)^2 + (y_i - y_j)^2 + (z_i - z_j)^2}}{d_i/2 + d_j/2} \right\}; i \neq j$$

Using this definition for  $B$ , reasonable  $B_{crit}$  values are between 1.0 and 1.7

# Particle Force Method Description

- We start with a Lennard-Jones potential (inspired by MD):

$$U_{LJ}(r) = 4\epsilon \left[ \left( \frac{\sigma}{r} \right)^{12} - \left( \frac{\sigma}{r} \right)^6 \right]$$

Lennard-Jones, J.E., "On the Determination of Molecular Fields," Proc. R. Soc. Lond. A, 106, 738, 463-477, (1924).

- We modify the repulsive term (the 12<sup>th</sup> power) such that the repulsion is not as non-linear:

$$U_{spring} = \frac{k}{2} (r - r_{min})^2 - \epsilon : r < r_{min}$$

- This model form maintains a smooth function (differentiable) across the  $r_{min}$  ( $r_{min} = 2^{1/6}\sigma$ ) threshold
- We are also modifying the particle drag term
- The need for this was motivated by the observation that some mass in the coupled methods was still in a regime where it was not dilute and dispersed, yet treated as such

7

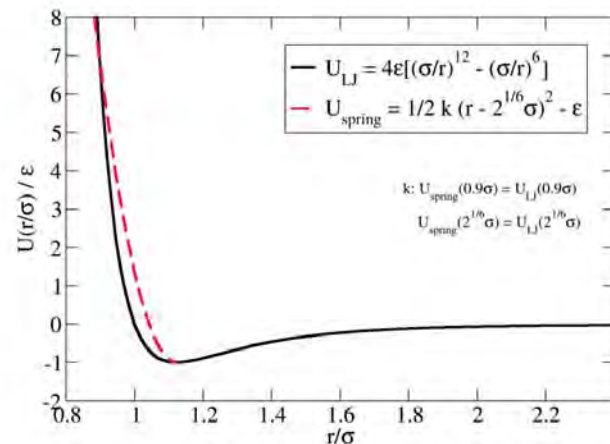
## Modified Potential Illustration

- A figure illustrating the modified potential shows the classical (black) and modified (red) model
- Surface tension  $\gamma$  relates to L-J parameters through:

$$\gamma_{LJ} = C \frac{\epsilon}{\sigma^2}$$

- Need to tune constant C
  - Hopefully universal
- $\sigma$  comes from geometry
- $\epsilon$  comes from C and  $\sigma$
- C is dimensionless

$$C = \frac{\gamma_{LJ} \sigma^2}{\epsilon}$$





# Drag Modification

- We don't want to have to recover the shape of the liquid to recover drag; unmodified the code will over-predict the drag
- Introduce a two-component scaling factor:

$$C_{Dm} = f_S C_{Do} \quad f_S = f_{S1} f_{S2}$$

- Define the terms numerically:

$$f_{S1} = \max \left( 1, \left( \frac{|k|N}{\sigma} \right)^K \right) \quad f_{S2} = \min(1, 1 + Proj)^L \quad Proj = \hat{u} \cdot \hat{k}$$

$$\hat{k} = \frac{\vec{k}}{|\vec{k}|}$$

- $\vec{k}$  is the mean connectivity vector:

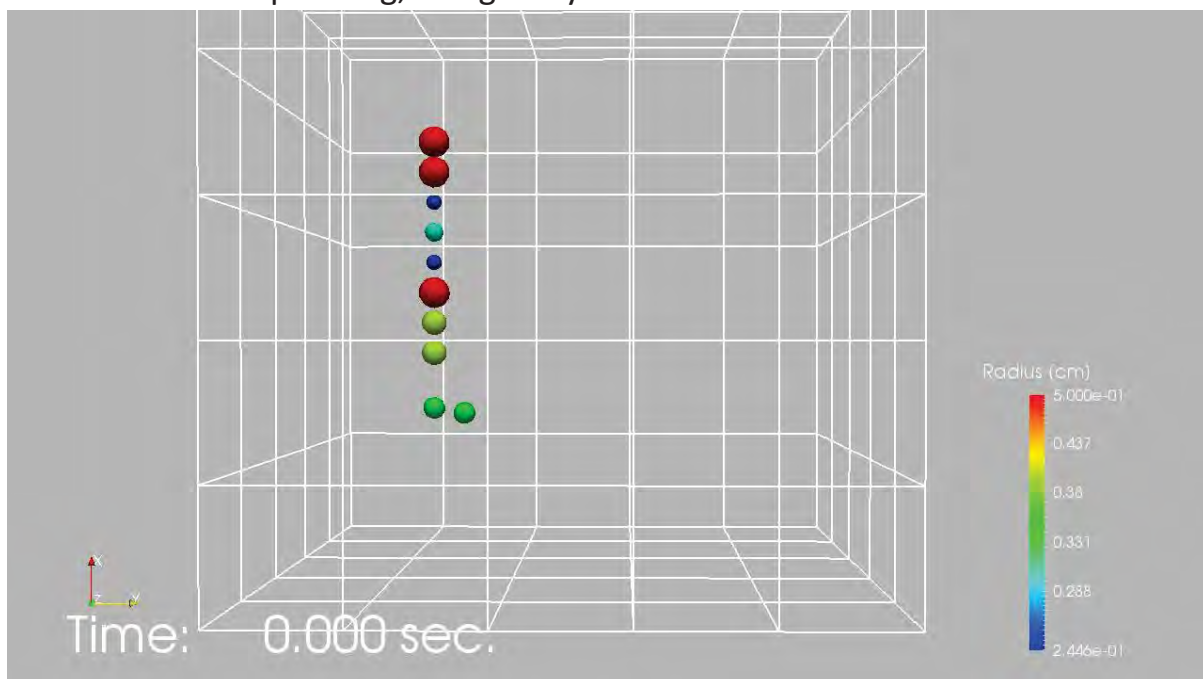
$$\vec{k} = \left\{ \frac{\sum_{i=1 \rightarrow N} (x_o - x_i)}{N}, \frac{\sum_{i=1 \rightarrow N} (y_o - y_i)}{N}, \frac{\sum_{i=1 \rightarrow N} (z_o - z_i)}{N} \right\}$$

- This work is still in progress (hasn't been tuned)

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# Lennard-Jones Model Verification

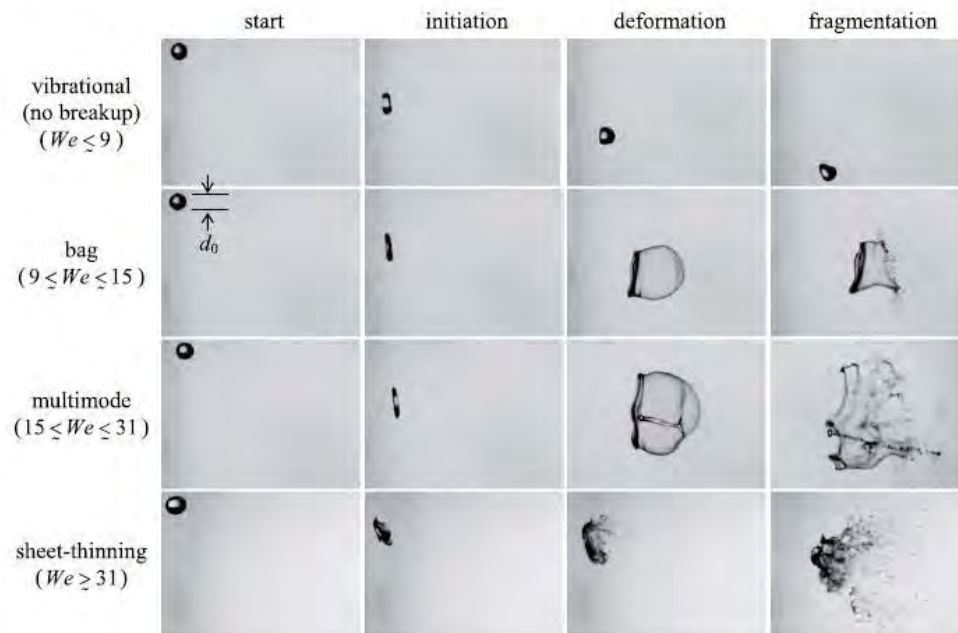
- Red drops are non-interacting
- Others interact with like colored drops, rebound on surface impact
- This is an evaporating, low gravity test scenario



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# Model Validation Problem

- Data suggests a critical Weber number of 9-12
- We want to match the data as best as possible

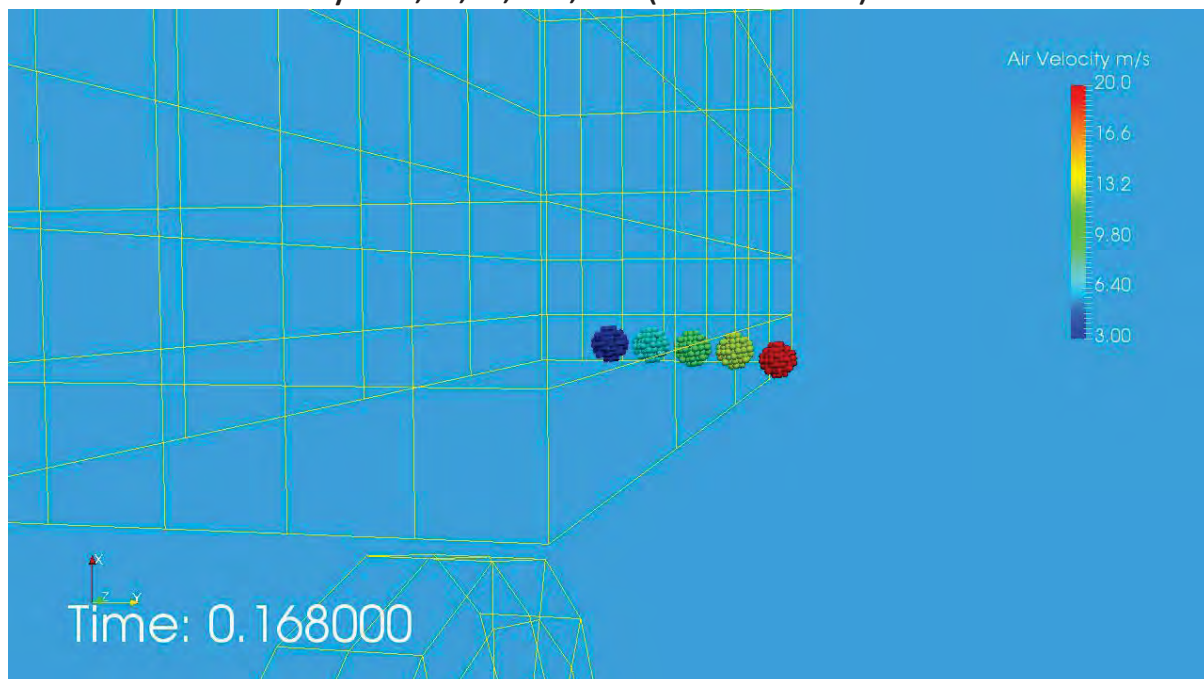


Flock, A.K., Gueldenbecher, D.R., Chen, J., Sojka, P.E., and Bauer, H.-J., "Experimental statistics of droplet trajectory and air flow during aerodynamic fragmentation of liquid drops," Int. J. Multiphase Flow, 47, pp. 37-49, (2012).

11

## Preliminary Validation Results

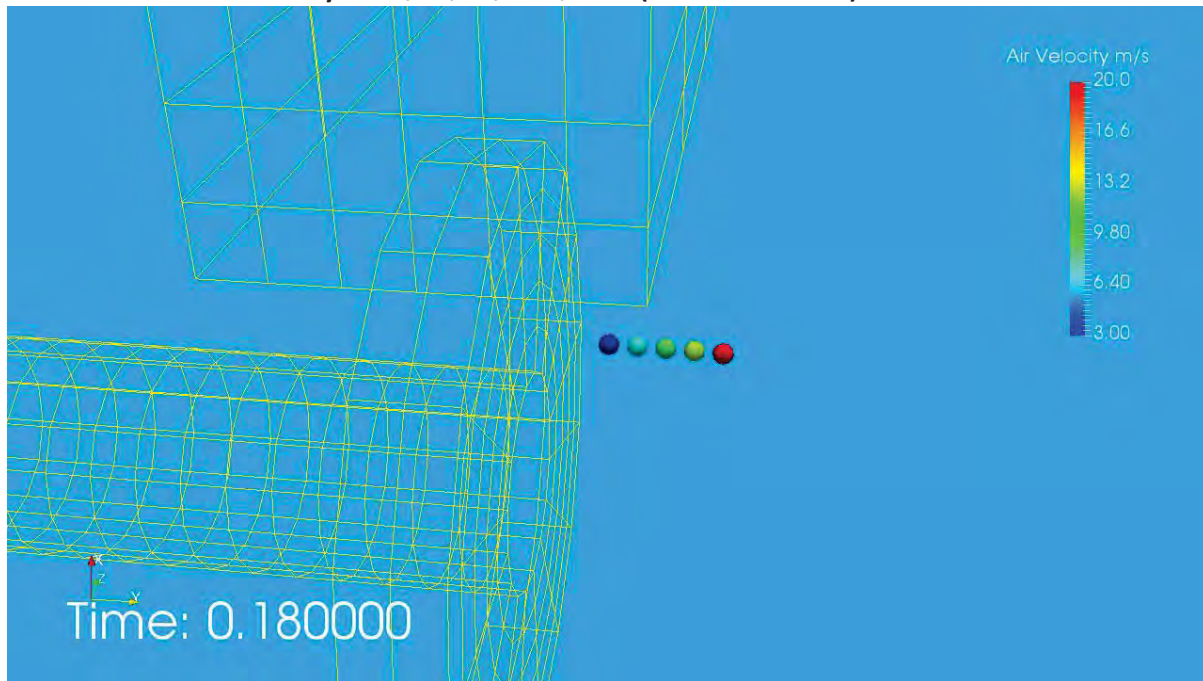
- $We = 1, 5, 11, 20, 55$  (blue to red)  $C=200$
- Nozzle velocity = 3, 6, 9, 12, 20 (blue to red)



12

# TAB model for drop break-up

- $We = 1, 5, 11, 20, 55$  (blue to red)
- Nozzle velocity = 3, 6, 9, 12, 20 (blue to red)



## Part 2: Liquid Pool Fire Modeling



# Mixed JetA-Composite Fire Test-9/14



Calorimeters



Heat Flux Gages



Thermocouple  
Rake in Pool



Wind

## Test Arrangement

**Composite Rubble Fire Test**  
**Assembly Time Lapse**  
**9/4/14**

## Composite Rubble Fire Test

9/5/14

### What do the results mean?

- A fire resulting from an aircraft crash can last an extraordinary length of time (hours to days) because of the reacting rubble
- It is possible to have a low-level burn with a significant increase in temperatures at later times
- Fibers may continue to react with no flaming present if there is sufficient material (i.e. at this scale)
- The epoxy never appeared to be a significant or distinguishing factor in this test as a fuel, is thought to be mostly consumed early in the burn
- Total composite mass loss is still not resolved for an unmitigated fire scenario, longer-term data would be helpful
  - Likely to exhibit significantly higher mass loss

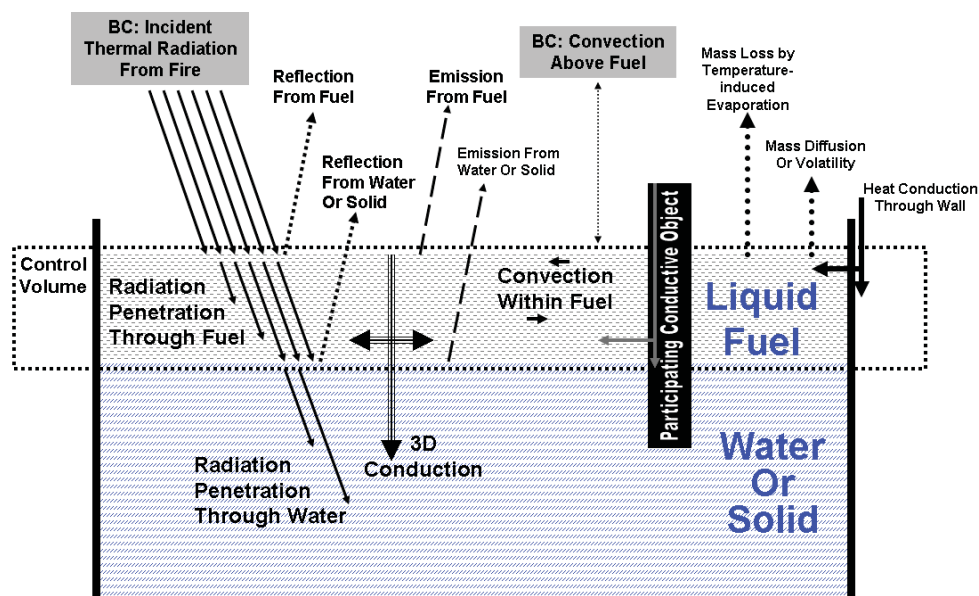


# Modeling this Test

- The significance of this problem is high for people who deal with transportation safety
  - Slow heat scenario
  - Can be challenging
- We generally lack the ability to model this scenario with existing tools
  - SIERRA/Aria does level-sets, not ideal in this regime
- I have not encountered a capability in any code for liquid soaked solid fuel fires
- We have performed some rubble soaked fuel tests in the past, but never had a scenario where the 'rubble' could also react
- An external proposal was funded to model this type of event (NSR&D)

# Existing PIRT

- We previously have described generically what is needed to model complex pool fire scenarios



# Approach

- Implement a volume of fluid capability in SIERRA/Fuego to model the receding fuel layer
  - Surface tension model for Marangoni forces:  $\tau_{mar} = -\nabla_s \sigma$
  - Evaporation dynamics at the interface
- Abstract the existing solid fuel modeling capabilities to predict the reactions in a rubble fuel bed
  - We leveraged this component of development to incorporate the CPD model for percolation theory based reaction kinetics
- Link the capabilities in Sandia's SIERRA/Fluid Mechanics code Fuego that has participating media radiation capability and solid reacting materials

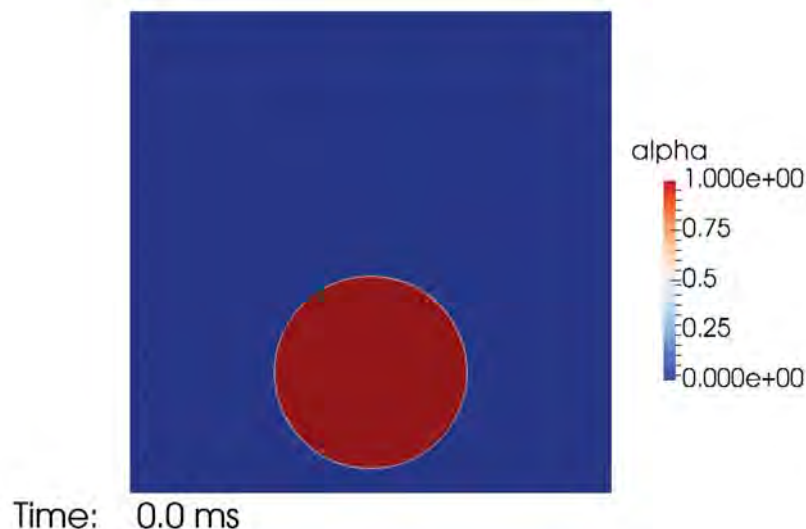
Sources for models:

Brackbill, J.U., D. B. Kothe, and C. Zemach, "A Continuum Method for Modeling Surface Tension," Journal of Computational Physics, 100, 335-354, 1992.

Hardt, S., and F. Wondra, "Evaporation model for interfacial flows based on a continuum-field representation of the source terms," Journal of Computational Physics, 227, 5871-5895, 2008.

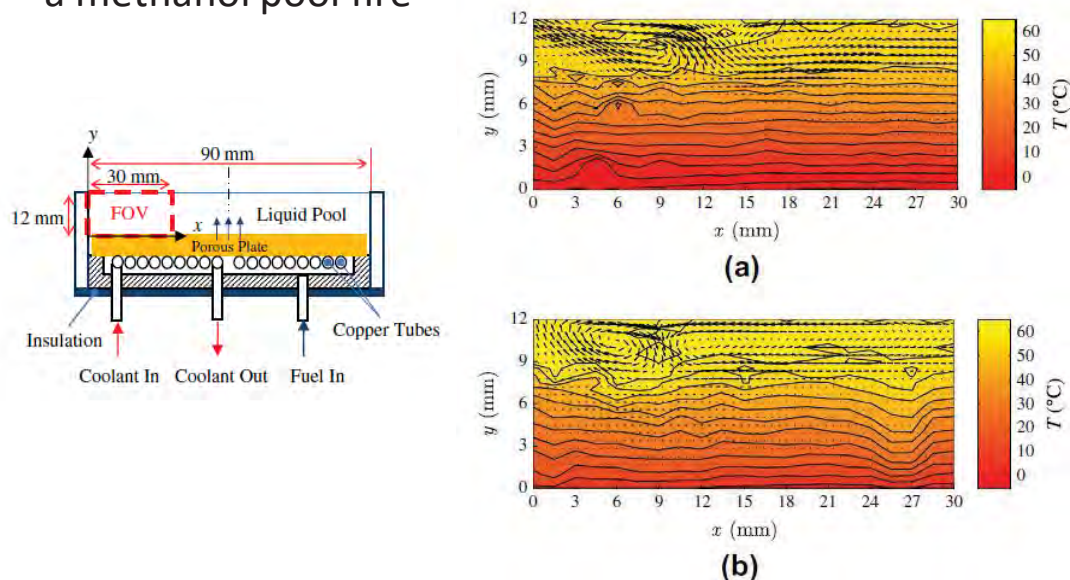
# Preliminary Verification Results

- A basic VOF has been implemented and verified (see video)
- Currently, we are working on modeling the surface forces and radiation interaction with the surface



# Validation Plan

- A recent dataset exists where flow is measured in a pool fire
- They show mixing at the top, stratification of temperature for a methanol pool fire



Vali, A., Nobes, D.S., and Kostiuk, L.W., "Transport phenomena within the liquid phase of a laboratory-scale circular methanol pool fire," *Combustion and Flame*, 161, pp. 1076-1084, 2014.

# Summary

- We have two applications right now actively developing unique multi-phase engineering models
  - A particle method for simulating drop break-up, shows promise
  - A VOF method for simulating fuel fires with solid fuel mixed in
- The MD inspired particle interaction method appears to be able to model the critical break-up of drops
  - Parameter tuning and model assessment is in progress
- A VOF method is being developed for modeling the pool dynamics in a mixed pool fire including solid fuel
  - Basic VOF is implemented, need radiation coupling, evaporation, and surface tension model

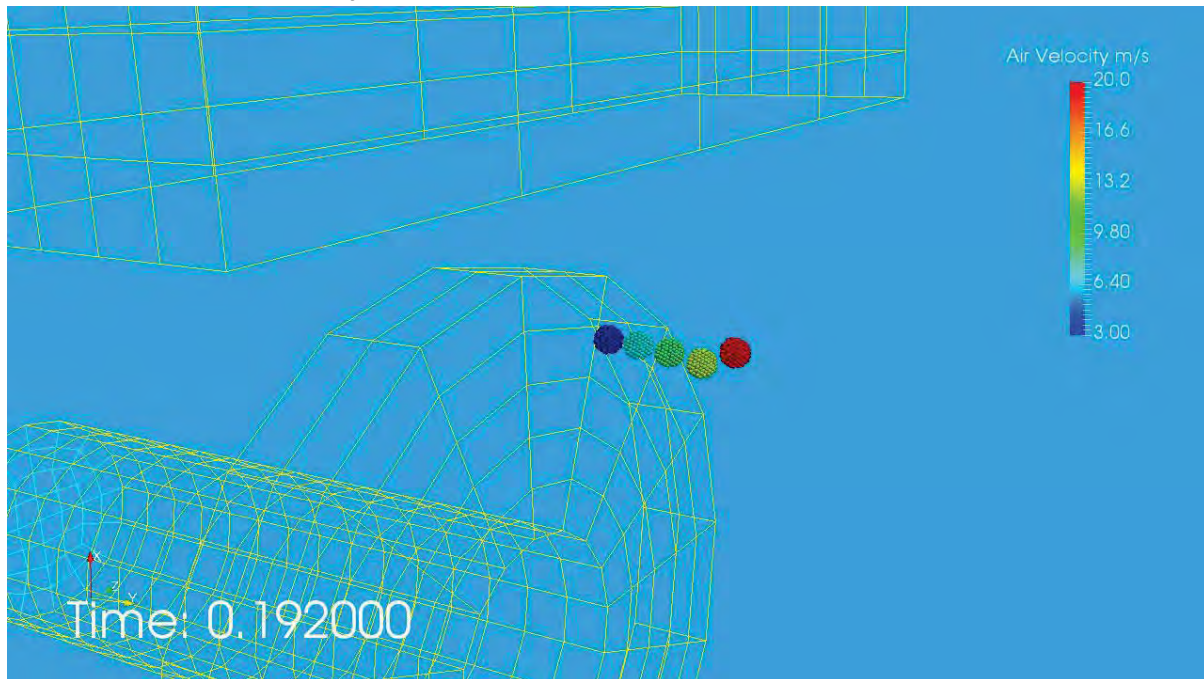
# Acknowledgements

- The test team including contributions from Brandon Servantes Sylvia Gomez-Vasquez, Travis Fitch, Randy Foster, Chuck Hanks, Jerry Koenig, William Dixon, Richard Simpson, Donald Mcmanaway, Deven Coddling, Paul Coddling, Steve Chandler, KD Pass, Craig Dickensheets, Ron Pederson, Jeff Kokos, Jim Nakos, and Randy Watkins.
- Major contributors to the code efforts including David Glaze, Stefan Domino, Flint Pierce, John Hewson, David Noble, Sam Subia, Amanda Dodd, Rekha Rao, Vern Nicolette, Anay Luketa, Sheldon Tieszen, Tyler Voskuilen
- VOF model implementation: Tyler Voskuilen
- LJ model implementation: Flint Pierce

# Extra Viewgraphs

# Preliminary Validation Results

- $We = 1, 5, 11, 20, 55$  (blue to red)  $C=200$
- Nozzle velocity = 3, 6, 9, 12, 20 (blue to red)



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## Existing Capabilities

Sandia develops the open source SIERRA mechanics. Fluid mechanics tools exist in this framework.

- The non-open source **Fuego** tool is primarily used for low-Mach number reacting flow problems and particle transport.
- **Aria** does heat transport and low-Reynolds number multi-phase transport and reaction
- **Aero** does shock, ablation, and high-Mach number simulations
- **Nalu** is a basic open source N-S flow code for testing new algorithms

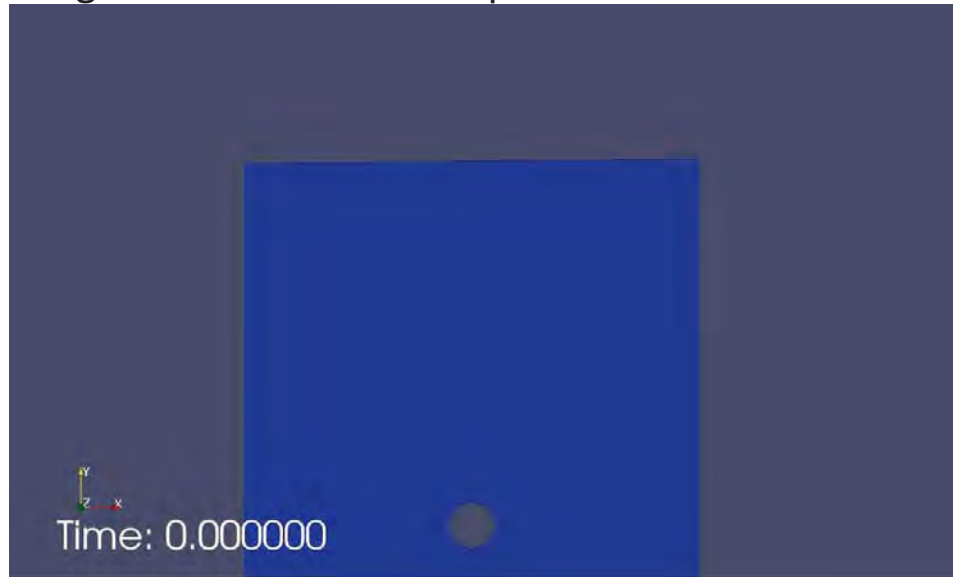
Capabilities potentially relevant to the composite fire scenario include:

- Adaptive boundary level-set method for multiphase flows (Aria)
- Particle combustion model (Fuego)
- 1-D solid reacting boundary condition model (Fuego)
- 1-D liquid pool model (Fuego)
- 3-D solid reacting material model (Fuego/Aria)



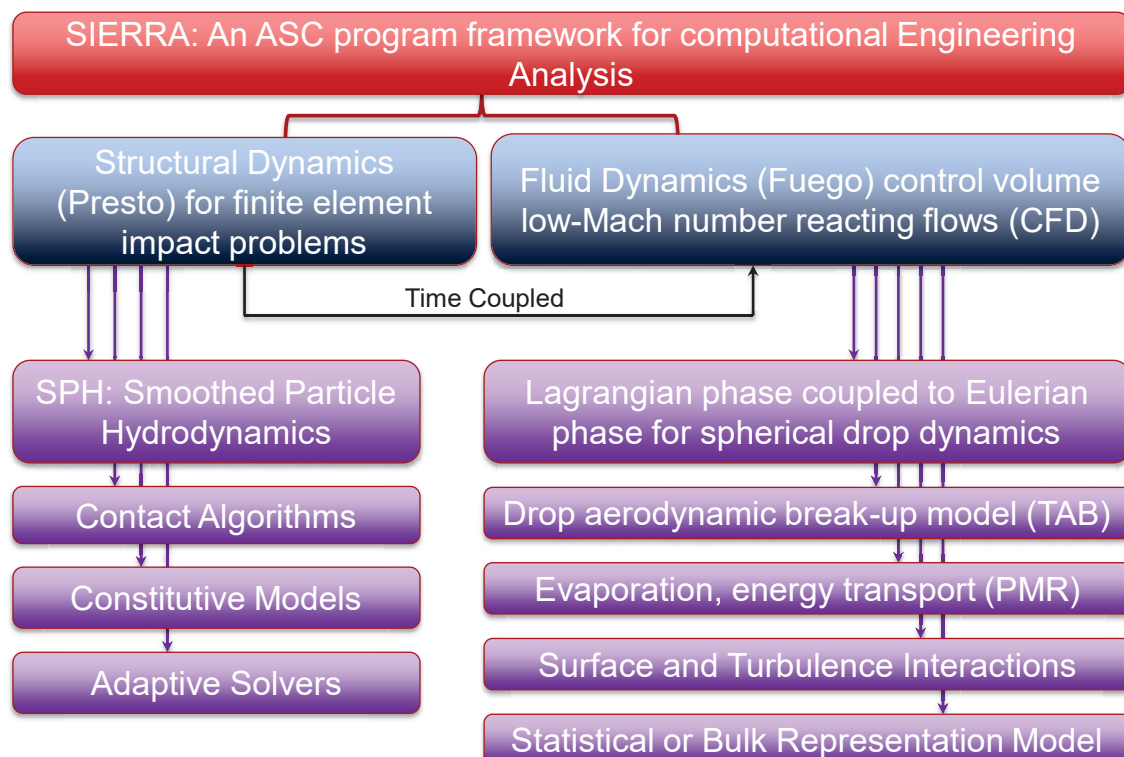
# Level-set methods

- CDFEM methods were recently implemented in Aria for resolving level-set multi-fluid interfaces
- The below video exhibits a 2-D prediction of a boiling drop rupturing on the surface of a liquid



## General Approach Schematic

Enabling Physics Modules Computational Tools

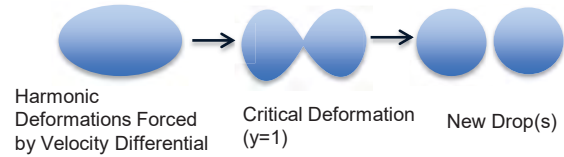


# Taylor Analogy Break-up (TAB) Model

- Originally by O'Rourke and Amsden (1987)
- Approximates the drop as a damped oscillator, formulated as a second-order differential equation, with  $y$  as a deformation parameter:

$$m_d \frac{d^2 y}{dt^2} = m_d \underbrace{\frac{C_F \rho_g}{C_b \rho_l} \frac{|u_d - u_g|^2}{r^2}}_{\text{Aerodynamic Forcing}} - m_d \underbrace{\frac{C_k \sigma}{\rho_l r^3} y}_{\text{Surface Energy Damping}} - m_d \underbrace{\frac{C_d \mu_l}{\rho_l r^2} \frac{dy}{dt}}_{\text{Viscous Damping}}$$

## Simplified Schematic



- Discretized solution for  $y$  is:

$$y(t + \Delta t) = \frac{We}{C} + \left\{ \left( y(t) - \frac{We}{C} \right) \cos(\omega \Delta t) + \frac{1}{\omega} \left( \dot{y}(t) + \frac{y(t) - We/C}{\tau_b} \right) \sin(\omega \Delta t) \right\} \exp(-\Delta t / \tau_b)$$

$$\omega = \sqrt{\frac{8C_k \mu}{\rho_l d_p^3} - \frac{1}{\tau_b^2}} \quad \text{Oscillation Frequency}$$

$$\dot{y}(t + \Delta t) = \frac{We - y(t)}{\tau_b} + \left\{ \frac{1}{\omega} \left( \dot{y}(t) - \frac{y(t) - We/C}{\tau_b} \right) \cos(\omega \Delta t) - \left( y(t) - We/C \right) \sin(\omega \Delta t) \right\} \omega \exp(-\Delta t / \tau_b)$$

$$\tau_b = \frac{1}{2} \frac{\rho_l d_p^2}{C_d \mu_l} \quad \text{Viscous Damping Time}$$

$$We = \frac{\rho_l d_p (u_p - u_g)^2}{\sigma} \quad \text{Weber Number}$$

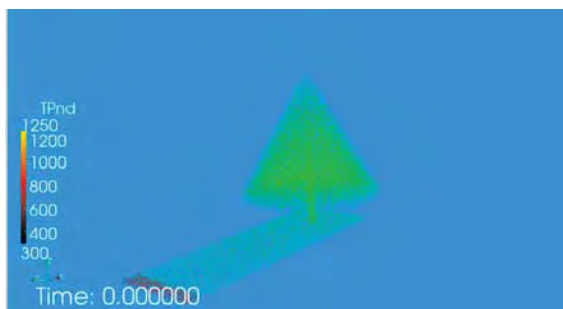
- New drop diameters can be calculated:

$$d(t + \Delta t) = d(t) \left[ 1 + \frac{C_k K}{20} + \frac{\rho_l d_p(t)^3}{8\sigma} \frac{6K - 5}{120} \dot{y}(t)^2 \right]$$

- We modified the algorithm to limit break-up for new particles

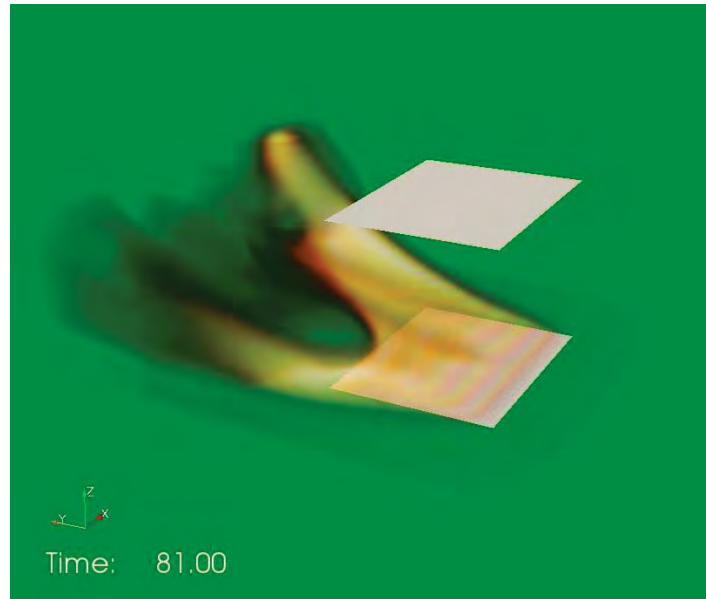
# Particle Combustion Model

- Primarily used in the past for two projects:
  - Wildland fire predictions for idealized trees
  - Aluminized propellant reactions



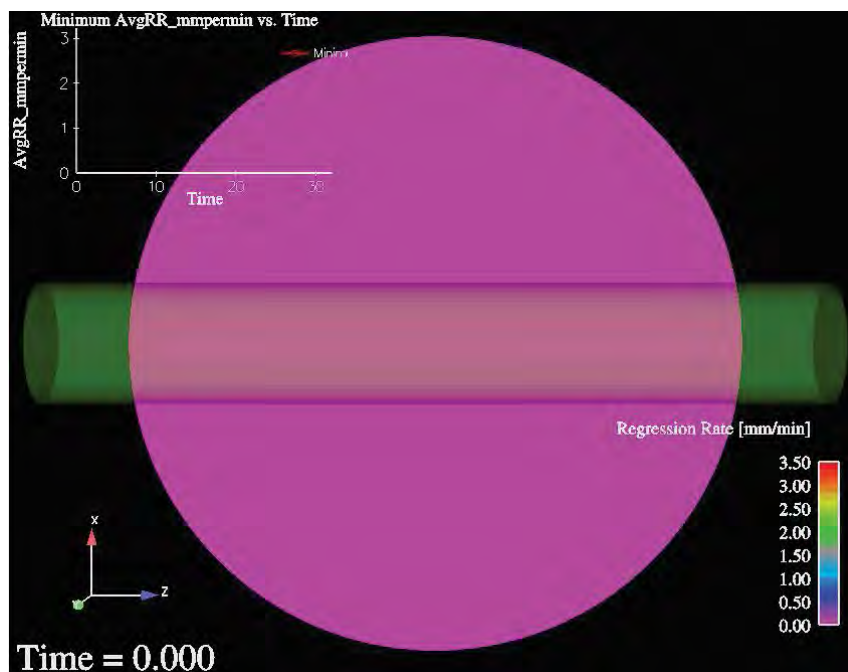
# 1-D Solid Reacting Boundary Condition

- Recent work demonstrates the verification of the method and compares to data in the context of a sensitivity analysis:



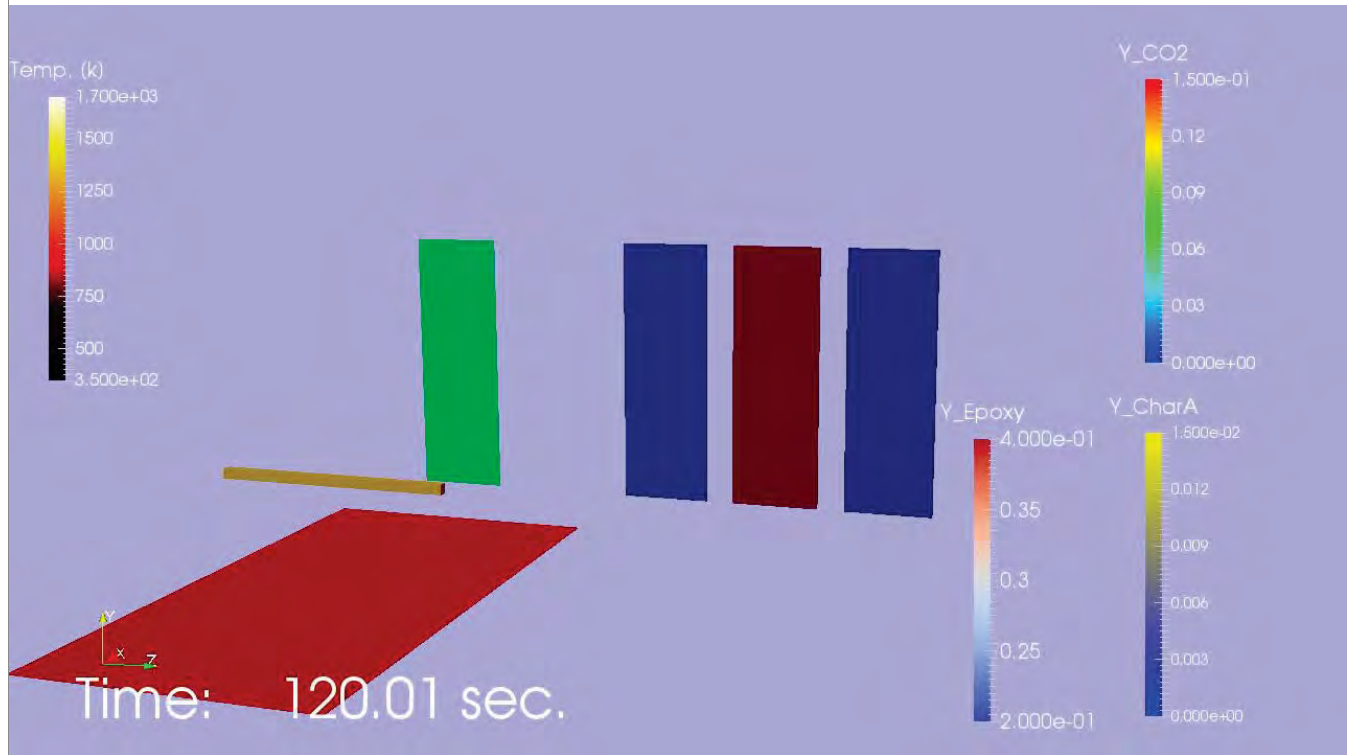
# 1-D Liquid Pool Model

- Historical model for predicting the burn rate for pool fires given heat transport from the flames (based on legacy SINTEF codes)



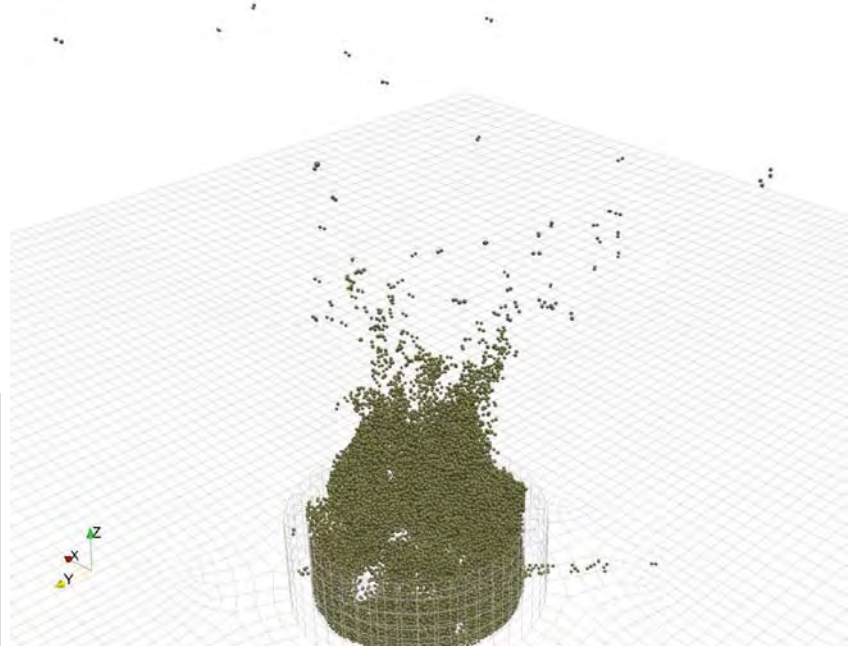
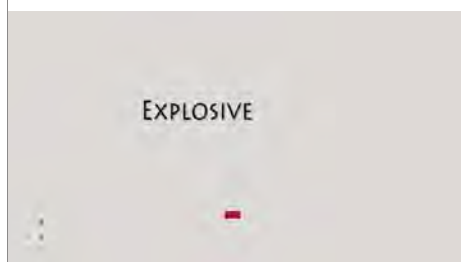
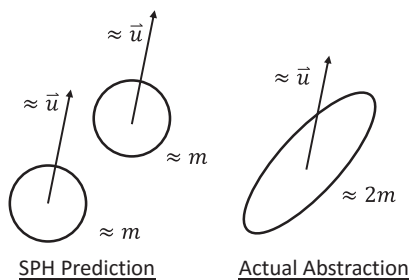
# 3-D Solid Reacting Material Model

- New model includes porous transport, charring reactions, oxidative reactions.



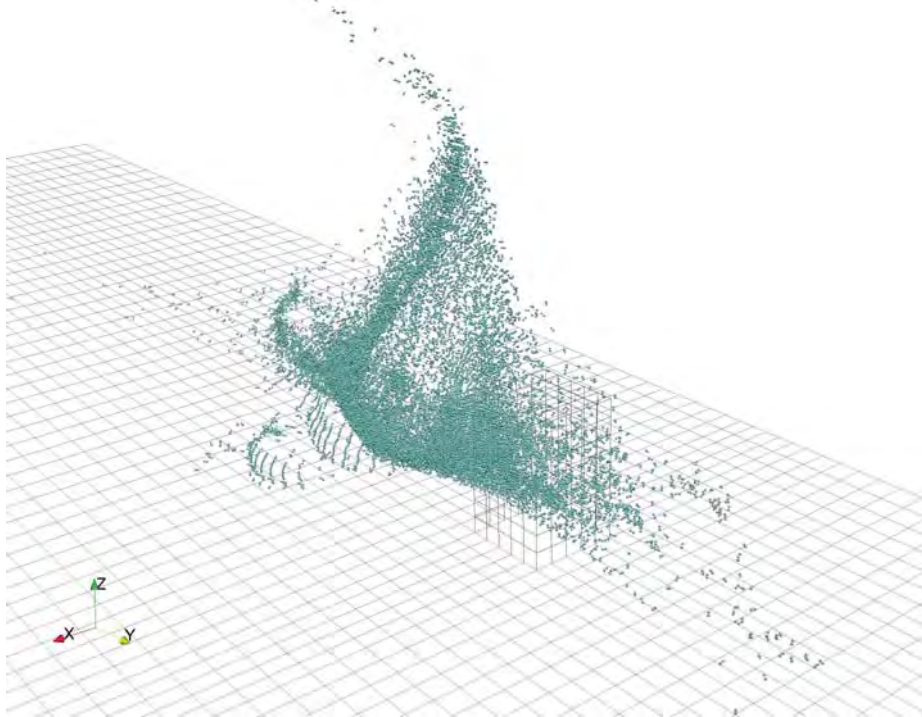
## Final Injection Illustration

- The last injection is populated with pairs in many cases (e.g. detonation inside tank scenario below)



# Final Injection Illustration

- Linear systems are also found in some scenarios (Detonation Outside Tank Scenario)



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## Numeric Model

- Introduce a dimensionless energy with a critical value  $E_{crit}$  for defining transfer

$$E_{dim} = SE/KE$$

$$SE = \sigma A = \sigma h \pi D_e$$

$$KE = (1/2)m|\vec{U}|^2$$

### Injection Criteria

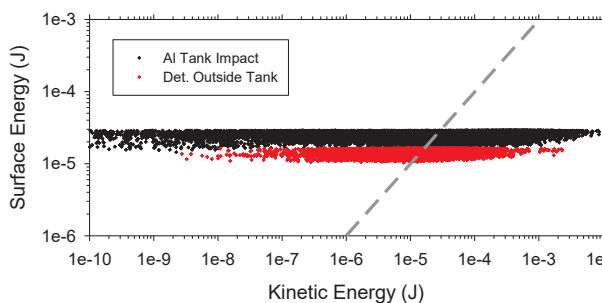
*Single Particle Criterion:*

$$B_{i,min} > B_{crit}$$

*Particle Pair Criterion:*

$$E_{dim} > E_{crit}$$

$$h = [(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2]^{1/2}$$



$$D_e = 2\sqrt{(4/3)h(r_1^3 + r_2^3)}$$

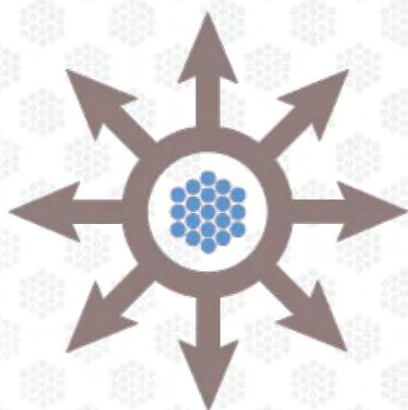
$$m = \rho V = (4/3)\pi\rho(r_1^3 + r_2^3)$$

$$|\vec{U}| = [(u_1 - u_2)^2 + (v_1 - v_2)^2 + (w_1 - w_2)^2]^{1/2}$$

- A reasonable  $E_{crit}$  value is 1.0 (or close)
- Particle pairs are combined if the system satisfies  $E_{crit}$  parameter

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## CARBON CAPTURE MULTIDISCIPLINARY SIMULATION CENTER

- this talk
1. our application: power generation from coal
  2. our predictive challenge: what is good enough?
- 
- Friday
3. particle dynamics: resolving particle location & rxn.
  4. eulerian polydispersity

## electric power generation

318GW U.S.A.coal



world  
coal

	January 2016 (MW)		Change since January 2015 (MW)		
	Announced + Pre-permit + Permitted	Construction	Announced + Pre- permit + Permitted	Construction	Newly Operating
East Asia	546,867	221,010	35,366	82,093	68,515
South Asia	253,009	76,595	-67,292	7,124	18,670
SE Asia	115,400	26,055	20,470	-2,879	10,099
Middle East and North Africa	9,688	2,036	4,960	0	0
Southern Africa	22,800	9,043	1,438	-1,085	1,545
Other Africa	10,665	0	-830	-600	0
Latin America	7,753	2,702	891	-573	935
US/Canada	2,860	600	-1,525	-830	618
Eurasia	16,370	1,700	2,930	-990	1,376
EU28	11,816	8,655	-8,448	-4,112	5,006
Non-EU Europe (incl Turkey)	75,253	3,265	3,843	-2,070	1,207

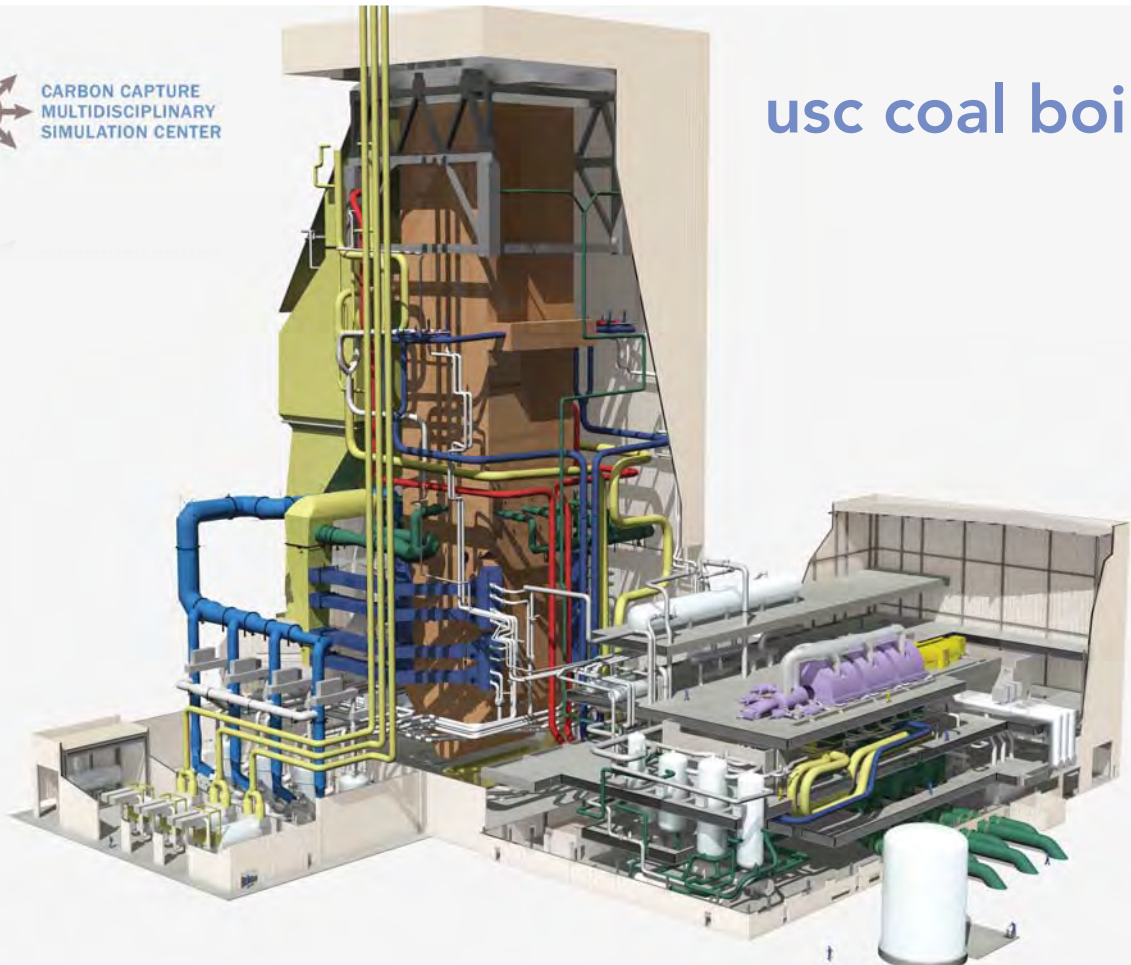
352GW coal  
under construction

# benefits of (A)USC (oxy-)coal combustion

technology	superheater temperature (°C)	carbon intensity (tonne CO <sub>2</sub> /MWh)	efficiency
avg. of existing U.S. units	540	0.95	32%
current technology	600	0.79	40%
USC	700	0.73	44%
AUSC	760	0.67	48%
AUSC w CCUS (oxy-coal)	760	0.07	42%



usc coal boiler

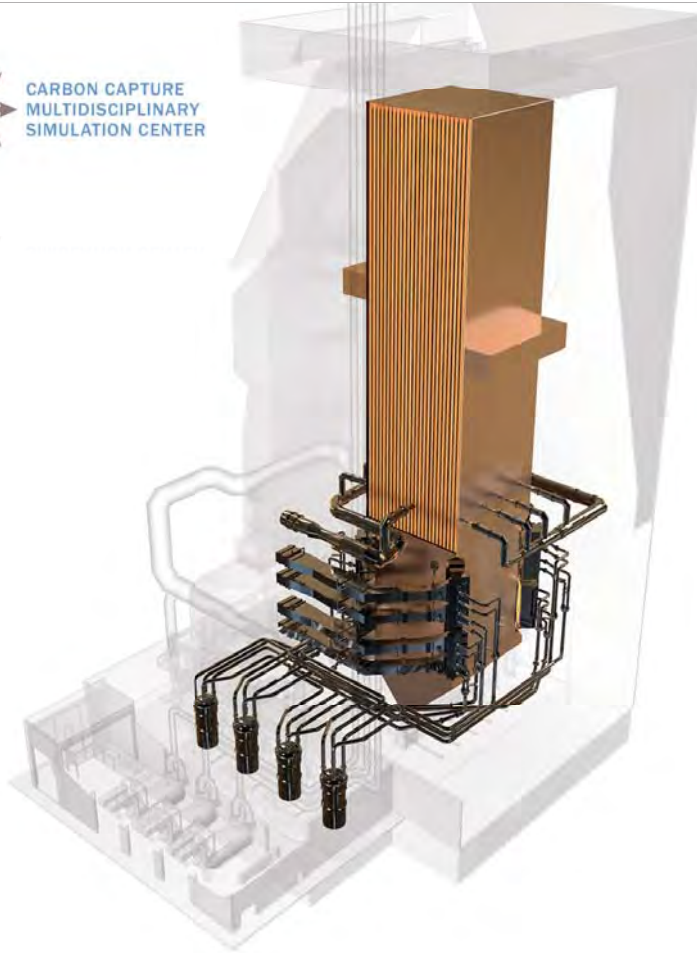






CARBON CAPTURE  
MULTIDISCIPLINARY  
SIMULATION CENTER

usc coal boiler

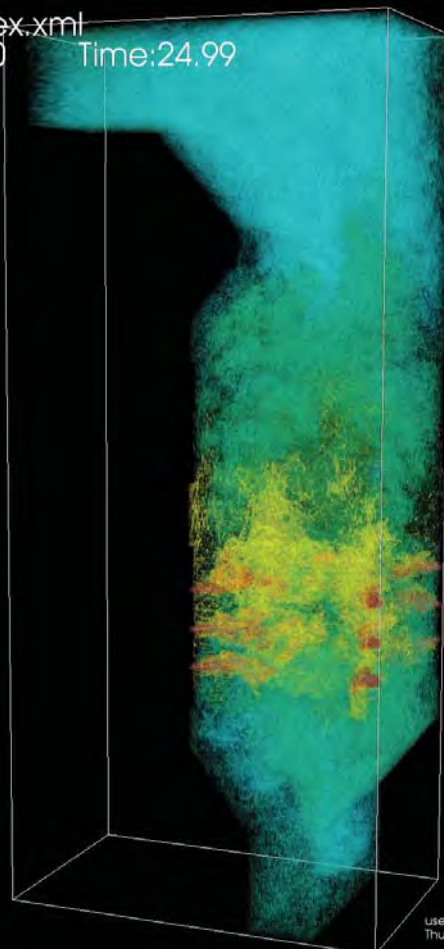
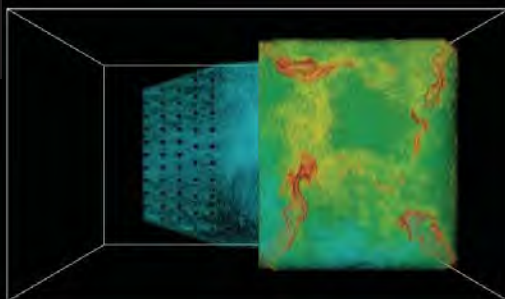


CARBON CAPTURE  
MULTIDISCIPLINARY  
SIMULATION CENTER

DB: index.xml  
Cycle: 0

Time: 24.99

Volume  
Var: w\_an0/0  
2.000  
1.500  
1.000  
0.5000  
-0.000  
Max: 28.19  
Min: 0.000



concentration of small particles

user: u0033047  
Thu Aug 21 13:55:48 2014

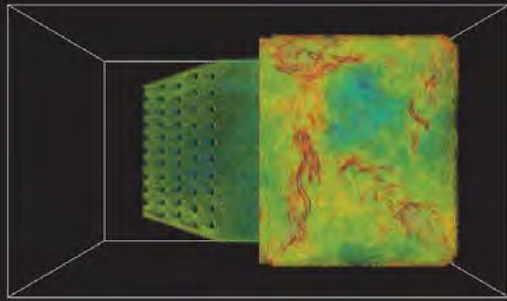


CARBON CAPTURE  
MULTIDISCIPLINARY  
SIMULATION CENTER

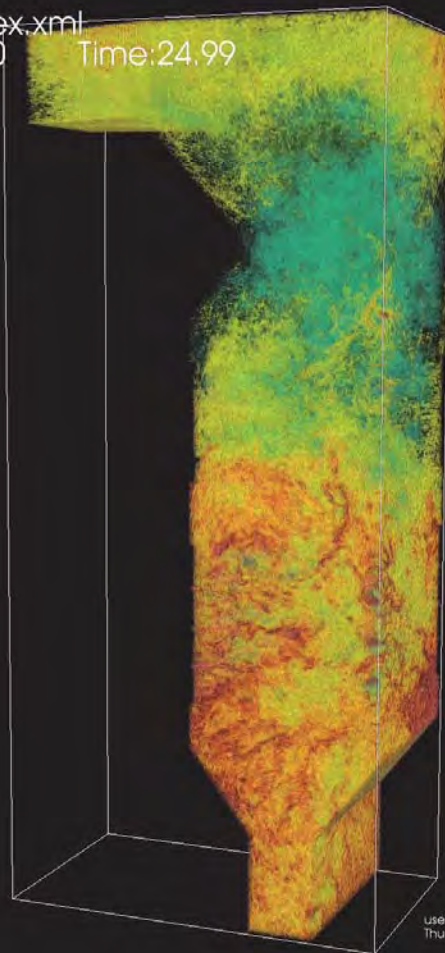
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Cycle: 0

Time: 24.99

Volume  
Var: w\_an2/9  
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0.02500  
0.01250  
0.000  
Max: 0.8620  
Min: 0.000

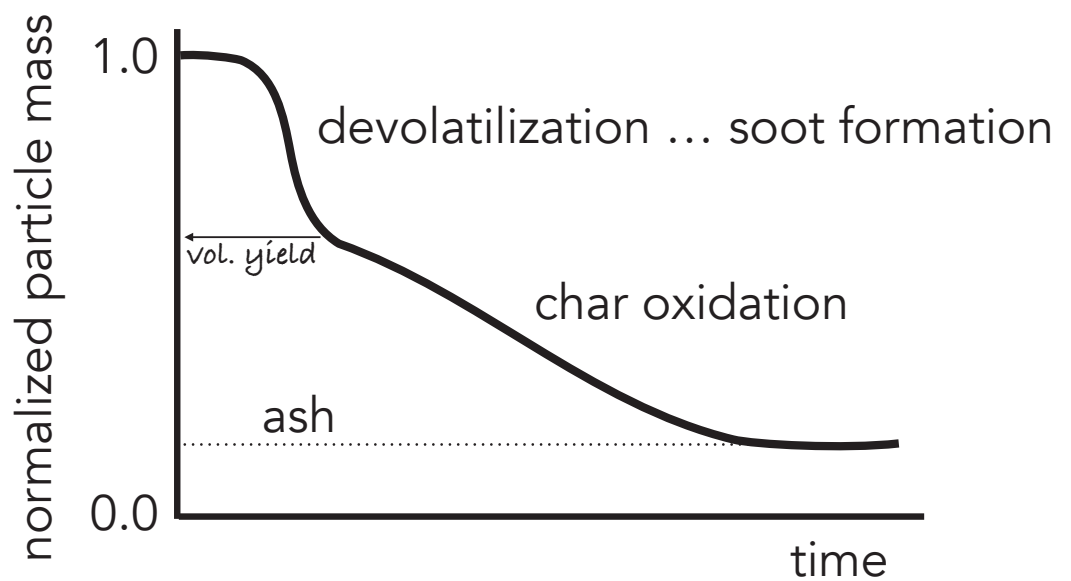


*concentration of large particles*

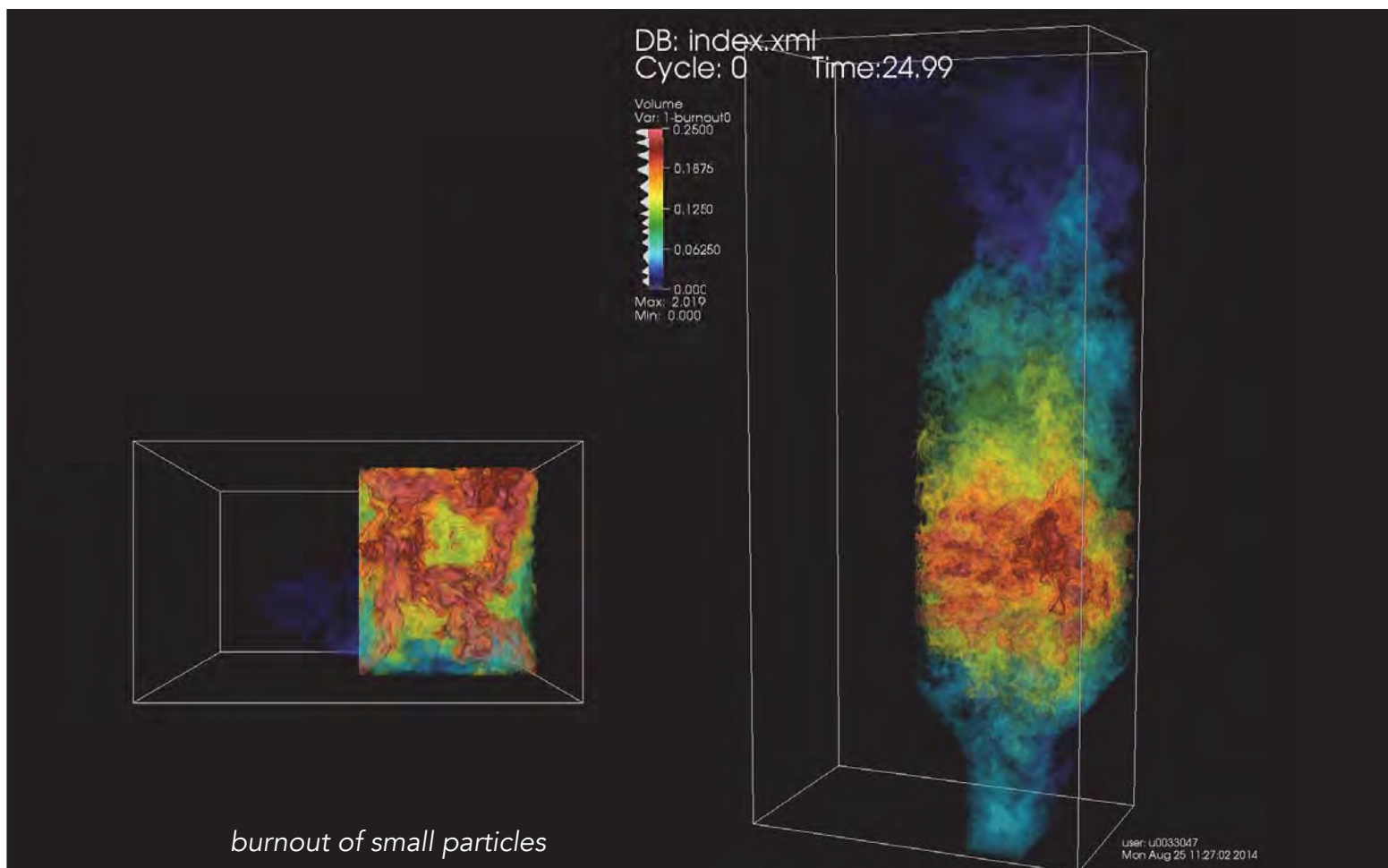
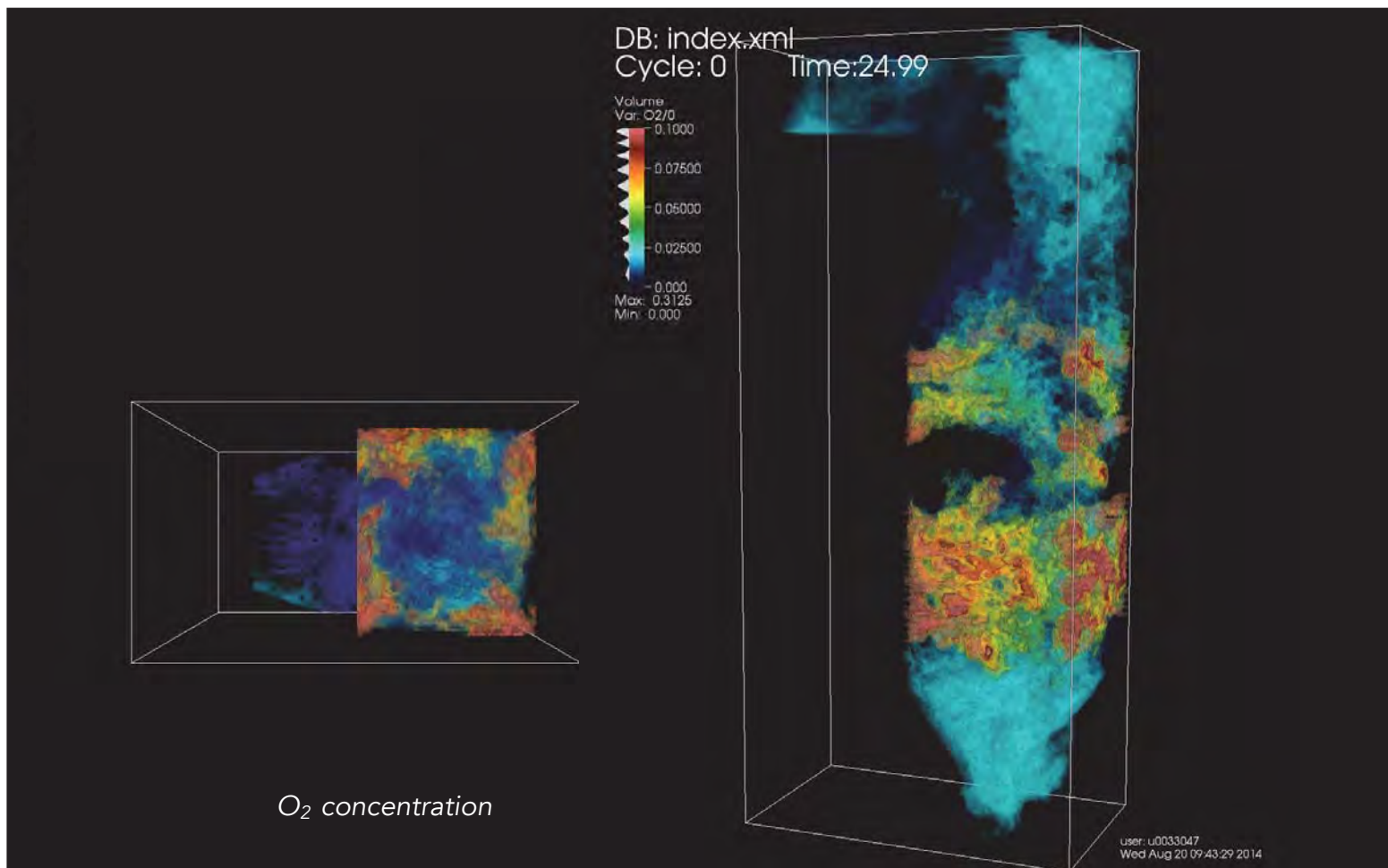


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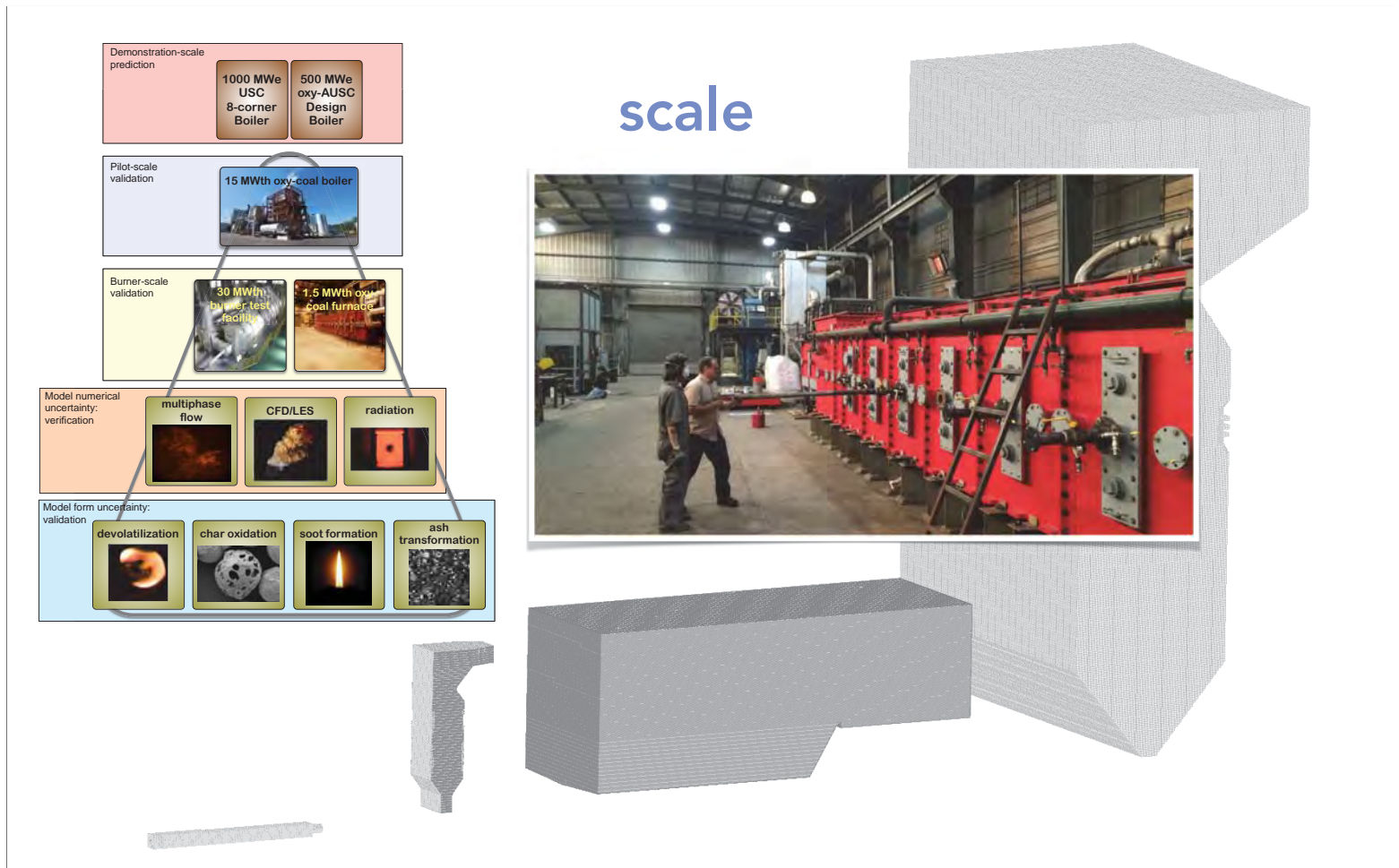
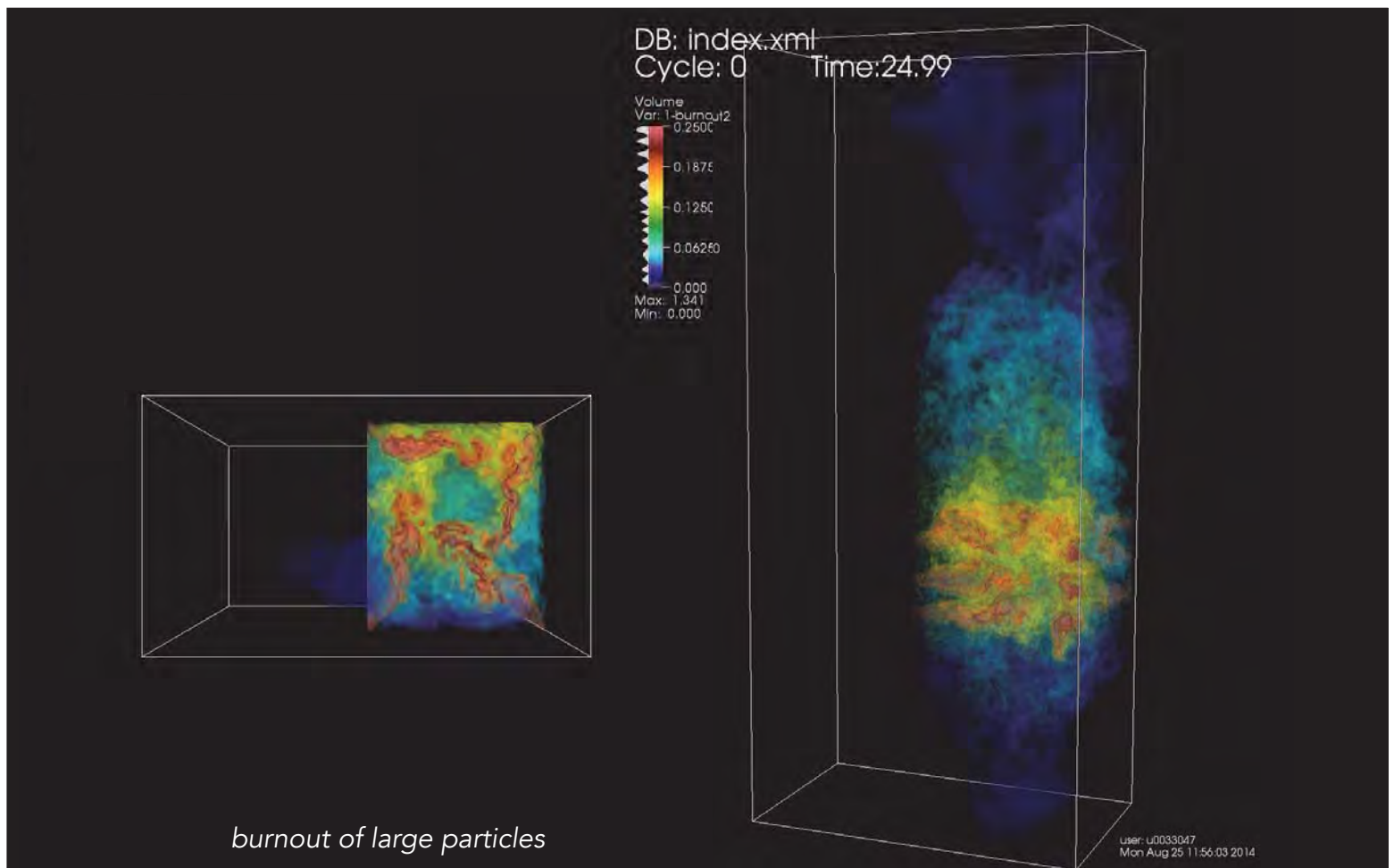
## particle transformations



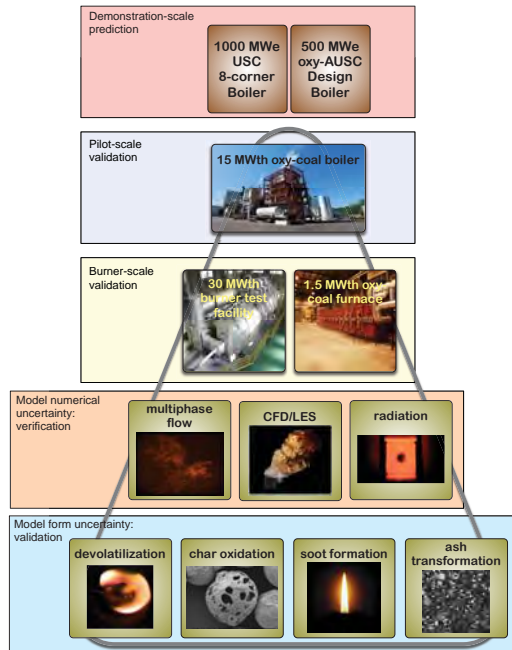
CARBON CAPTURE  
MULTIDISCIPLINARY  
SIMULATION CENTER



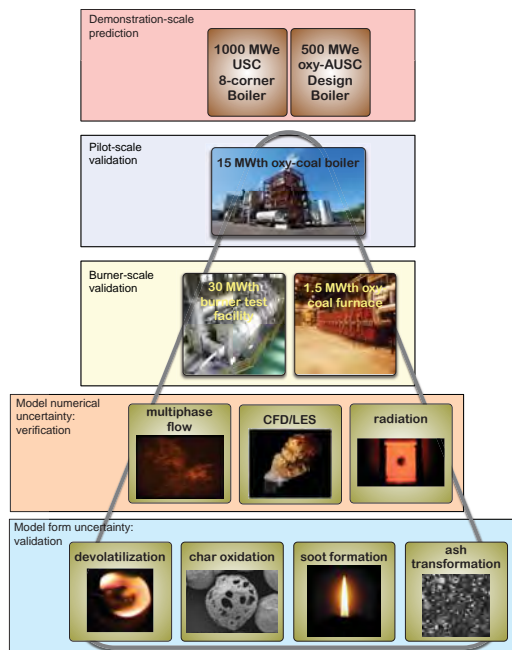




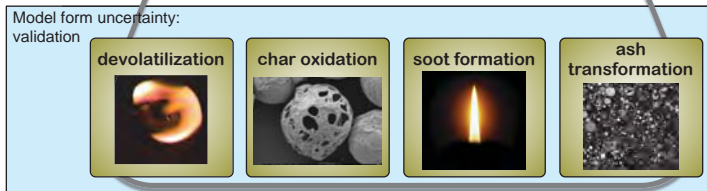
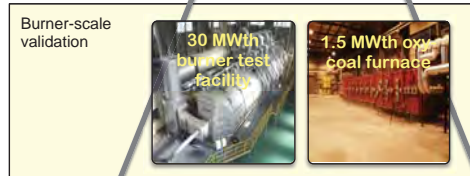
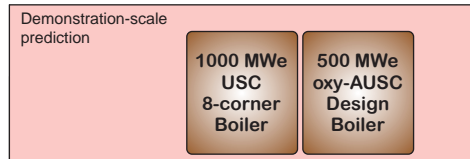
scale



scale







Provide evidence of predictivity

## objective predictivity



### PSAAPii:

- demonstrate positive societal impact of extreme computing
- accelerate the deployment of a new technology: high efficiency carbon capture for pulverized-coal power generation (AUSC oxy-coal)
- optimize state-of-the-art coal boilers for evolving energy market demands

### GE:

- Model Form Uncertainty
- Validation/UQ
  - expt. & sim. obtain predictivity

$$y_e = y_m(x) + \epsilon + b$$

$$\text{evidence: } |\mathbb{E}(b)| \leq \sigma_e \quad \forall e$$

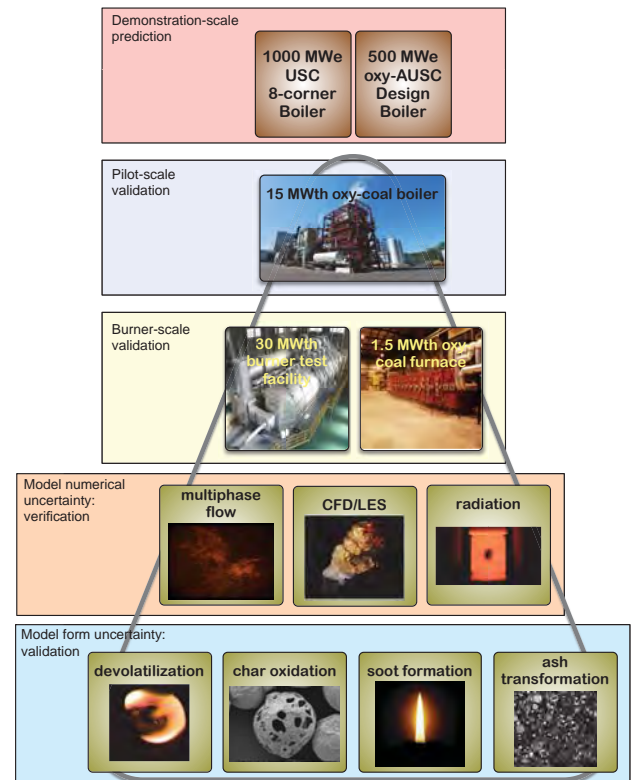
$$|\overline{y_m(\mathbf{x})} - \overline{y_e}| \leq \sigma_e$$

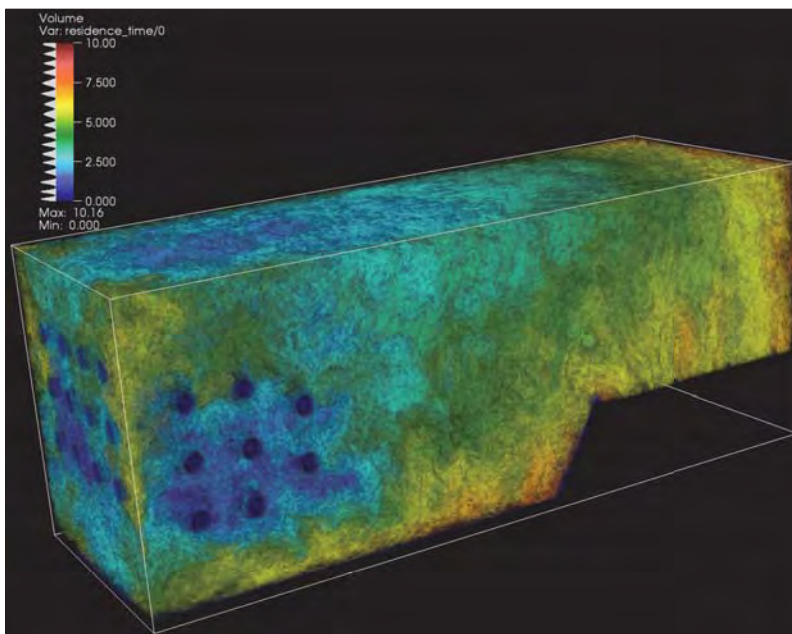
## predictive objective:

$\sigma_{y_p}$

- find  $\sigma_{x_m}$  for each brick to meet predictive objective:  $\sigma_{y_p}$
- find corresponding  $\mathbb{E}(x|y_e), \text{Var}(x|y_e)$
- find model form that is 'good enough'
- this is an inverse problem
- focus on intended use
- top-down UQ methodology:
  - given prediction constraint  $\sigma_{y_p}$
  - do inverse problem on prediction to find  $\sigma_{x_m}$
  - for each brick:
    - given  $\sigma_{x_m}$ , do forward problem to find  $\sigma_{min}$
    - if  $\sigma_{y_e} > \sigma_{min}$ : inadequate experiment
    - do inverse problem to find  $\mathbb{E}(x|y_e), \text{Var}(x|y_e)$
    - update model form until  $\sqrt{\text{Var}(x|y_e)} < \sigma_{x_m}$  for each  $e \in \mathcal{E}$

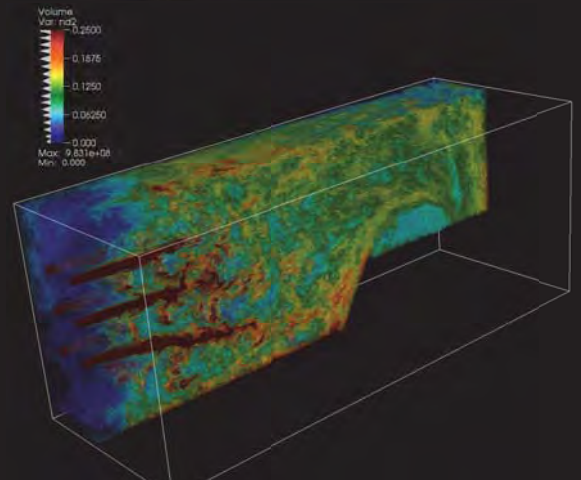
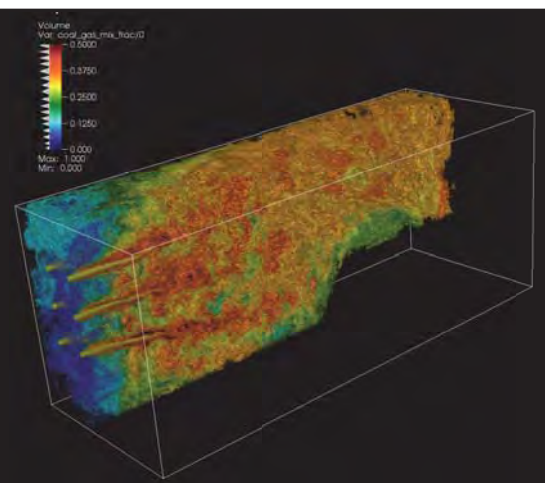
$$T_{midwall} \leq 870 \pm 15K$$





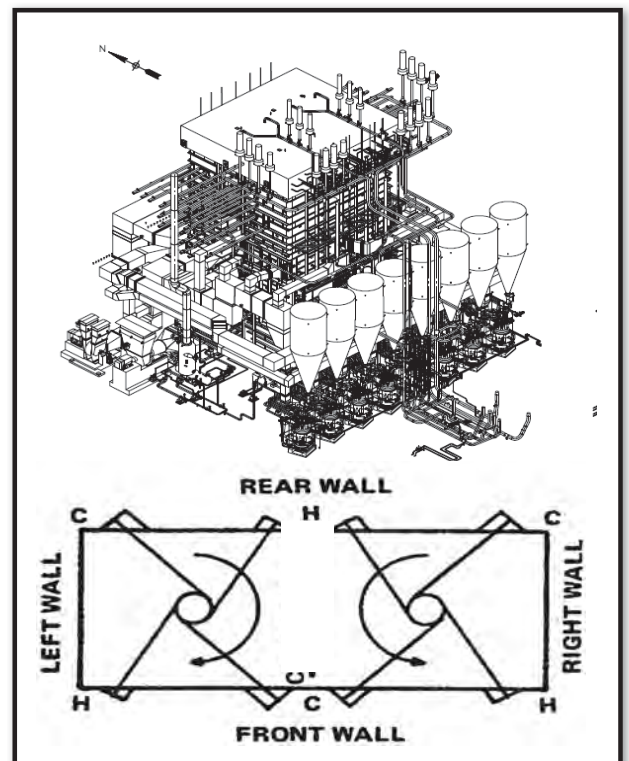
## Yr 2 DESIGN

250 million cells  
128K cores  
~35 million cpu hours  
2.5 cm resolution  
1.7 sec per timestep  
1.4e-4 second time step size



- GE/Alstom power “flagship” boiler
- USC
- 1090 MW ~ 1 million U.S. homes
- ~430 inlets
- height: ~65m
- cross section: ~35m x 15m
- feed: 130 kg/s of coal (100 train cars of coal per day)
- O<sub>2</sub> from 1000 kg/s of air
- division panels, platens, super-heaters and re-heater tubing ~210 miles of piping
- walls, and tubing made of 11 different metals with varying thickness

## 8-Corner Boiler

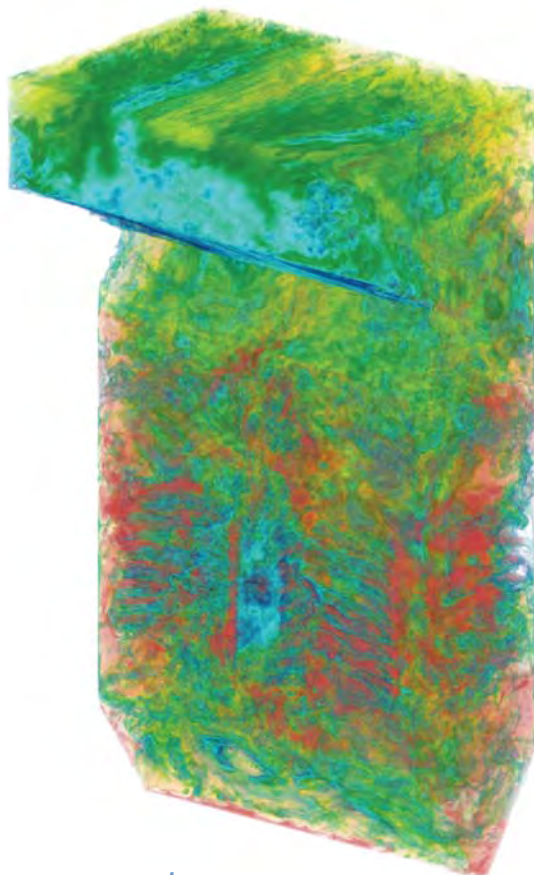




*8 corner geometry*

### *computational scale*

- ▶ 2016 INCITE Award
  - ▶ 350 million core hours
  - ▶ 4 simulations
- ▶ each simulation:
  - ▶ 256,000 cores
  - ▶ 23 days on MIRA
  - ▶ 8 days on TITAN
- ▶ resolution:
  - ▶  $2.5 \text{ cm}^3$
  - ▶ 2 billion cells



*large particles*

### *computational scale*

- ▶ 2016 INCITE Award
  - ▶ 350 million core hours
  - ▶ 4 simulations
- ▶ each simulation:
  - ▶ 256,000 cores
  - ▶ 23 days on MIRA
  - ▶ 8 days on TITAN
- ▶ resolution:
  - ▶  $2.5 \text{ cm}^3$
  - ▶ 2 billion cells



# Overview of Multiphase Flow Physics & Challenges at CCMT

S. Balachandar

**UF** UNIVERSITY of  
FLORIDA



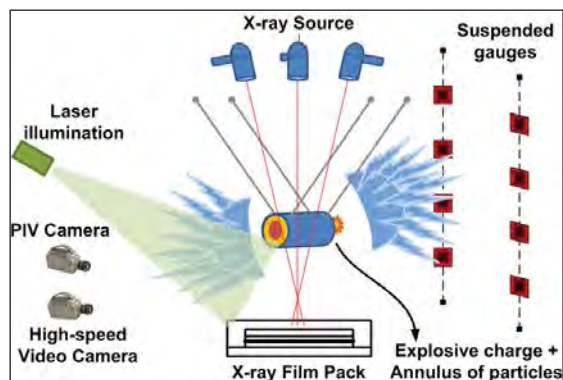
**NASA**

**UF** UNIVERSITY of  
FLORIDA

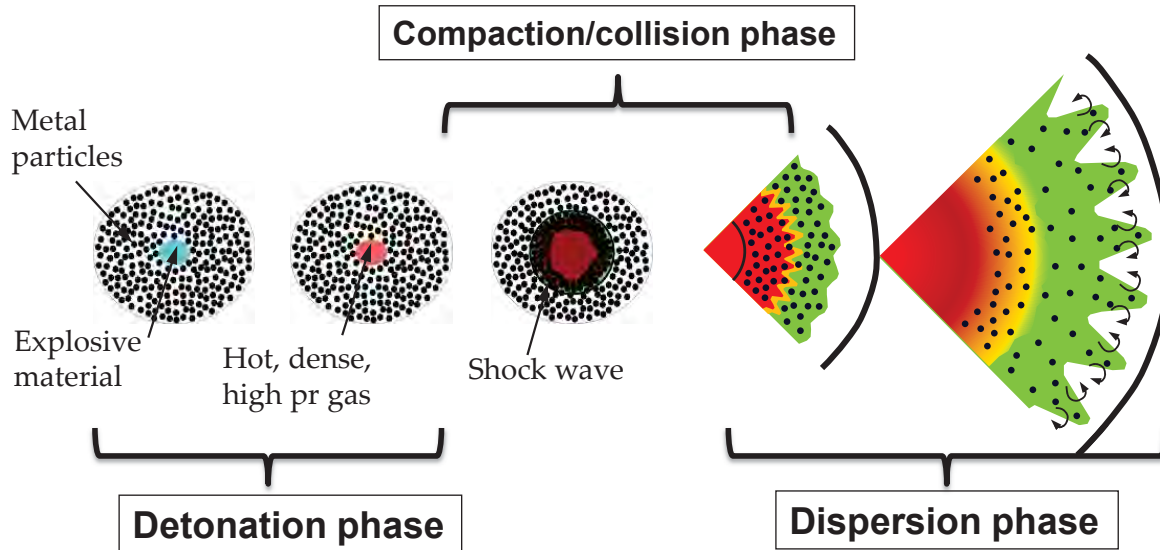
## Demonstration Experiment Configuration



From  
David Frost



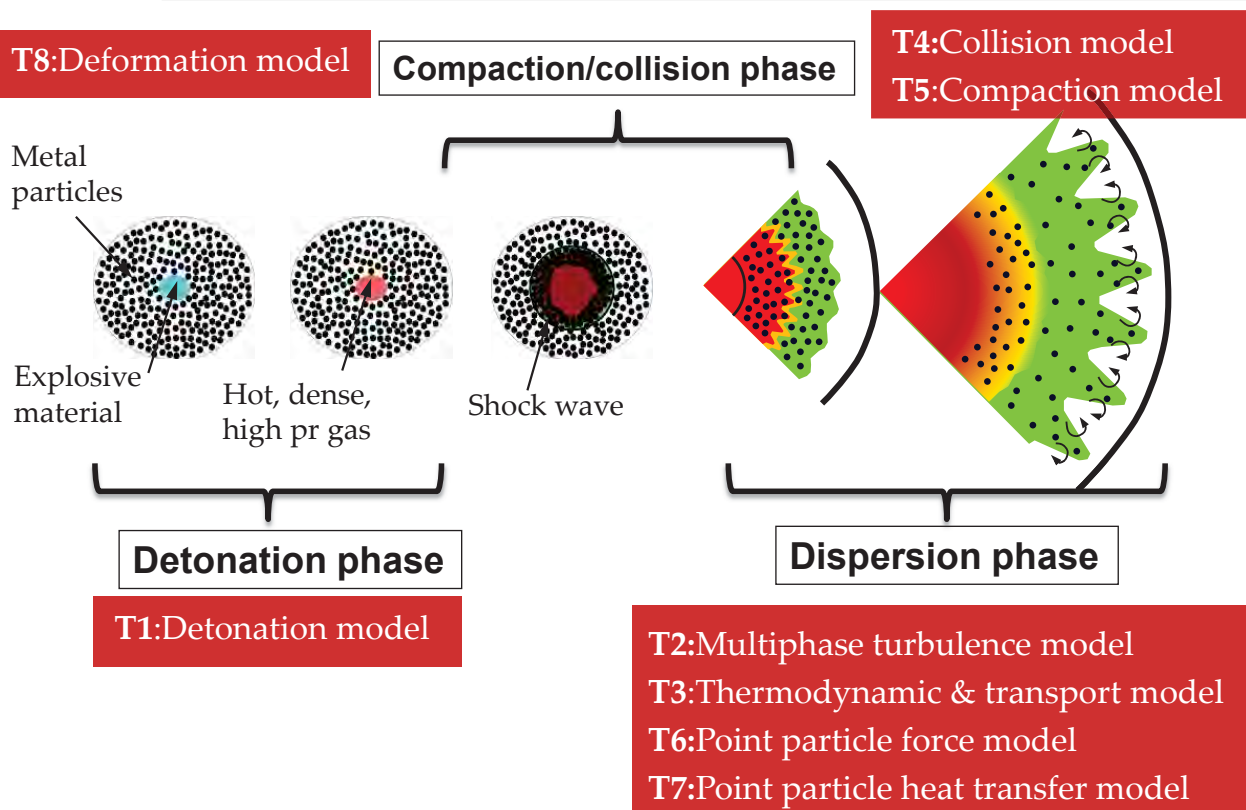
# Sequence of Events



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3

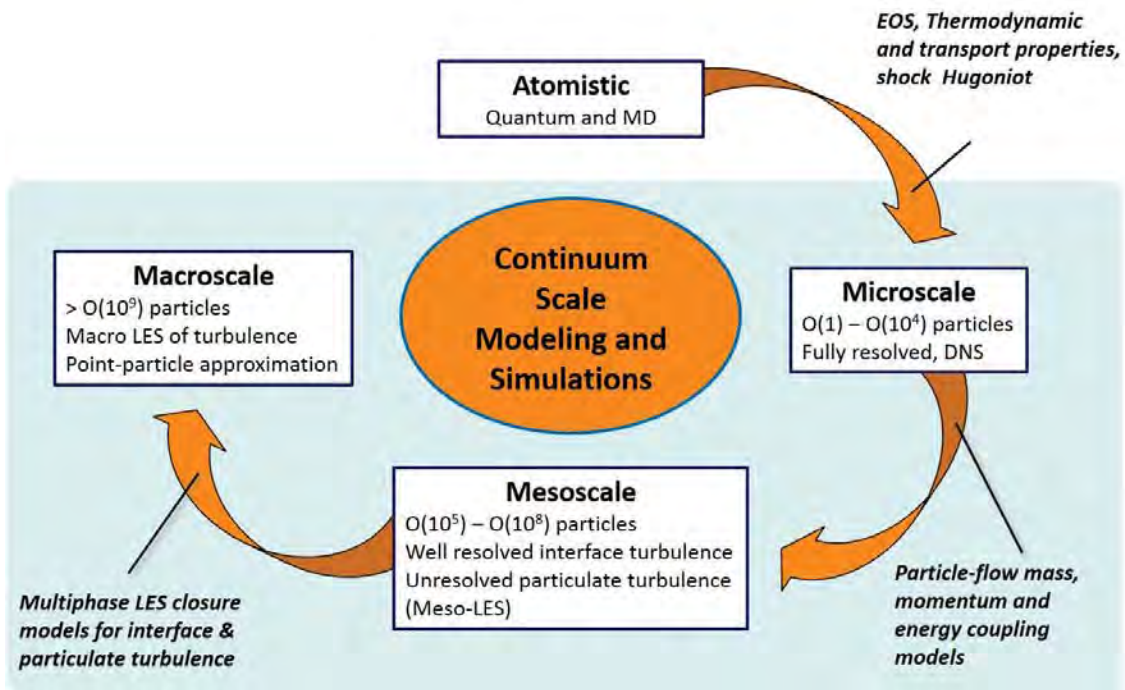
# Physical Models – Sources of Errors & Uncertainties



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4

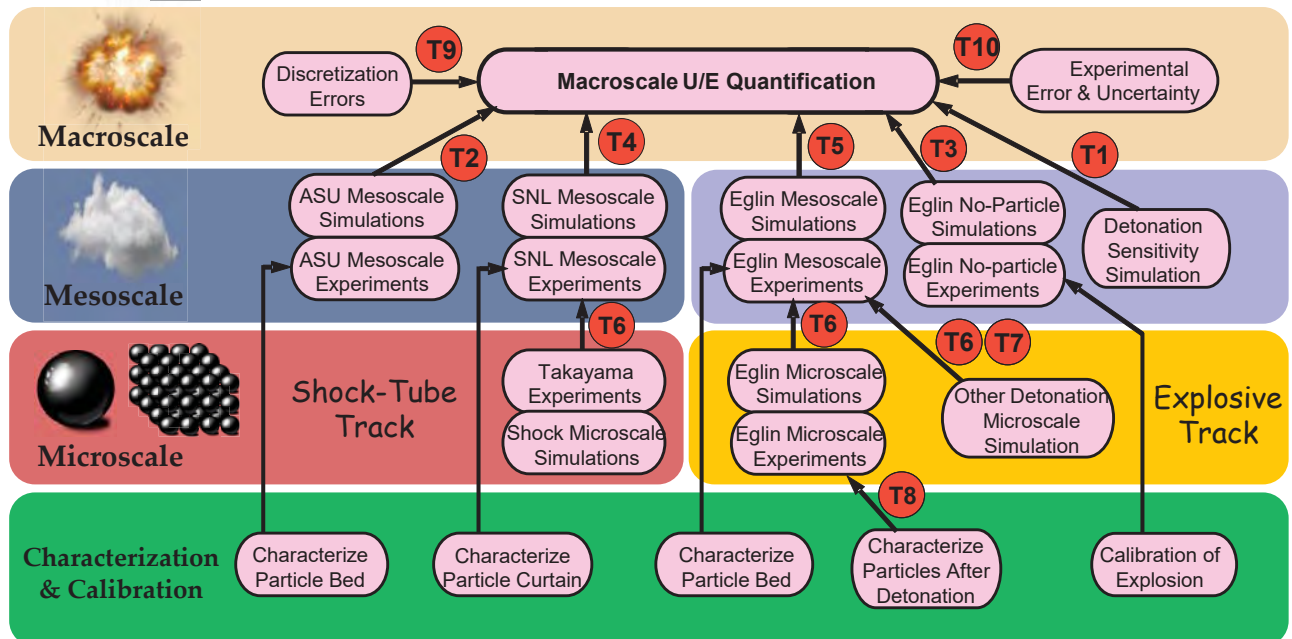
# Multiscale Integration Strategy



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5

# Uncertainty Budget – Overall Plan



- Integrates all the center activities
- Uncertainty reduction through iterative improvement

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6

# T1 to T9: Influence on Macro Simulation

- Gas equations

$$\frac{\partial \mathbf{U}_g}{\partial t} = \mathbf{G}_{inv} + \mathbf{G}_{vis} + \mathbf{G}_{turb} + \mathbf{f}_{gp} + \mathbf{f}_{ext}$$

Fluxes (T3)

Turbulence LES closure (T2)

$$\mathbf{U}_g = \begin{Bmatrix} \alpha_g \rho_g \\ \alpha_g \rho_g \mathbf{u}_g \\ \alpha_g \rho_g E_g \end{Bmatrix}$$

Detonation source (T1)

- $i^{th}$  Particle equations

$$\frac{d\mathbf{U}_{p,i}}{dt} = \mathbf{f}_{pp} - \mathbf{f}_{gp,i}$$

Point particle coupling (T6, T7)

$$\mathbf{U}_p = \begin{Bmatrix} \rho_p \\ \rho_p \mathbf{u}_p \\ \rho_p E_p \end{Bmatrix}$$

③ T9: Numerical errors

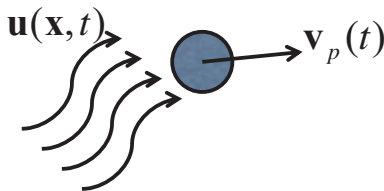
Collision & Compaction Model (T4, T5)

②

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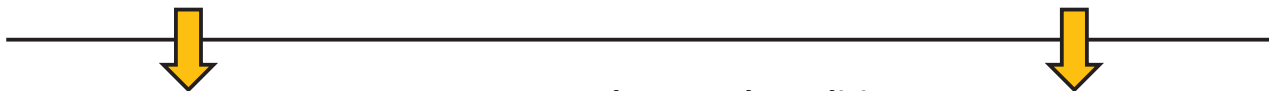
7

## Point-Particle Coupling Models



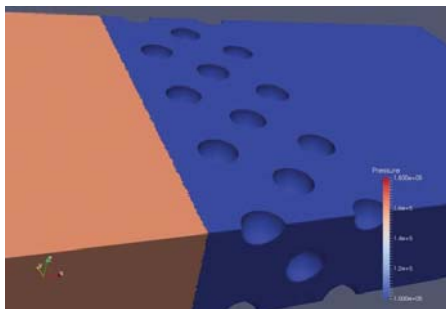
Commonly used models are based on:

- Uniform flow
- Quasi-steady flow
- Isolated particles
- Low Mach number, modest Reynolds number



But the actual conditions are:

- Strong non-uniform
- Highly unsteady
- Very large Mach and Reynolds numbers
- Particle-Particle interaction
  - Fluid-mediated
  - Direct collision



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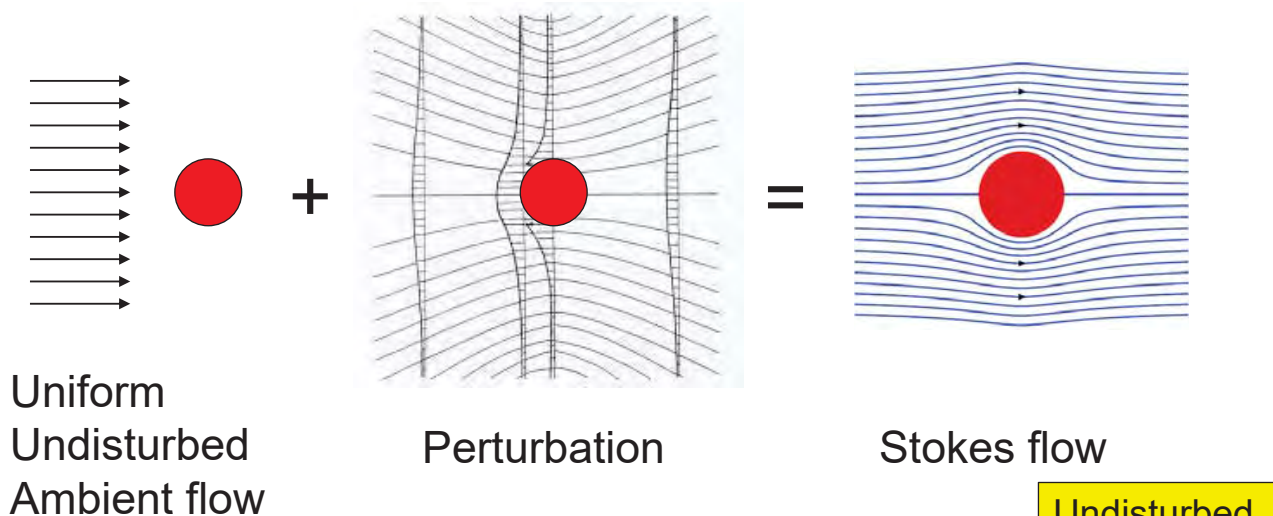


# Faxén Theorem (1924)

- Stokes flow ( $Re = 0$  limit)

$$\text{Stokes Drag} = 3\pi \mu d \bar{u}$$

Undisturbed flow at the particle



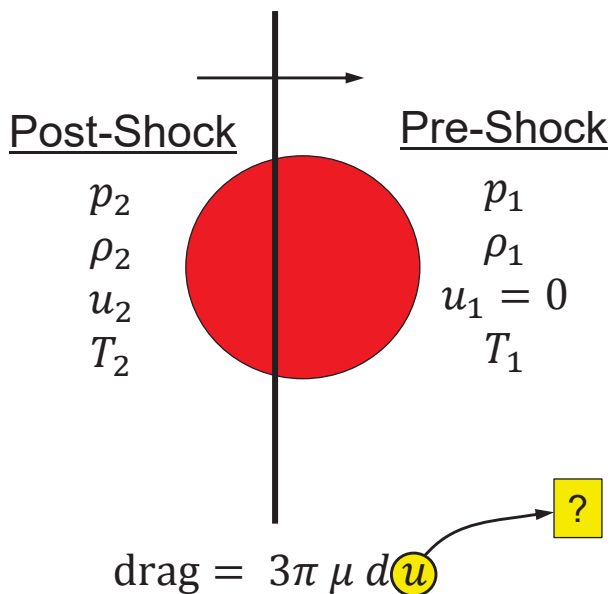
$$\text{Faxén Theorem: Drag} = 3\pi \mu d \bar{u}^S$$

Undisturbed flow

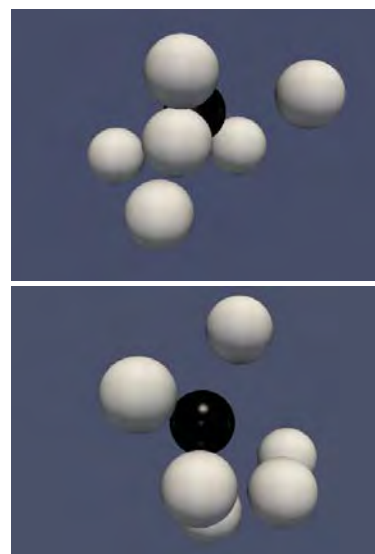
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# Why We Need Faxén Theorem

- Shock-particle interaction



- Flow-mediated particle-particle interaction



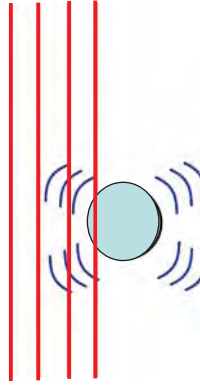
A generalized Faxén Theorem is needed at finite  $Re, Ma$

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# Generalized Faxén Theorem

- Expressed in terms of undisturbed flow seen by the particle
  - Can be any spatially varying flow ( shock thickness  $\ll d$  )
  - Can be rapidly varying flow
- Framework applies for force, heat transfer, pressure, volume, etc.

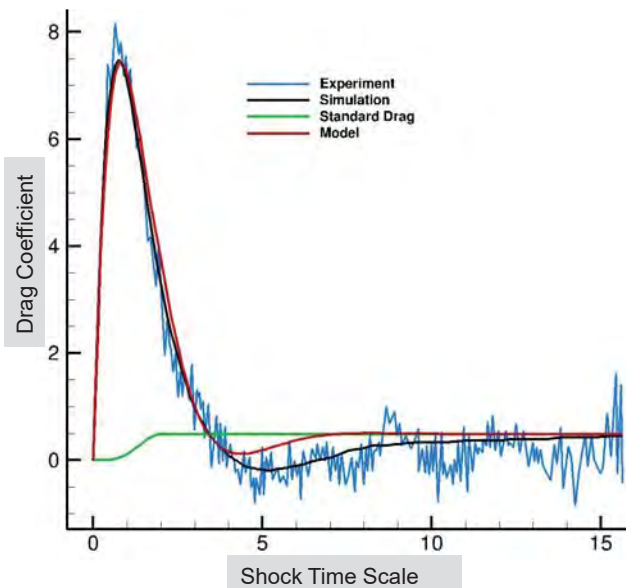
$$m_p \frac{d\mathbf{v}_p}{dt} = F_{gp,i} = V \overline{\rho} \frac{D\mathbf{u}}{Dt} + 3\pi\mu d \overline{\mathbf{u}}^S + V \int_{-\infty}^t (K_i + K_{vV}) \left( \frac{D\overline{\rho\mathbf{u}}_r^S}{Dt} \right) d\xi + V \int_{-\infty}^t K_{vS} \left( \frac{D\overline{\rho\mathbf{u}}^S}{Dt} \right) d\xi$$


The diagram shows a circular particle in a flow field. A shock wave, represented by four vertical red lines, is moving from left to right. Blue curved lines around the particle represent the flow field. The particle is positioned to the right of the shock.

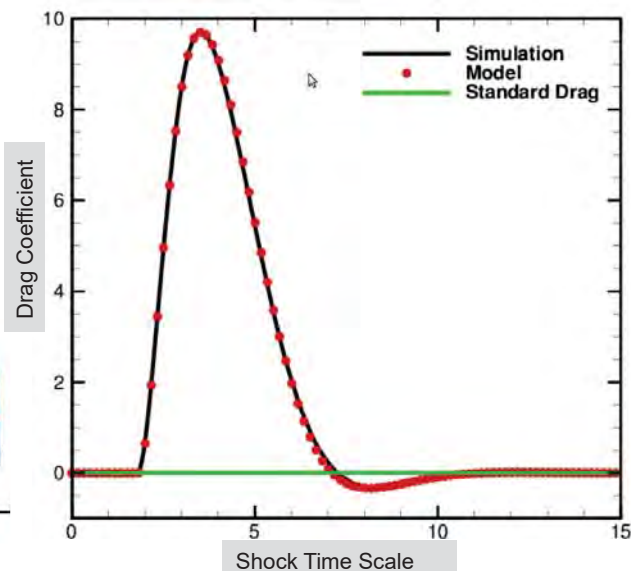
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# Point-Particle Models for Single Particle



- Air Shock over a Particle ( $M_s = 1.22$ )  
(viscous simulation)



- Fan-Particle Interaction ( $M_s = 1.22$ )  
(inviscid simulation)

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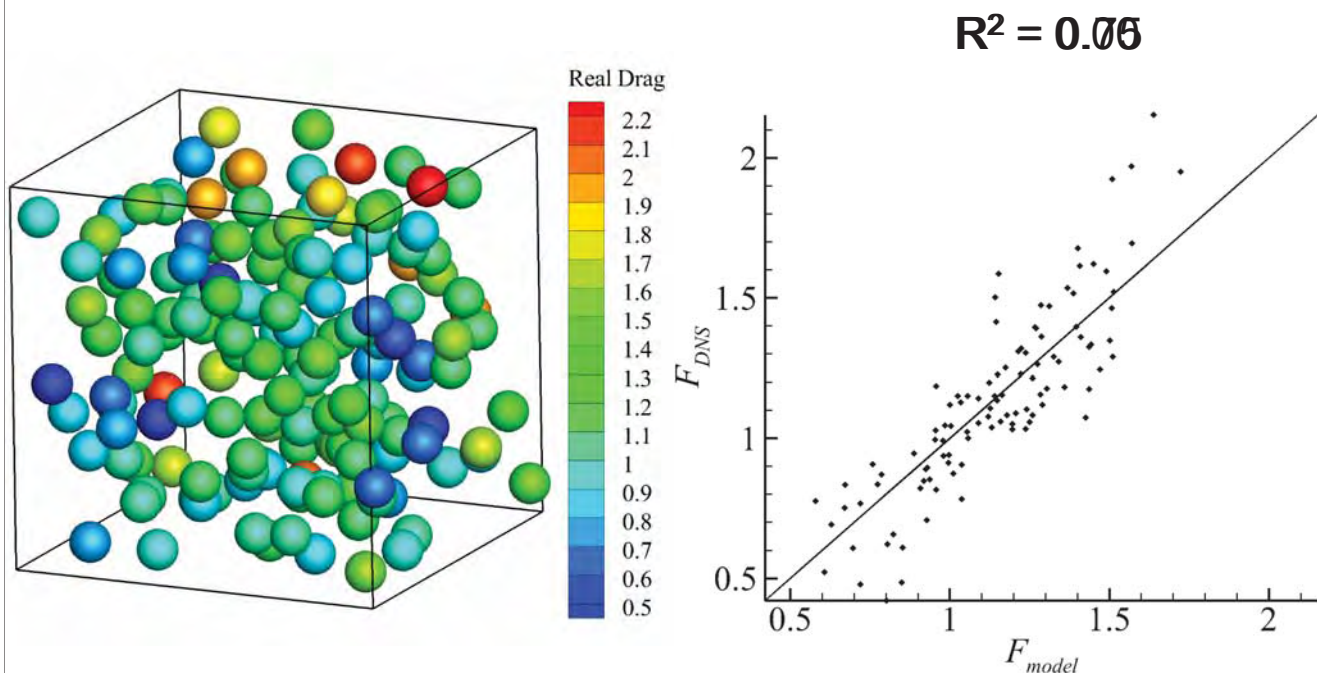
12

- The best single particle point-particle (PP) force model
- Beyond force model (single particle)
  - Particle pressure, volume and shape
  - Particle temperature
- Beyond single particle (PIEP Model)
  - Systematic inclusion of neighbor effects
    - Microscale velocity fluctuations
    - For inter-particle collisions, Reynolds stress closure, etc.

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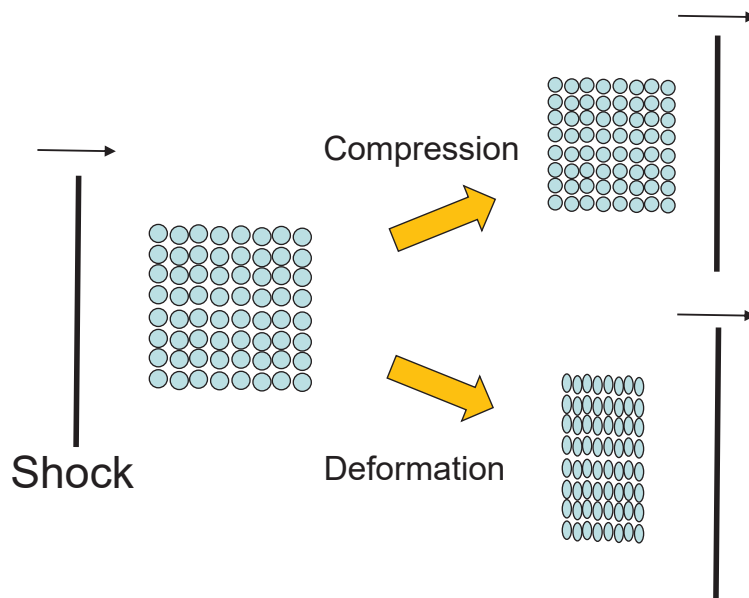
13

## PIEP Model vs DNS (drag)



$$F_{model} = \langle F \rangle + F_{PIEP}$$

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- Particle-particle-contact (PPC) drives a faster shock through the bed
- Gas-pressure and PPC-forces contribute to compression & deformation
- Both compression & deformation contribute to volume fraction evolution

CCMT (or compaction)

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- Compaction equation (Dense limit: Baer-Nunziato model)

$$\frac{\partial \alpha_p}{\partial t} + \mathbf{u}_i \cdot \nabla \alpha_p = \frac{1}{\mu} (p_p - p_g)$$

- Volume fraction equation (Dilute incompressible limit)

$$\frac{\partial \alpha_p}{\partial t} + \nabla \cdot (\mathbf{u}_p \alpha_p) = 0$$

- Questions:

- Can we obtain Compaction Equation from first principle?
- How to smoothly transition from one limit to the other?
- What is the appropriate interfacial velocity,  $\mathbf{u}_i$ ?
- How do we implement compaction in Euler-Lagrange framework?

# Compaction Phase Model

- Compaction equation (Dense limit: Baer-Nunziato model)

$$\frac{\partial \alpha_p}{\partial t} + \mathbf{u}_i \cdot \nabla \alpha_p = \frac{1}{\mu} (p_p - p_g)$$

- Volume fraction equation (Dilute incompressible limit)

$$\frac{\partial \alpha_p}{\partial t} + \nabla \cdot (\mathbf{u}_p \alpha_p) = 0$$

- Pressure evolution equation (like BBO or MRG eqn):

$$\frac{dp_p}{dt} = -\frac{1}{\tilde{\mu}} (p_p - p_g) + \frac{dp_g}{dt} + \dots$$

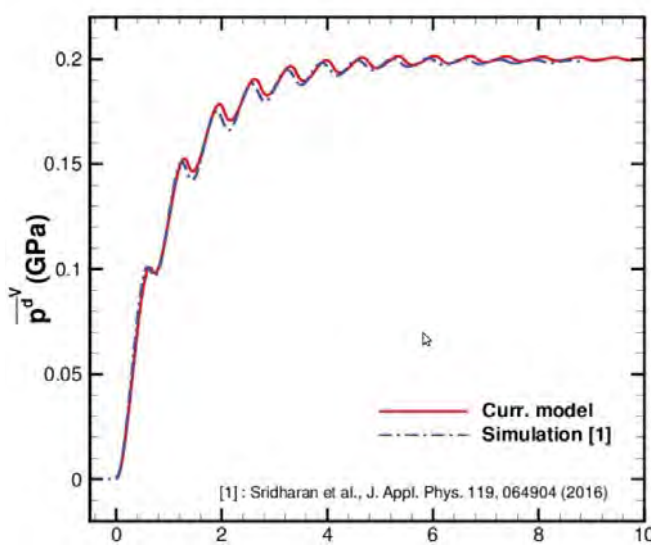
- Equations for particle volume or density can be derived
- Accounts for ONLY gas-induced compression effect

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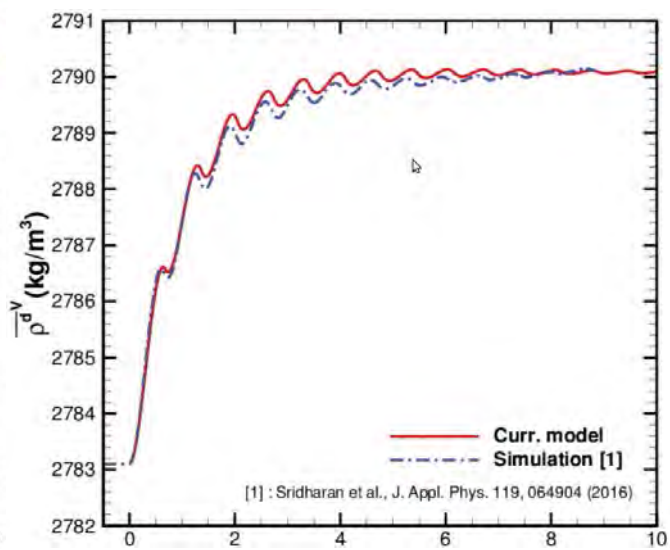
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# Pressure Evolution Equation

- 0.2 GPa NM shock over a deformable Al Particle



Mean pressure evolution



Particle density evolution

Need to include PPC-force and deformation effects

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- Key question: How do you allow shock waves faster than particle velocity to propagate by inter-particle contact?
- Answer: Introduce particle-contact pressure; Eigen-decompose the Lagrangian particle equations of motion

$$\begin{aligned} \frac{du_{p,i}}{dt} = & -\frac{p_p}{2\alpha_p\rho_p} \left[ \frac{\partial\alpha_p}{\partial x} \Big|_R + \frac{\partial\alpha_p}{\partial x} \Big|_L \right] + \frac{a_p}{2} \left[ \frac{\partial u_p}{\partial x} \Big|_R - \frac{\partial u_p}{\partial x} \Big|_L \right] \\ & - \frac{1}{2\rho_p} \left[ \frac{\partial p_p}{\partial x} \Big|_R + \frac{\partial p_p}{\partial x} \Big|_L \right] \\ & + \text{gas forces} \end{aligned}$$

- Particle pressure takes care of collisional forces as well

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- Key question: How do you allow shock waves faster than particle velocity to propagate by inter-particle contact?
- Answer: Introduce particle-contact pressure; Eigen-decompose the Lagrangian particle equations of motion

$$\begin{aligned} \frac{du_{p,i}}{dt} = & -\frac{p_p}{2\alpha_p\rho_p} \left[ \frac{\partial\alpha_p}{\partial x} \Big|_R + \frac{\partial\alpha_p}{\partial x} \Big|_L \right] + \frac{a_p}{2} \left[ \frac{\partial u_p}{\partial x} \Big|_R - \frac{\partial u_p}{\partial x} \Big|_L \right] \\ & - \frac{1}{2\rho_p} \left[ \frac{\partial p_p}{\partial x} \Big|_R + \frac{\partial p_p}{\partial x} \Big|_L \right] \\ & + \text{gas forces} \end{aligned}$$

- Particle-contact pressure takes care of collisional forces as well

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- Propagating particle fronts tend to diffuse
  - The diffusion coefficient can be quite large
  - Depends on volume fraction
- Diffusion is due to correlated velocity fluctuation
  - Collision-induced velocity fluctuation
  - Gas-mediated particle-particle interaction
  - Also due to number density fluctuation
- Can be modeled as diffusional velocity

$$-\mathcal{D} \frac{\nabla \alpha_p}{\alpha_p}$$

where

$$\mathcal{D} \sim 7.3 d_p |\mathbf{u}_r| f(\alpha_p)$$

Segre *et al.*, Nature, 2001  
Yu *et al.*, JGR, 2014

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- Rigorous convergence of Euler-Lagrange simulations is elusive
- Grid size = O(particle size) is problematic
- Grid to particle interpolation is not a problem
  - Higher order accuracy can be achieved
- Particle to grid injection or projection is problematic
  - Higher order accuracy is not easy
  - Is it a stochastic or a deterministic particle
- Particle number density naturally forms cell-to-cell fluctuation
  - How to obtain reasonable smooth particle volume fraction
  - How many particles per cell is optimal?
  - Dynamic load balancing

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# *Do you have any questions?*



## Sources of Errors & Uncertainties

- T1: Detonation modeling
- T2: Multiphase turbulence modeling
- T3: Thermodynamics & transport properties

②

- T4: Particle-particle collision modeling
- T5: Compaction modeling (dense-to-dilute transition)

①

- T6: Point-particle force modeling
- T7: Point-particle thermal modeling

- T8: Particle deformation and other complex physics

③

- T9: Discretization and numerical approximation errors

- T10: Experimental and measurement errors & uncertainties

Advance state-of-the-art in multiphase numerics and point-particle models

# Equations and Closures for Deformation and Flow of Continuous and Disperse Materials

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Oct. 6 - 7, 2016 , St. Petersburg, FL 33716



LA-UR-16-27348

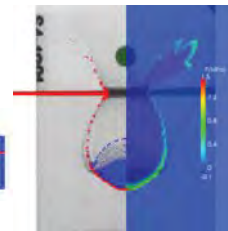
1/11

## Transitions between Continuous and Disperse Phases

Transitions between disperse and continuous states are common phenomena. Often these transitions occur in the presence of other materials, such as air or water.



Disperse to continuous



Continuous to disperse

How to build a numerical tool to model these transition process?

Two issues: **model equations** and **numerical methods**.

Zhang & Jayaraman, (2013), Int. J. Multiphase Flow. **56** pp 149 - 159.

Long et al. (2016) Comput. Methods Appl. Mech. Engrg. **300** pp 611-627.



LA-UR-16-27348

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## Averaging Methods

Most of these processes are stochastic and multiscale. **Volume averaging** method are often used.

- What is the size of the representative volume element (RVE)? Does such a size exist?
- Does the RVE size change with time (because of e.g. crack growth or reduction of particles sizes due to burning)?
- How to related the closure quantities to physics at lower length and time scales? (A lot of “effective” quantities are used).

**Ensemble averaging** methods have been used to derive the averaged equations.

- No need for RVE.
- More directly related to physics at lower length and time scales. Less human interpretation of physics.

## Ensemble Phase Average

For phase  $i$ , let  $C_i$  be the phase indicator function:

$$C_i(\mathbf{x}, t, R) = \begin{cases} 1 & \text{if } \mathbf{x} \in \text{phase } i \text{ in realization } R \\ 0 & \text{otherwise} \end{cases}$$

Let  $\mathcal{P}$  be the probability measure for realizations.

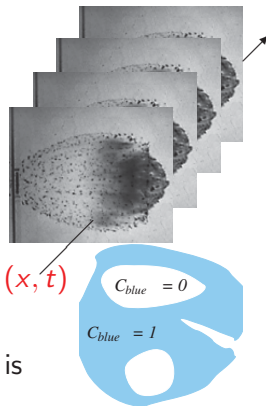
$$\text{Volume fraction: } \theta_i(\mathbf{x}, t) = \int C_i(\mathbf{x}, t) d\mathcal{P}$$

For any quantity  $q_i$  pertaining to phase  $i$ , its average is

$$\langle q_i \rangle(\mathbf{x}, t) = \frac{1}{\theta_i(\mathbf{x}, t)} \int q_i(\mathbf{x}, t, R) C_i(\mathbf{x}, t, R) d\mathcal{P}, \quad \nabla \langle q_i \rangle \neq \langle \nabla q_i \rangle$$

With this average we have the general transport equation:

$$\frac{\partial}{\partial t}(\theta_i \langle q_i \rangle) + \nabla \cdot (\theta_i \langle \mathbf{v}_i q_i \rangle) = \theta_i \left\langle \frac{\partial q_i}{\partial t} + \nabla \cdot (\mathbf{v}_i q_i) \right\rangle + \int \dot{C}_i q_i d\mathcal{P}.$$



## Kinematics

$$q_i = 1 : \quad \frac{\partial \theta_i}{\partial t} + \nabla \cdot (\theta_i \langle \mathbf{v}_i \rangle) = \theta_i \langle \nabla \cdot \mathbf{v}_i \rangle + \int \dot{C}_i d\mathcal{P}.$$

$$q_i = \rho_i^0 : \quad \frac{\partial}{\partial t} (\theta_i \langle \rho_i^0 \rangle) + \nabla \cdot (\theta_i \langle \mathbf{v}_i \rho_i^0 \rangle) = \int \dot{C}_i \rho_i^0 d\mathcal{P}.$$

Using  $\tilde{\mathbf{v}}_i = \langle \mathbf{v}_i \rangle + \langle \mathbf{v}_i' \rho_i^0 \rangle / \langle \rho_i^0 \rangle$ , we define  $d/dt = \partial/\partial t + \tilde{\mathbf{v}}_i \cdot \nabla$ .

Without phase change,

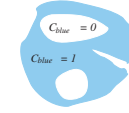
$$\frac{d\theta_i}{dt} = \theta_i (\langle \nabla \cdot \mathbf{v}_i \rangle - \nabla \cdot \langle \mathbf{v}_i \rangle) = - \int (\mathbf{v}_i - \langle \mathbf{v}_i \rangle) \cdot \nabla C_i d\mathcal{P},$$

$$\frac{d\langle \rho_i^0 \rangle}{dt} = - \langle \rho_i^0 \rangle \langle \nabla \cdot \mathbf{v}_i \rangle.$$

For a solid material, such as metal, let  $\theta_v = 1 - \theta_s$  be the porosity

$$\nabla \cdot \langle \mathbf{v}_s \rangle = - \frac{1}{\rho_s^0} \frac{d\rho_s^0}{dt} + \frac{1}{1 - \theta_v} \frac{d\theta_v}{dt}.$$

elastic                  plastic



The elastic part causes density and pressure changes. The plastic part is associated with void growth — essential part of the tension plasticity (TEPLA) model for ductile metals.

## Momentum Equation

With  $q_i = \rho_i^0 \mathbf{v}_i$  in the transport equation, after some manipulations we have the momentum equation:

$$\theta_i \langle \rho_i^0 \rangle \frac{d\tilde{\mathbf{u}}_i}{dt} = \theta_i \nabla \cdot \boldsymbol{\sigma}_{Ai} + \nabla \cdot [\theta_i (\langle \boldsymbol{\sigma}_i \rangle - \boldsymbol{\sigma}_{Ai})] + \nabla \cdot (\theta_i \boldsymbol{\sigma}_i^{Re}) + \mathbf{f}_i.$$

$$\mathbf{f}_i = - \int (\boldsymbol{\sigma}_i - \boldsymbol{\sigma}_{Ai}) \cdot \nabla C_i d\mathcal{P},$$

where  $\mathbf{f}_i$  is the interfacial force, and  $\boldsymbol{\sigma}_{Ai}$  is an auxiliary stress.

Different fields or authors use different auxiliary stresses:

- For Rayleigh-Taylor mixing,  $\boldsymbol{\sigma}_{Ai} = \mathbf{0}$
- For soil mechanics or flows in porous media,  $\boldsymbol{\sigma}_{Ai} = -\langle p_i \rangle \mathbf{I}$ .
- For most disperse multiphase flows,  $\boldsymbol{\sigma}_{Ai} = \langle \boldsymbol{\sigma}_{fluid} \rangle$ .

All choices are legitimate as long as they are consistent in all the places. However, this consistence is **never** considered in current models.

## Stresses in Solid

Let  $\sigma_{Ai} = -\langle p_i \rangle \mathbf{I}$ . Using  $\sigma_{sij} = \partial_k (r_i \sigma_{skj}) - r_i \partial_k \sigma_{skj}$ , we have

$$\theta_s \langle \sigma_s \rangle = \sum_{g=1}^N P_g(\mathbf{x}, t) \left\langle \int_{S_{gf}+S_{gg}} \mathbf{r} \mathbf{n} \cdot \sigma_s dS \right\rangle_g + O(\ell^2/L^2),$$

$S_{gf}$ : grain-fluid interface.

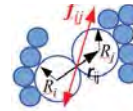
$S_{gg}$ : the grain-grain contact surface. On  $S_{gg}$ ,  $\mathbf{n} \cdot \sigma_s dS = \mathbf{f}_{gg'}$ .

For isotropic grain structures, on  $S_{gf}$ ,  $\mathbf{n} \cdot \sigma_s = -\langle p_f \rangle \mathbf{n}$ ,

$$\sum_{g=1}^N P_g(\mathbf{x}, t) \left\langle \int_{S_{gf}} \mathbf{r} \mathbf{n} dS \right\rangle_g = \alpha \theta_s \mathbf{I}.$$

$$\theta_s \langle \sigma_s \rangle = \sum_{g=1}^N P_g(\mathbf{x}, t) \sum_{g'=1}^N \mathbf{r} \mathbf{f}_{gg'} - \alpha \theta_s \langle p_f \rangle \mathbf{I}.$$

Skeleton stress  $\sigma_{sk}$



$\theta_i (\langle \sigma_i \rangle - \sigma_{Ai}) = \sigma_{sk} + (1 - \alpha) \theta_s \langle p_f \rangle \mathbf{I}$ , — a common stress in soil mechanics.

## Scale Separation and Closures

### Small particle approximation

$$\mathbf{f}_c(\mathbf{x}, t) = - \int (\sigma_c - \langle \sigma_c \rangle) \cdot \nabla C_c d\mathcal{P},$$

$$\mathbf{f}_d(\mathbf{x}, t) = n(\mathbf{x}, t) \int (\langle \sigma_c \rangle_1 - \langle \sigma_c \rangle) \cdot \mathbf{n} dS,$$

$$\mathbf{T}_c(\mathbf{x}, t) = n \int \mathbf{n} \cdot (\langle \sigma_c \rangle_1 - \langle \sigma_c \rangle) \mathbf{n} dS,$$

$$\mathbf{T}_c(\mathbf{x}, t) = n \int \mathbf{n} \cdot (\langle \sigma_c \rangle_1 - \langle \sigma_c \rangle) \mathbf{n} dS,$$

$$\mathbf{f}_c = -\mathbf{f}_d + \nabla \cdot (\theta_d \mathbf{T}_c)$$

The small particle approximation provides an example of the capability or easiness of using the ensemble phase averaging method to connect macroscopic closures to interactions at lower scales.

For dilute Stokes flow:

$$\mathbf{f}_d = \frac{9}{2a^2} \mu_c (\tilde{\mathbf{v}}_d - \tilde{\mathbf{v}}_c) + \frac{3}{4} \mu_c \nabla^2 \tilde{\mathbf{v}}_c$$

$$\mathbf{T}_c = 3\mu_c \dot{\epsilon}_c, \quad \dot{\epsilon}_c = \frac{\dot{\epsilon}_m}{1 - \theta_d}$$

The **mixture** stress:

$$\langle \sigma_c \rangle + \theta_d \mathbf{T} = 2\mu_c \left( 1 + \frac{5}{2} \theta_d \right) \dot{\epsilon}_m$$

For dilute potential flow:

$$\mathbf{f}_d = \frac{\langle \rho_c \rangle}{2} \left[ \frac{d\tilde{\mathbf{v}}_c}{dt} - \frac{d\tilde{\mathbf{v}}_d}{dt} + \frac{\nabla \cdot (\theta_d \langle \mathbf{v}'_d \mathbf{v}'_d \rangle)}{\theta_d} \right]$$

$$\mathbf{T} \propto (\tilde{\mathbf{v}}_d - \tilde{\mathbf{v}}_c)^2.$$

Zhang and Prosperetti, (1994) J. Fluid Mech. **267**, pp 185

Irving and Kirkwood, (1950) J. Chem. Phys. **18**, pp 817

Zhang and Prosperetti, (1997) Int. J. Multiphase Flow. **23**, pp 425

Zhang et. al (2002) PRE. **66**, pp 051806

## Commonly Used Models

$$\theta_i \langle \rho_i^0 \rangle \frac{d\tilde{\mathbf{u}}_i}{dt} = \theta_i \nabla \cdot \boldsymbol{\sigma}_{Ai} + \nabla \cdot [\theta_i (\langle \boldsymbol{\sigma}_i \rangle - \boldsymbol{\sigma}_{Ai})] + \nabla \cdot (\theta_i \boldsymbol{\sigma}_i^{Re}) + \mathbf{f}_i.$$

$$\mathbf{f}_i = - \int (\boldsymbol{\sigma}_i - \boldsymbol{\sigma}_{Ai}) \cdot \nabla C_i d\mathcal{P},$$

Theoretically  $\boldsymbol{\sigma}_{Ai}$  needs to be consistent in these equations. In typical engineering calculation we use

$$\mathbf{f}_d = -\mathbf{f}_c = \text{drag} + \text{added mass force} + \text{Basset force}, \quad \mathbf{T} = \mathbf{0}$$

each of these forces are modeled separately, without consideration of  $\boldsymbol{\sigma}_{Ai}$ . The stress difference  $\langle \boldsymbol{\sigma}_i \rangle - \boldsymbol{\sigma}_{Ai}$  is often ignored.

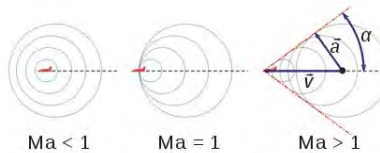
### Unanswered questions:

- Are these forces independent? For instance, why added mass is independent of viscosity? How about buoyancy?
- How  $\mathbf{T}$  affects the solution?
- $\boldsymbol{\sigma}_{Ai} = -p_c \mathbf{I}$  or  $\langle \boldsymbol{\sigma}_c \rangle$ ? (e.g. viscous effects?)
- Should the effective or normal viscosity be used in  $\langle \boldsymbol{\sigma}_c \rangle$ ?

## Some Possible “Low Hanging Fruits”

Most, if not all, the models for phase interaction forces are for incompressible flows, in which any perturbation affects the entire domain.

- For many high speed cases we need to study effect of compressibility to drag, added mass, and Basset force.



- To understand effective properties (e.g. effective viscosities) we can calculate closures, such as  $\mathbf{T}$ , by numerically simulating particle suspensions. Complex mesh may be needed around particles, or the material point method can be used.
- In fluid mixing problems, we need to investigate effects of pseudo turbulence at low particle Reynolds numbers introduced by random particle arrangements.



## Concluding Remarks

- Currently models for multiphase flows, granular flows, porous media flows, and material failure and pulverization are developed independently. Relations among these models and their validities in more general situations need to be examined.
- The ensemble phase averaging method could be used to study the connections, validities and possible modifications of these models.
- The method provides a bridge between the macroscopic closures and the lower length and time scale physics.
- With today's computing technology (both numerical method and hardware), it is now feasible to calculate many of the closures from the ensemble phase averaging method.

# 3-Phase Model of Explosion Fields

A. Kuhl, J. Bell<sup>1</sup>, V. Beckner<sup>1</sup> & D. Grote



CCMT-Multiphase Flow Deep Dive

October 6-7, 2016  
St. Petersburg, FL

LLNL-PRES-703462

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<sup>1</sup> Lawrence Berkeley Lab



## Overview— *3 Phase Model*

**GAS PHASE:** compressible 2-phase Navier-Stokes equations

**PARTICLE PHASE:** dilute heterogeneous continuum model

**FRAGMENT PHASE:** Discrete Lagrangian Particles (DLP) model

### NUMERICAL METHODS

- Unsplit high-order Godunov schemes on uniform meshes
- Adaptive Mesh Refinement (AMR) to follow turbulent mixing
- Complete EOS based on tabulated Cheetah values

# 3-Phase Model

## 1. GAS PHASE: conservation laws of 2-phase compressible Navier-Stokes equations

Mass  $\partial_t \rho + \nabla \cdot \rho \mathbf{u} = 0$

Momentum:  $\partial_t \rho \mathbf{u} + \nabla \cdot (\rho \mathbf{u} \mathbf{u} + p) = \nabla \cdot 2\mu \mathbf{e}_{ij} + \nabla \lambda e_{ii} - D_p - D_F$

Energy:  $\partial_t \rho E + \nabla \cdot (\rho E \mathbf{u} + p \mathbf{u}) = \nabla \cdot \mathbf{u} 2\mu \mathbf{e}_{ij} + \nabla \lambda e_{ii} + \nabla \cdot (\kappa / c_p) \nabla h - (Q_p + Q_F) - (D_p + D_F) \cdot \mathbf{u}$

Products:  $\partial_t \rho Y_D + \nabla \cdot (\rho Y_D \mathbf{u}) = \nabla \cdot (\rho D_D \nabla Y_D)$

EOS:  $p, T, \Gamma = f_i(\rho, u, Y_D)$  (Cheetah code)

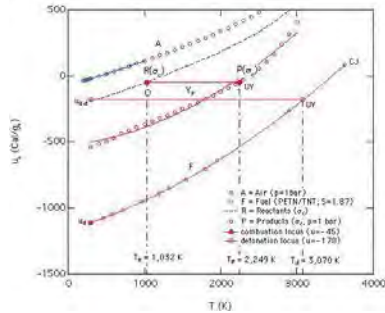
Transport:  $\mu, \lambda, \kappa, D = g_i(\rho, u, Y_D)$  &  $Le = 1$  (Cheetah code)

Combustion:

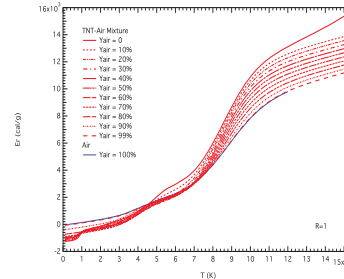
(i) *Quadratic Model*: frozen Fuel & Air with Products at thermodynamic equilibrium ( $300 \text{ K} < T < 4,000 \text{ K}$ )

(ii) *Cheetah*: based on Thermodynamic equilibrium of Detonation Products-Air mixture ( $300 \text{ K} < T < 15,000 \text{ K}$ )

Quadratic Model



Equilibrium Model



Numerical Method:

- Unsplit high-order Godunov method for hyperbolic terms (Bell, Colella, Trangenstein, 1989)
- 2<sup>nd</sup> order Runge-Kutta method for viscous terms

# 3-Phase Model—continued

## 2. PARTICLE PHASE—conservation laws of a dilute heterogeneous continuum

Mass:  $\partial_t \sigma + \nabla \cdot \sigma \mathbf{v} = 0$

Momentum:  $\partial_t \sigma \mathbf{v} + \nabla \cdot \sigma \mathbf{v} \mathbf{v} = D_p$

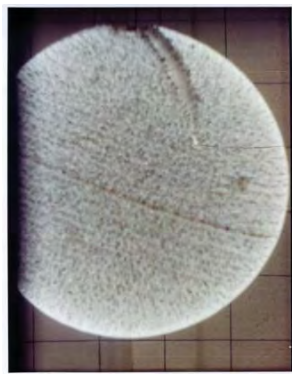
Energy:  $\partial_t \sigma E_s + \nabla \cdot \sigma E_s \mathbf{v} = Q_p + D_p \cdot \mathbf{v}$

Drag:  $D_p = (1/8)\pi d^2 \rho (\mathbf{u} - \mathbf{v}) |\mathbf{u} - \mathbf{v}| C_D$

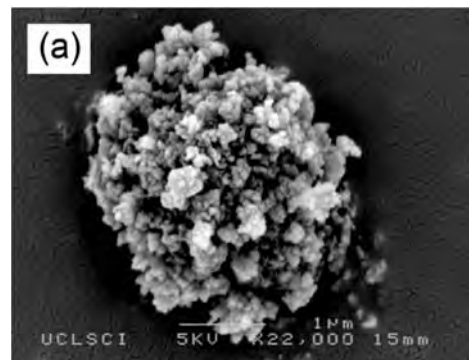
Heat Transfer:  $Q_p = \pi d \mu C_p \text{Pr}^{-1} (T - T_p) Nu$

where  $C_D = 0.48 + 28 \text{Re}^{-0.85}$  &  $Nu = 2 + 0.6 \text{Pr}^{1/3} \text{Re}^{1/2}$

Carbon Particles in Detonation Products



DND (Detonation Nano Diamond) Comp B



Numerical Method: 2<sup>nd</sup> order Godunov method for dilute particle systems (Collins et al., 1994)

# 3-Phase Model—concluded

## 3. FRAGMENT PHASE: Discrete-Lagrangian-Particles (DLP) model

Position:  $\dot{\mathbf{x}}_i = \mathbf{v}_i(\mathbf{x}_i)$  for all particles  $i$

Velocity:  $m_i \dot{\mathbf{v}}_i = D_F(\mathbf{x}_i) + \mathbf{g}$  for all particles  $i$

Energy:  $m_i \dot{e}_i = Q_F(\mathbf{x}_i)$  for all particles  $i$  (where  $e_i = c_s T_i$ )

Mass:  $\dot{m}_i = -\dot{s}_i(\mathbf{x}_i)$  for all particles  $i$

Drag:  $D_F = (1/8)\pi d^2 \rho (\mathbf{u} - \mathbf{v}) |\mathbf{u} - \mathbf{v}| C_D$  where  $C_D = 0.48 + 28 \text{Re}^{-0.85}$

Heat Transfer:  $Q_F = \pi d \mu C_p \text{Pr}^{-1} (T - T_p) \text{Nu}$  where  $\text{Nu} = 2 + 0.6 \text{Pr}^{1/3} \text{Re}^{1/2}$

Combustion Model: combustion of fragment wake with air

Air:  $\partial_t \rho Y_A + \nabla \cdot (\rho Y_A \mathbf{u}) = -\alpha \dot{s}_i$

Fuel:  $\partial_t \rho Y_F + \nabla \cdot (\rho Y_F \mathbf{u}) = -\dot{s}_i$

Products:  $\partial_t \rho Y_p + \nabla \cdot (\rho Y_p \mathbf{u}) = (1 + \alpha) \dot{s}_i$

Burning Rate:  $\dot{s}_i = s d_i^2$



### Numerical Method:

- 2<sup>nd</sup> order Runge-Kutta method
- fully (2-way) coupled to the fluid dynamics at all AMR levels

# Numerical Methods

- High-order Godunov method for the hyperbolic terms: PPM (Colella & Glaz, 1985; Colella & Woodward, 1984; Bell, Colella & Trangenstein, 1989)
- A 2<sup>nd</sup> order explicit Runge-Kutta method for the viscous terms
- A 2<sup>nd</sup> order Godunov method for the particle phase (Collins et al., 1994)
- Adaptive Mesh Refinement (AMR) to resolve steep gradients (Bell, Berger, Saltzman, Welcome, 1994)



## Conservation laws of compressible Navier-Stokes equations

Mass  $\partial_t \rho + \nabla \cdot \rho \mathbf{u} = 0$

Momentum:  $\partial_t \rho \mathbf{u} + \nabla \cdot (\rho \mathbf{u} \mathbf{u} + p) = \nabla \cdot 2\mu \mathbf{e}_{ij} + \nabla \lambda e_{ii}$

Energy:  $\partial_t \rho E_T + \nabla \cdot (\rho E_T \mathbf{u} + p \mathbf{u}) = \nabla \cdot \mathbf{u} 2\mu \mathbf{e}_{ij} + \nabla \mathbf{u} \lambda e_{ii} + \nabla \cdot (\kappa / c_p) \nabla h$

Products:  $\partial_t \rho Y_D + \nabla \cdot (\rho Y_D \mathbf{u}) = \nabla \cdot (\rho \tilde{D}_D \nabla Y_D)$

General Form:

$$\partial_t U + \nabla \cdot W(U) = \nabla \cdot \mathcal{D}$$

where  $U = \begin{Bmatrix} \rho \\ \rho \mathbf{u} \\ \rho E_T \\ \rho Y_D \end{Bmatrix}$   $W(U) = \begin{Bmatrix} \rho \mathbf{u} \\ \rho \mathbf{u} \mathbf{u} + p \\ \rho E_T \mathbf{u} + p \mathbf{u} \\ \rho Y_D \end{Bmatrix}$   $\mathcal{D} = \begin{Bmatrix} 0 \\ 2\mu \mathbf{e}_{ij} + \lambda e_{ii} \\ \mathbf{u} 2\mu \mathbf{e}_{ij} + \mathbf{u} \lambda e_{ii} + \kappa / c_p \nabla h \\ \rho \tilde{D}_D \nabla Y_D \end{Bmatrix}$

## SOLUTION

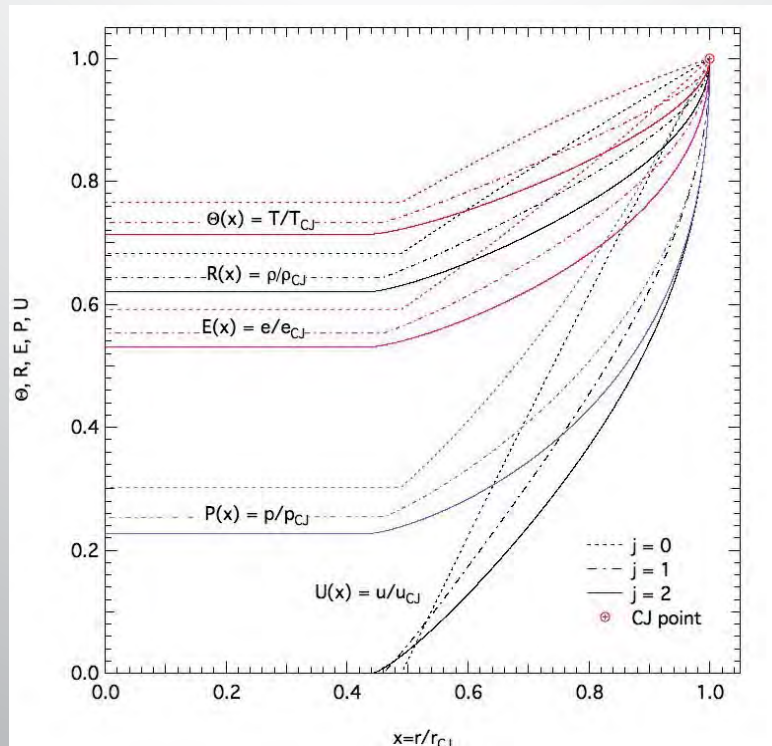
Hyperbolic Terms:  $\partial_t U + \nabla \cdot W(U) = 0$

Predictor:  $\tilde{U}_{ijk}^{n+1} = U_{ijk}^n - \Delta t \left[ W_{i+1/2,jk}^{n+1/2} - W_{i-1/2,jk}^{n+1/2} \right] / \Delta x - \Delta t \left[ W_{i,j+1/2,k}^{n+1/2} - W_{i,j-1/2,k}^{n+1/2} \right] / \Delta y - \Delta t \left[ W_{ij,k+1/2}^{n+1/2} - W_{ij,k-1/2}^{n+1/2} \right] / \Delta z$

Diffusion:  $\partial_t U = \nabla \cdot \mathcal{D}$

Corrector:  $U_{ijk}^{n+1} = \tilde{U}_{ijk}^{n+1} + \Delta t * (\nabla \cdot \mathcal{D})_{ijk}^{n+1/2}$

## Initial Conditions\*



\* A. Kuhl, On the Structure of Self-Similar Detonation Waves in TNT Charges, *Combustion, Explosion and Shock Waves*, 2015 **51** (1) pp. 72-79

# Initial Conditions

## TNT Charge Mass

- \* 1-g
- \* 1-kg
- \* 1-T

## Mesh Size:

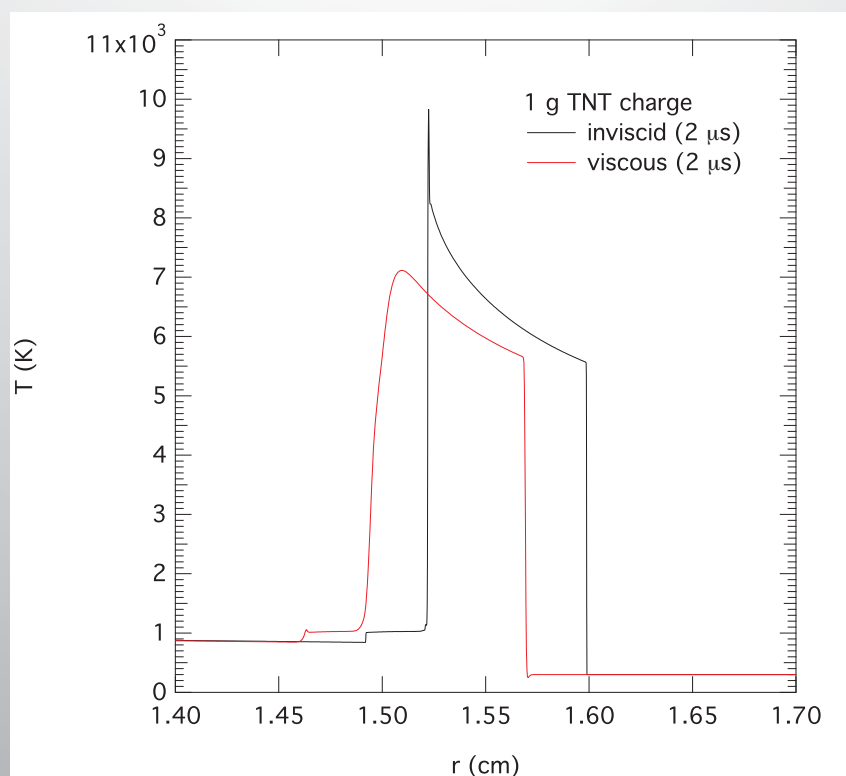
$$\Delta r = 1 \mu m$$

TABLE 2. CJ and UV states for TNT ( $\rho_0 = 1.654 \text{ g/cc}$ )

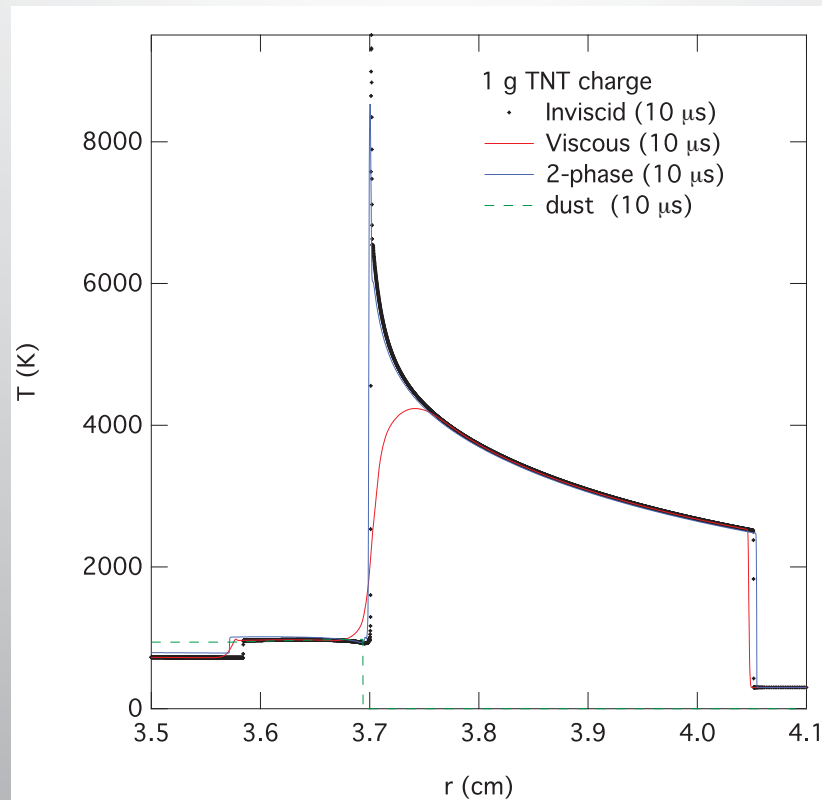
Variable	CJ State	UV State
$p(\text{kbar})$	197.59759	90.13
$\rho(\text{g/cm}^3)$	2.1616	1.654
$e(\text{cal/g})^*$	1,352.46	1,064.8
$u(\text{cal/g})$	273.16	-66.5
$T(\text{K})$	3,237.875	2,866.9
$s(\text{cal/g-K})$	1.58447	1.623
$u_r(\text{km/s})$	1.68595	0
$W(\text{km/s})$	7.18	0
$a(\text{km/s})$	5.4939	4.0593
$\Gamma = W_{CJ} / u_{r,CJ} - 1$	3.2586	—

\*  $e \equiv u + 1,131$

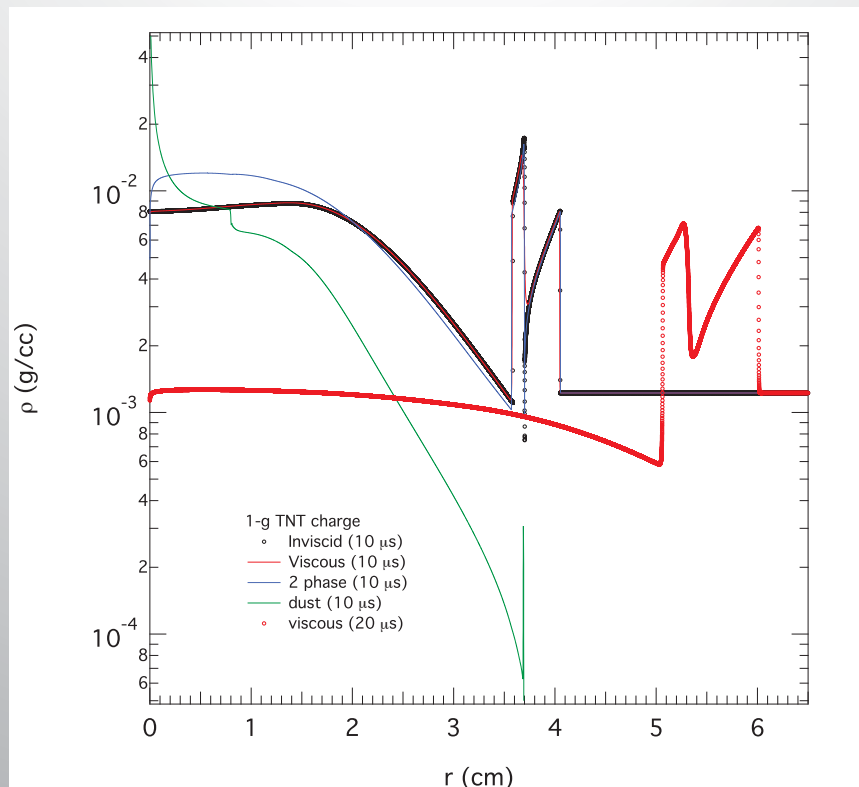
# Temperature Profiles at 2 $\mu s$



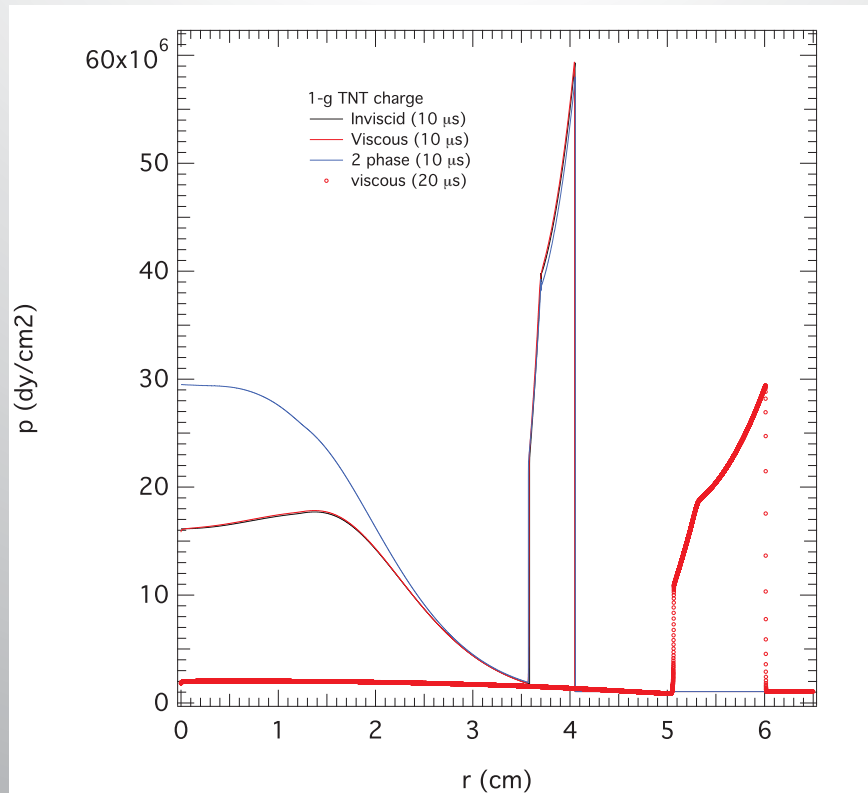
# Temperature Profiles for 3 Models



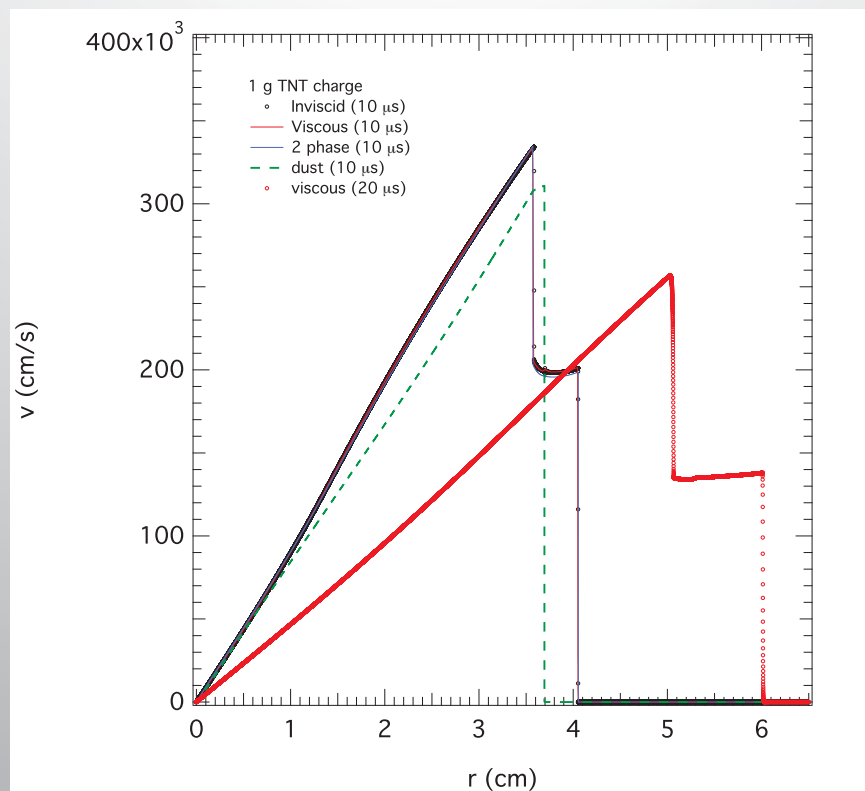
# Density Field for the 3 Models



# Pressure Field for the 3 Models

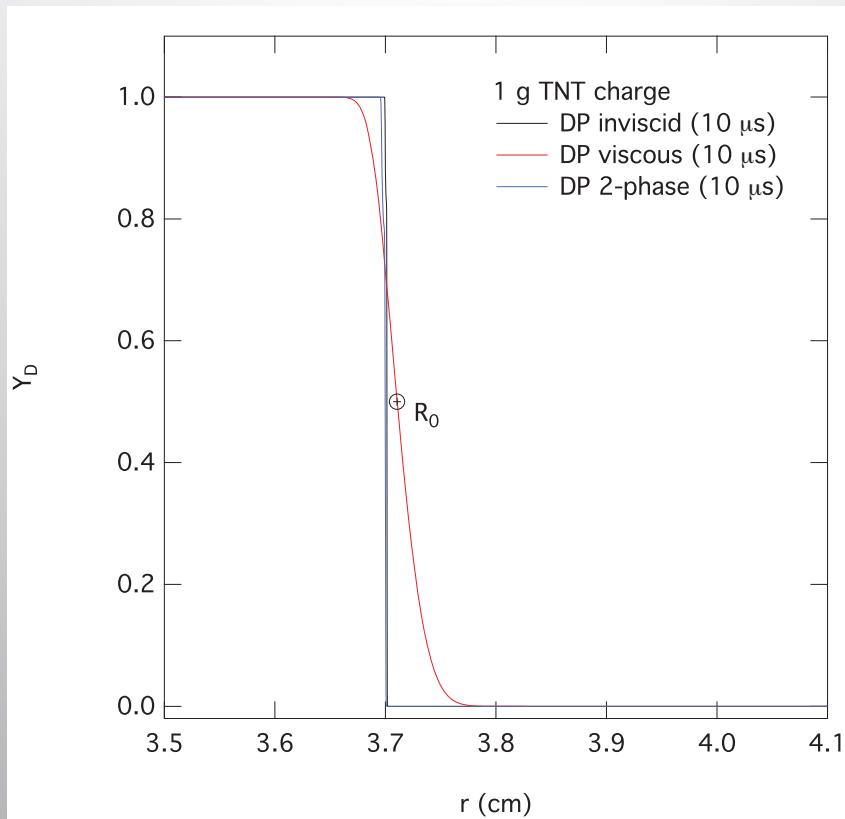


# Velocity Field for the 3 Models

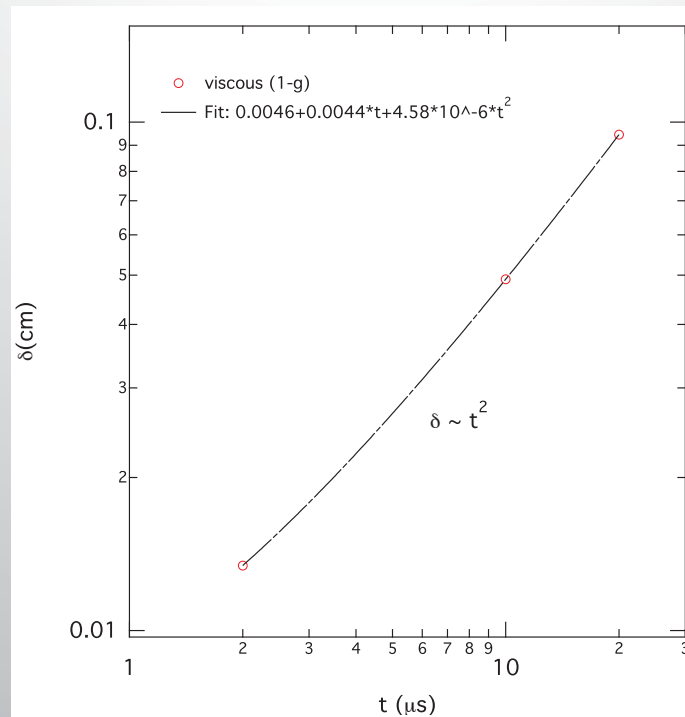




# DP-Air Interface for the 3 Models



## Mixing Layer Growth



$\delta \sim t^2$  : at  $t = 0.63 \mu$ s

$N_\delta = 144$  cells for 1g &  $N_\delta = 164$  cells for 1kg

# Similarity Solution

**Conservation Law:**  $\partial_t \rho Y_D + \nabla \cdot (\rho Y_D \mathbf{u}) = \nabla \cdot (\rho D_D \nabla Y_D)$

**Hyperbolic:**  $\partial_t \rho Y_D + \nabla \cdot (\rho Y_D \mathbf{u}) = 0$   $(\rho \tilde{Y}_D)_i^{n+1} = (\rho Y_D)_i^n - \Delta t [F_{i+1/2}^{n+1/2} - F_{i-1/2}^{n+1/2}]$

**Diffusion:**  $\partial_t (\rho Y_D) = \nabla \cdot (\rho D_D \nabla Y_D)$   $(\rho Y_D)_i^{n+1} = (\rho \tilde{Y}_D)_i^{n+1} + \Delta t [\nabla \cdot \rho D_D \nabla Y_D]_i^{n+1/2}$

**Model Law:**  $\partial_t Y_D = D_0 \nabla^2 Y_D$

**Similarity Variable:**  $\xi = r / \delta$

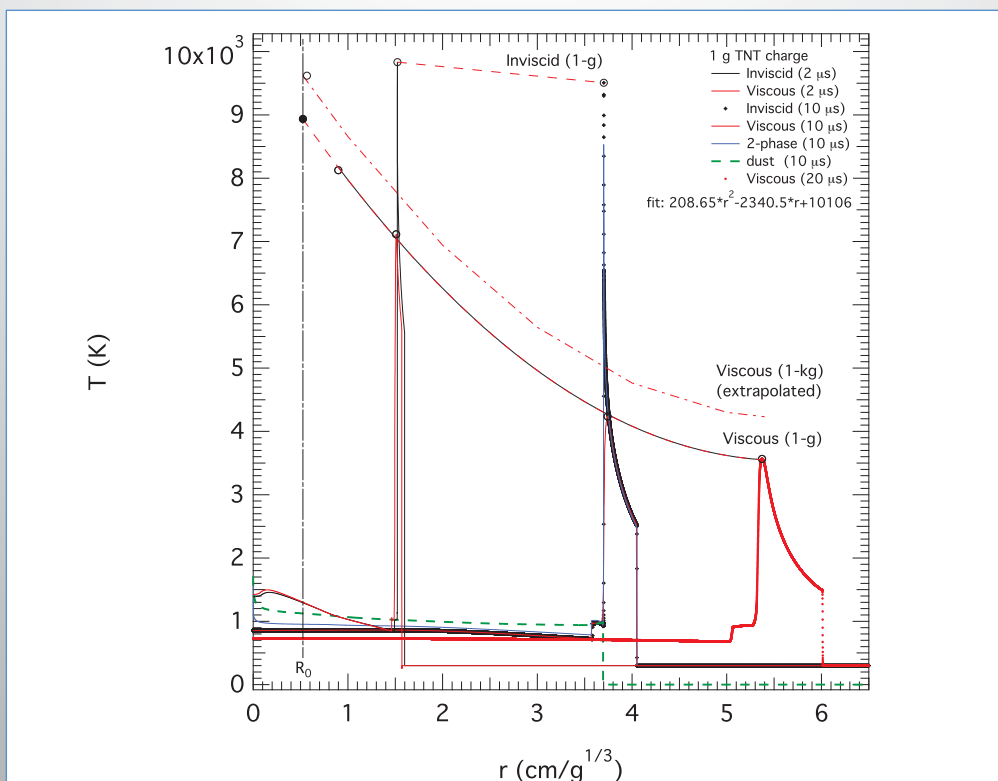
**Mixing Layer Width:**  $\delta \equiv Y_{10\%} - Y_{90\%} = at^2$

where interface:  $R_0(t) \equiv r @ Y_{50\%}$  is advected with the flow;  $\xi_0 \equiv R_0 / \delta$

**Similarity Solution:**

$$Y_D = f(\xi) = \frac{1}{2} [1 + \operatorname{erfc}(\xi - \xi_0)]$$

## Temperature Profiles—viscous vs inviscid



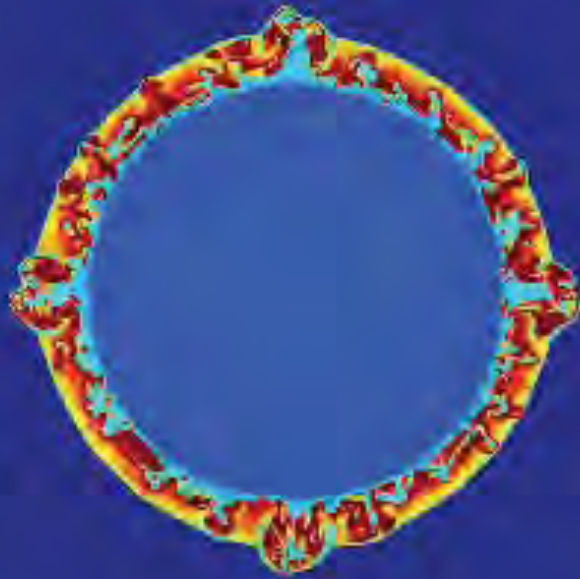
# SUMMARY

1. **Mass Diffusivity**—causes DP to diffuse  $\sim t^2$  ; the profile  $Y_D(r) \rightarrow \text{erfc}(\xi - \xi_0)$
2. **Thermal Diffusivity**—causes heat to diffuse, lowering the temperature at the DP-air interface from 9 -10,000 K to 4 - 7,000 K
3. **Viscosity**—causes shocks to thicken to 100's of cells—but does not change the shock jump conditions
4. The **2-Phase Model** preserves the identity of the carbon particles (i.e., protects them from combustion with air) so that the evolution of the particle phase can be followed throughout the explosion field
5. **Scaling Laws:**
  - The diffusion layer  $\delta$  grows as  $t^2$  —independent of the explosion scale
  - The inviscid blast wave time and length scale as  $t / kg^{1/3}$  and  $r / kg^{1/3}$
  - The diffusion effects in the blast wave scale with  $Re$  &  $Pe$  numbers
  - As the explosion scale gets larger, the diffusion effects decrease

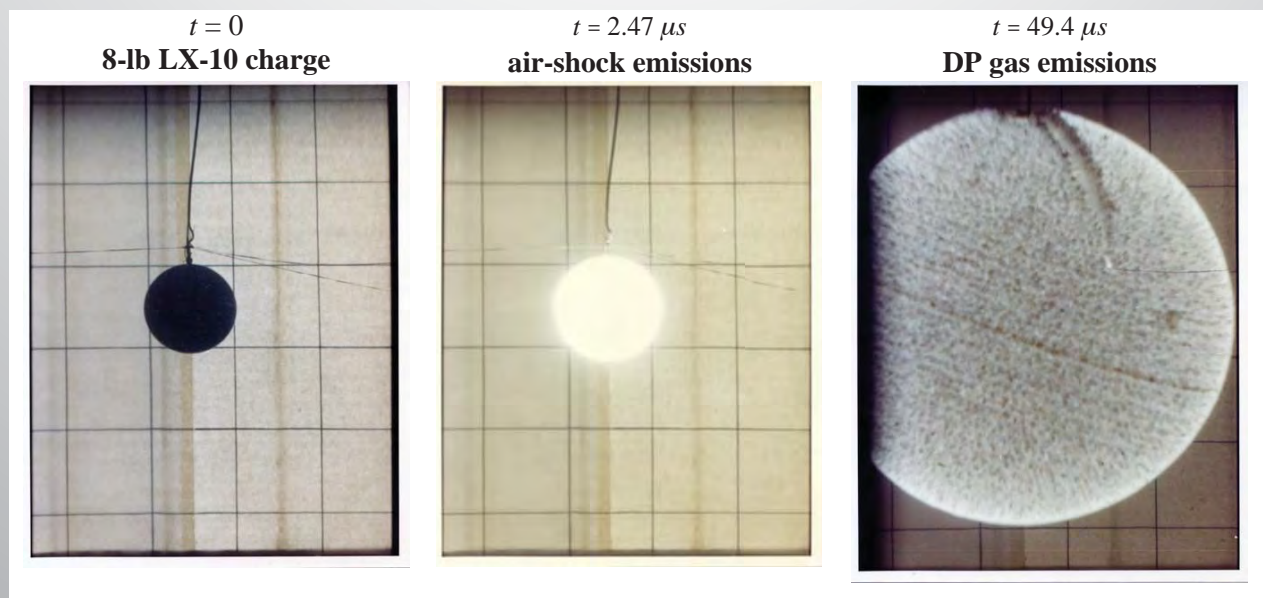
# CONCLUSIONS

1. The **Complete Model** in 1D spherical coordinates (page 6) is recommended:
  - Inviscid terms — accurately model the underlying compressible flow
  - Viscous terms — diffuse the species and temperature
  - 2-phase terms — follow the carbon particle dynamics
2. The **Complete Model** must be used at early times ( $t \leq 20 \mu s / kg^{1/3}$ ) to capture all physical effects.
3. This 1D solution can be used to initialize the 3D AMR multi-phase code to compute the evolution of turbulent mixing and combustion in the fireball.

Temperature field showing transition to turbulent combustion at  $20 \mu\text{s/g}^{1/3}$



# Physical Mechanisms of Optical Emissions

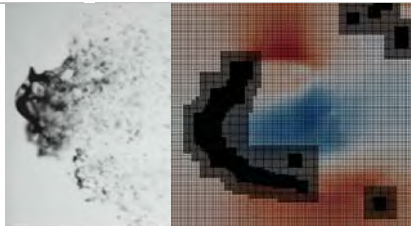
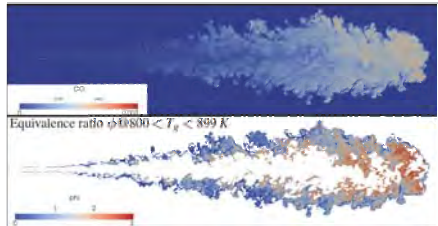


## Sectioned View of LX-10 Fireball @ 49 $\mu s$





*Exceptional service in the national interest*



## Multiphase Flow Simulation Strategies at the CRF

Marco Arienti, F. Doisneau, J. Oefelein  
Sandia National Laboratories, California



Sandia National Laboratories is a multi-program laboratory managed and operated by Sandia Corporation, a wholly owned subsidiary of Lockheed Martin Corporation, for the U.S. Department of Energy's National Nuclear Security Administration under contract DE-AC04-94AL85000. SAND NO. 2011-XXXXP

## Presentation layout

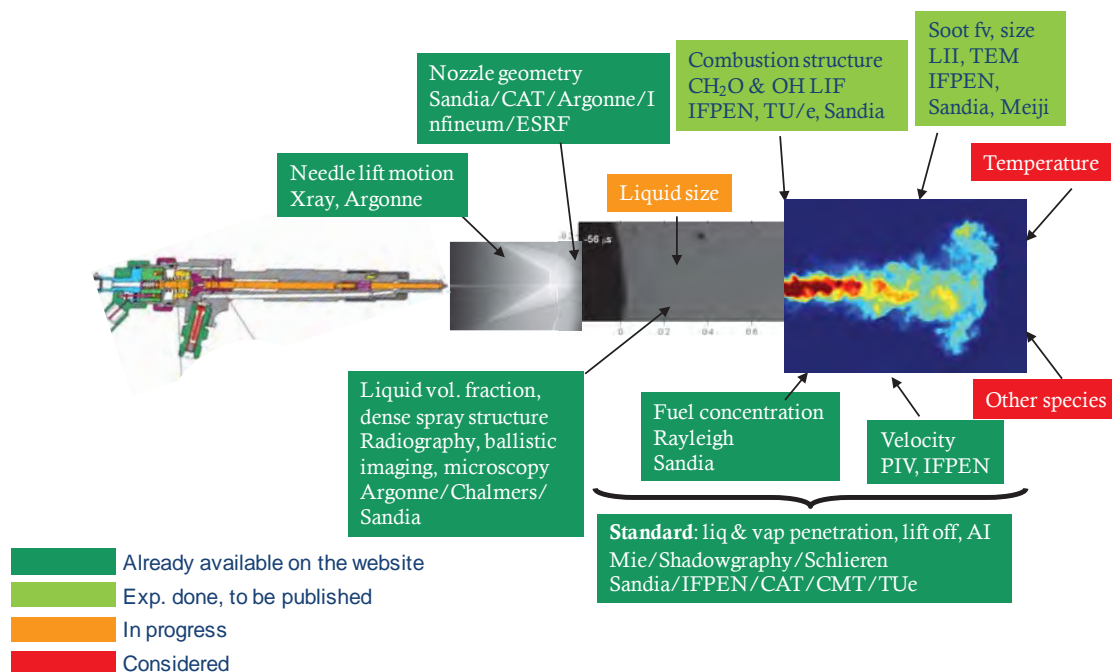


- From injector to spray flames
  - One-fluid formulation with interface capturing
  - Demonstration: Spray A
  - A new Eulerian Multi-Fluid approach for dense sprays
  - Continuation: spray flame auto-ignition
- Molten metal and drop fragmentation
  - *A priori* test for molten aluminum in crossflow
  - The opportunity of new experimental diagnostics/data

# FROM INJECTOR TO SPRAY FLAMES

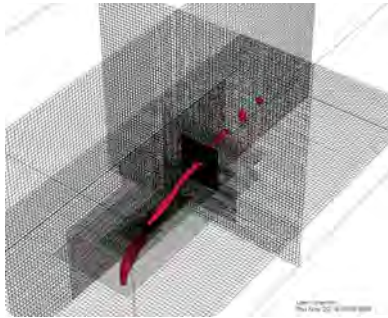
3

## Spray A and the Engine Combustion Network (ECN)

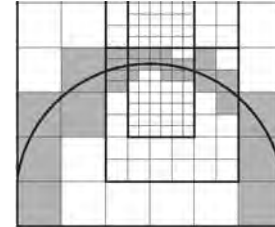


4

# Parallel multiphase solver (CLSVOF) + adaptive mesh refinement for primary atomization



Block-structured adaptive mesh refinement  
(John Bell)



Solved in the variables:

$$(\rho, \rho \mathbf{u}, E)^m, F^m, \mathbf{x}_c^m, \phi^m$$

for each phase/material:

$$m = 1 \dots M$$

$$\frac{\partial(\mathbf{u}\rho)}{\partial t} + \nabla \cdot (\mathbf{u} \otimes \mathbf{u}\rho + p\mathbf{I}) = \nabla \cdot \boldsymbol{\tau} + \rho \mathbf{g} - \sum_{m=1}^M \gamma_m \kappa_m \nabla H(\phi_m)$$

$$\rho = \sum_{m=1}^M \rho_m H(\phi_m) \quad H(\phi) = \begin{cases} 1 & \phi \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

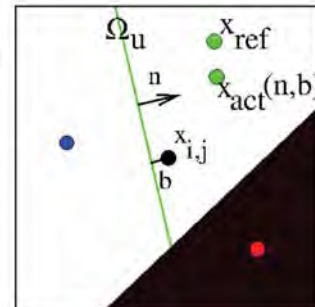
$$\boldsymbol{\tau} = \sum_{m=1}^M \tau_m H(\phi_m) \quad \tau_m = 2\mu_m \left( D - \frac{\text{TR}(D)\mathbf{I}}{3} \right) \quad D = \frac{\nabla \mathbf{u} + \nabla \mathbf{u}^T}{2}$$

## New feature: The moment of fluid (MOF)<sup>[1]</sup> method

- MOF tracks volume fractions through advection, but also uses the centroid of a material region  $\mathbf{x}_{ref} = \mathbf{x}_c^m$  in determining the orientation of the interface

$$(\mathbf{n}, b) = \arg \min_{|F_{ref} - F_{act}(\mathbf{n}, b)|=0} \|\mathbf{x}_{ref} - \mathbf{x}_{act}(\mathbf{n}, b)\|_2$$

$$\mathbf{n} \cdot (\mathbf{x} - \mathbf{x}_{i,j}) + b = 0 \quad \mathbf{n} = \begin{pmatrix} \sin(\Phi) \cos(\Theta) \\ \sin(\Phi) \sin(\Theta) \\ \cos(\Phi) \end{pmatrix}$$

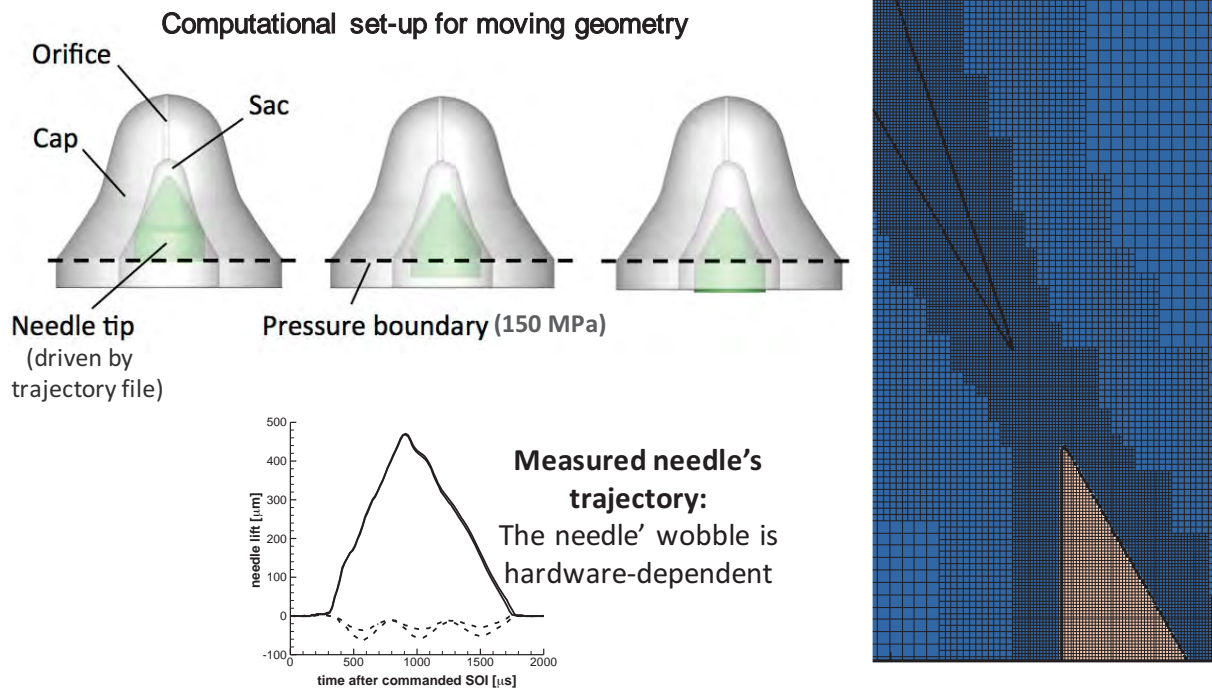


- The interface is chosen as the piece-wise linear reconstruction that contains the volume fraction and **minimizes the variation** in centroid position

[1] V. Dyadechko M. Shashkov LA-UR-05-7571 (2007)

## New feature:

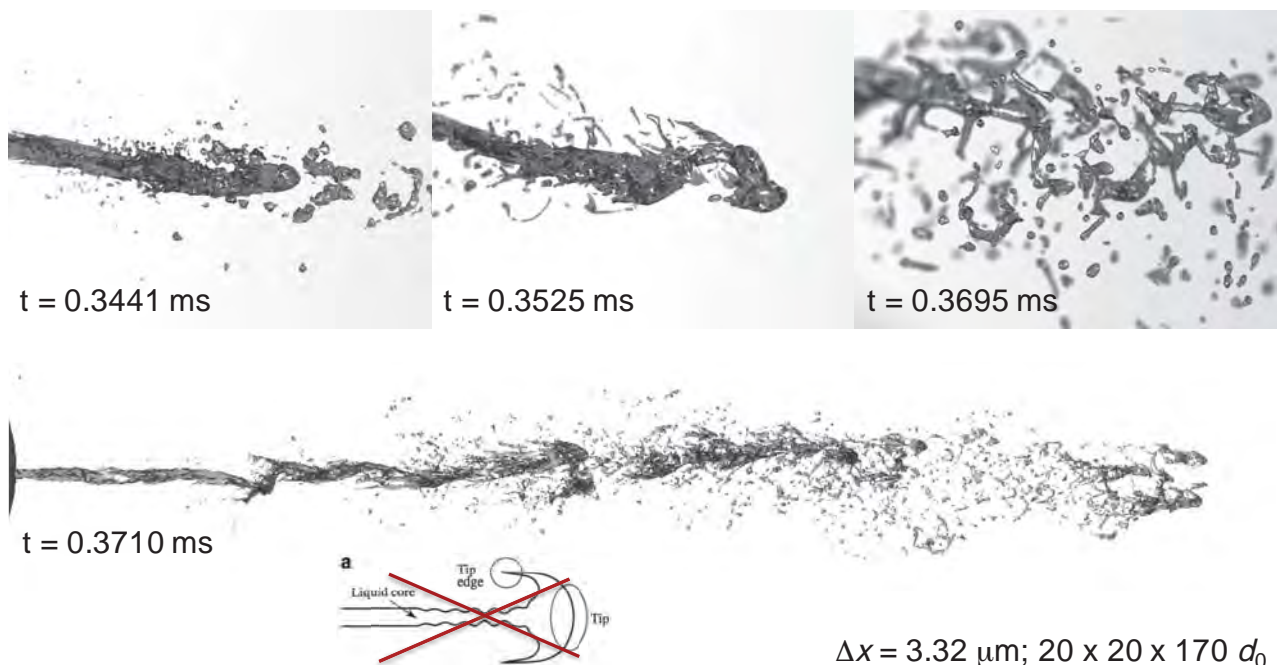
## Moving boundaries with non-conformal mesh



7

## Open needle: jet tip structure

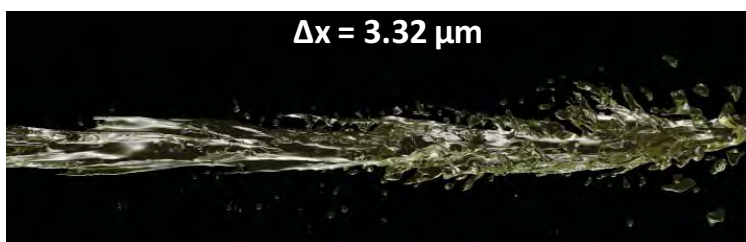
*No “mushroom” jet tip because of asymmetric internal flow*



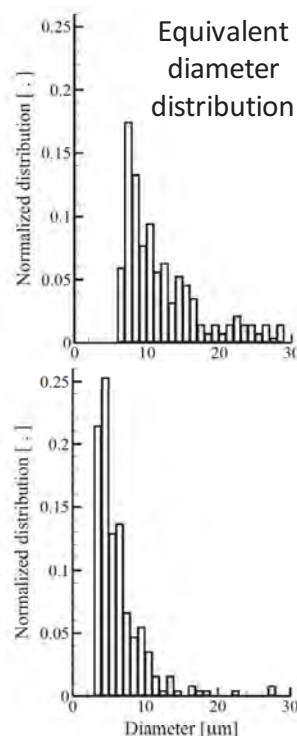
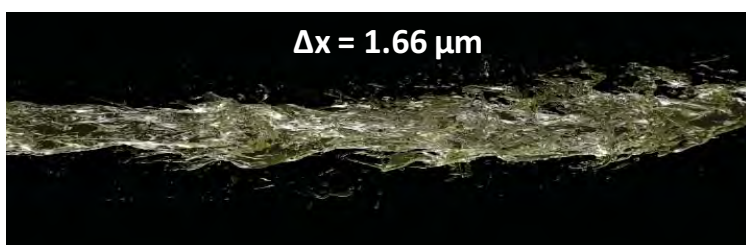
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# Challenge: grid convergence

*Same simulation repeated at twice the grid resolution: is the size distribution converging?*

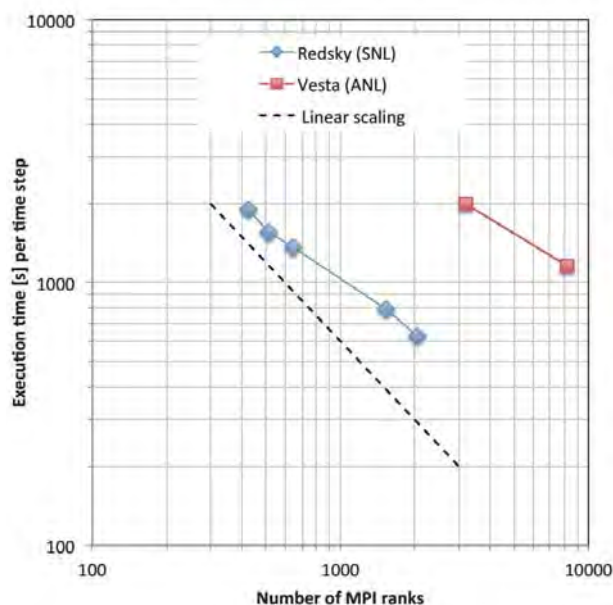
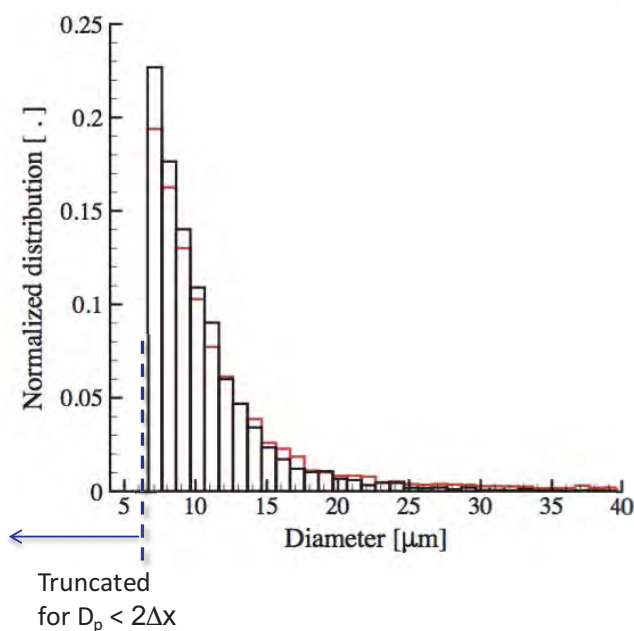


Snapshots of the jet at  $t_{ASOI} = 6 \mu s$



## The limits of the DNS framework

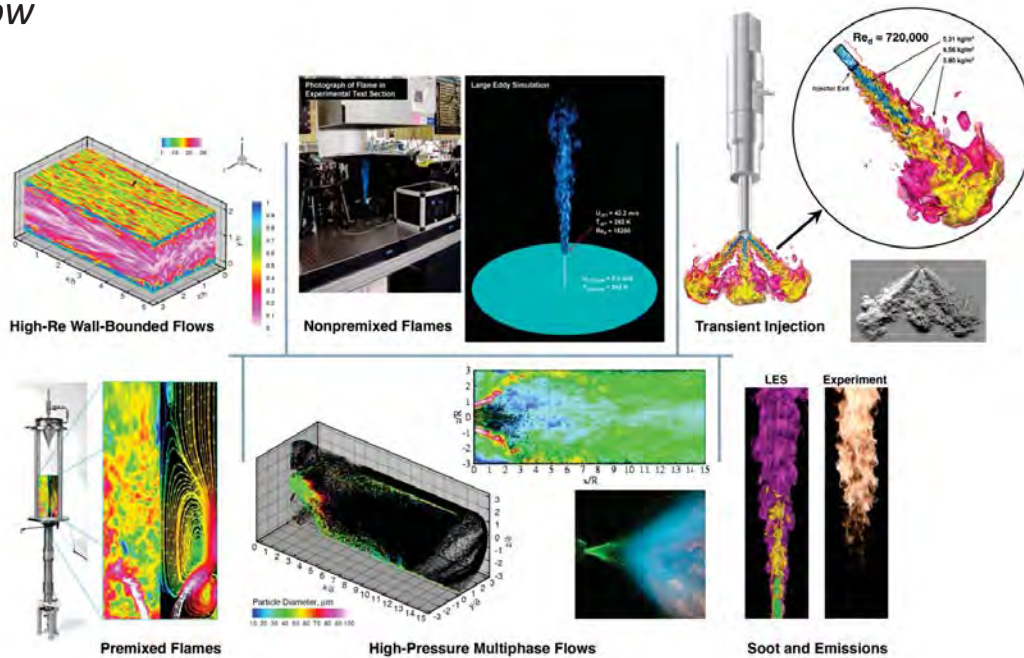
*Inherently under-resolved + very high cost for a very small time-step*





# LES framework: RAPTOR

*Single-fluid, diffuse interface approach applied to super-critical flow*



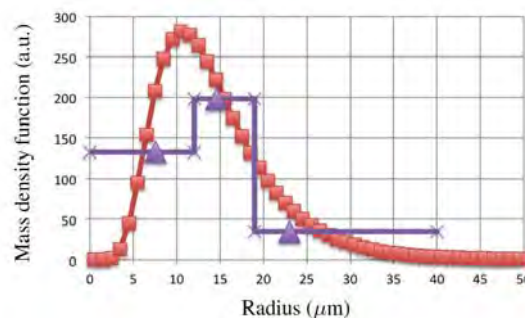
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## A new coupling approach from kinetic modeling: the Eulerian Multi-Fluid (EMF)

- Number density function (NDF):  $dN = f(t; \mathbf{x}; \mathbf{c}, \theta, r) d\mathbf{x} d\mathbf{c} dr d\theta$
- NDF satisfies a Boltzmann-like PDE [Williams, 1958]:

$$\begin{cases} \text{Multicomponent Navier-Stokes + spray source terms} \\ \partial_t f + \partial_x c f + \partial_c \mathbf{F} f + \partial_\theta \mathbf{H} f + \partial_r \mathbf{E} f = \mathbf{B} + \mathbf{C}, \end{cases}$$

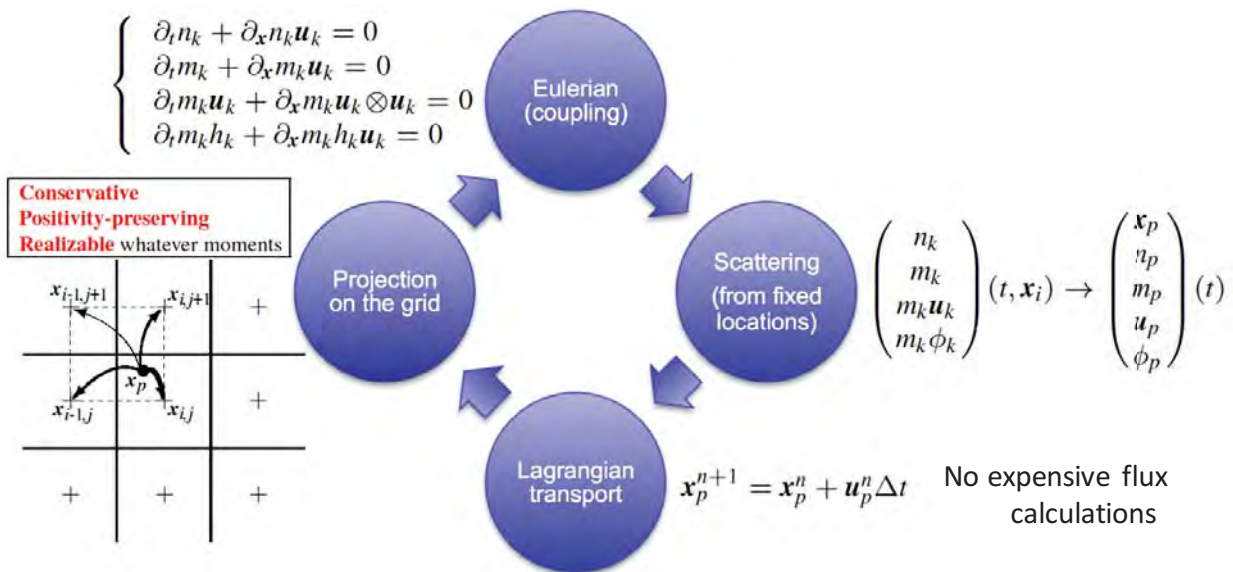
- Multiple section can accommodate a given spray distribution:



12

# A semi-Lagrangian transport scheme

Information localized at cell nodes – good for coupling/easier load balancing

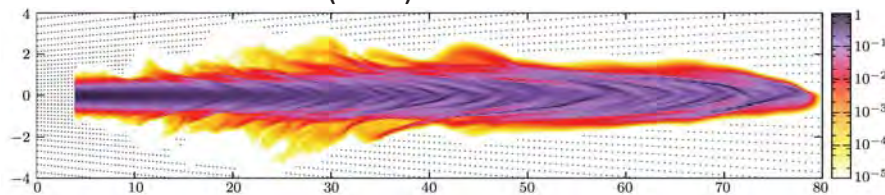


13

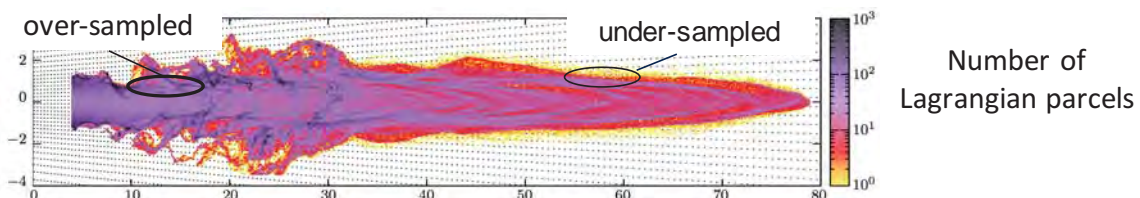
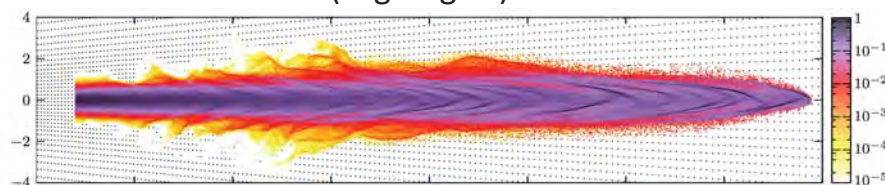
## EMF vs Lagrangian

Example with 1-way coupling test

Mass concentration (EMF)



Mass concentration (Lagrangian)



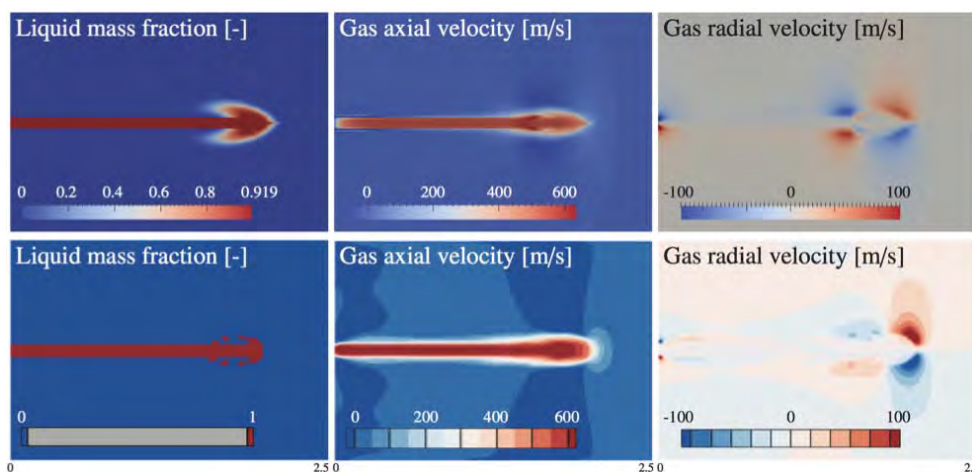
14

# Comparison: EMF vs CLSVOF

*Test on momentum-coupling algorithm in a supersonic, laminar injection test case at high mass-loading conditions*

## RAPTOR + EMF

$\Delta x = 12.5 \mu\text{m}$   
 $\Delta t = 8 \text{ ns}$



Density overshoot results from pressure-less assumption

Good agreement on jet tip trajectory

Good agreement on gas entrainment

Doisneau, Arienti, and Oefelein, accepted by JCP

15

# Spray A flame autoignition

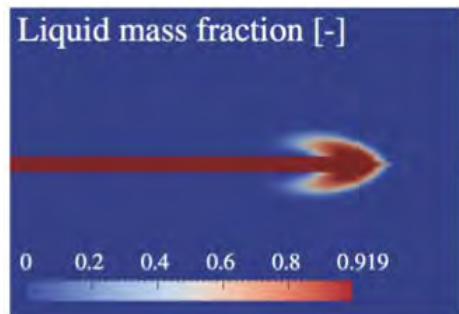


Doisneau, Arienti, and Oefelein, Combustion Symposium, 2016

16

# EMF issues and continuing work

- Density overshoot, no surface tension
  - Add inter-particle interaction in the Forward Lagrangian step?



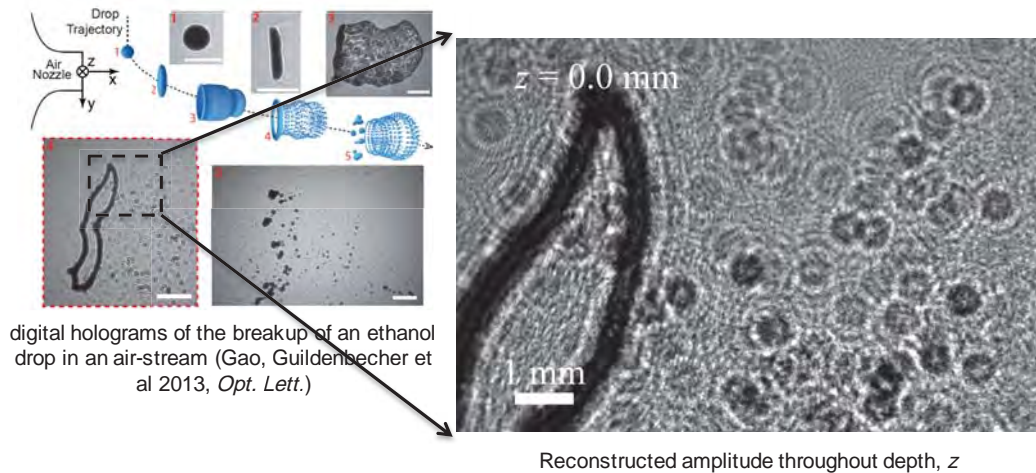
- Inclusion of collision, coalescence, and break-up models in the transition from dense to dilute spray
- What to do to prime the near field with spray data?

## MOLTEN METAL DROP FRAGMENTATION



# Morphology and fragmentation of molten metal

- Motivation: rocket failures can lead to propellant fires
  - Sandia Laboratories is interested in predicting the response of objects in this environment
- A new technology for validation: Digital in-line holography (DIH)



Daniel R. Guildenbecher, SNL

19

## Computational set-up for 2D drop in crossflow

$$We_g = \frac{\rho_g U_g^2 d_0}{\sigma} \quad (\text{at post-shock condition})$$

Water @ 300 K,  $d_0 = 1$  mm  
(Tait liquid,  $\sigma = 0.072$  N/m,  $\rho = 1000$  kg/m<sup>3</sup>, Oh = 0.0030)

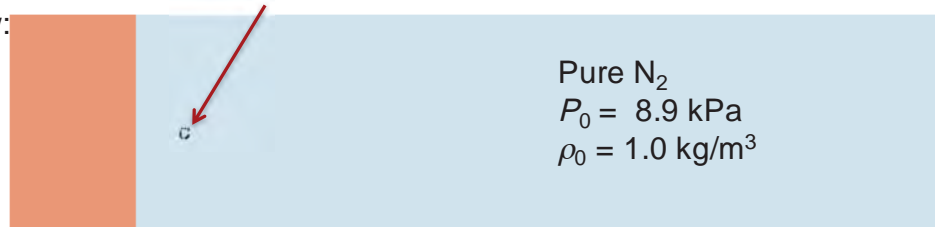
Uniform crossflow:

P.G.

$$We_g = 106$$

$$U_g = 77.3 \text{ m/s}$$

$$M = 1.14$$



Aluminum @ 1200 K,  $d_0 = 1$  mm  
(Tait liquid,  $\sigma = 1.1$  N/m,  $\rho = 2300$  kg/m<sup>3</sup>, Oh = 0.00022)

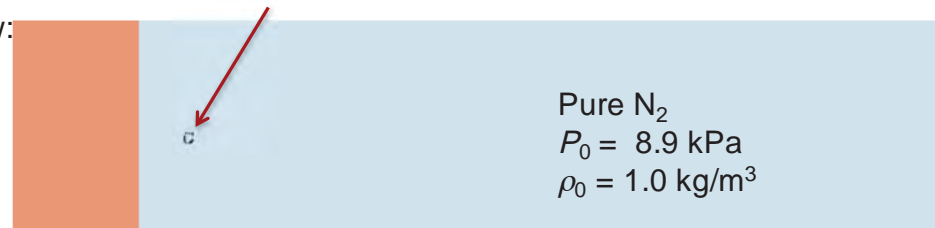
Uniform crossflow:

P.G.

$$We_g = 174$$

$$U_g = 302 \text{ m/s}$$

$$M = 1.64$$

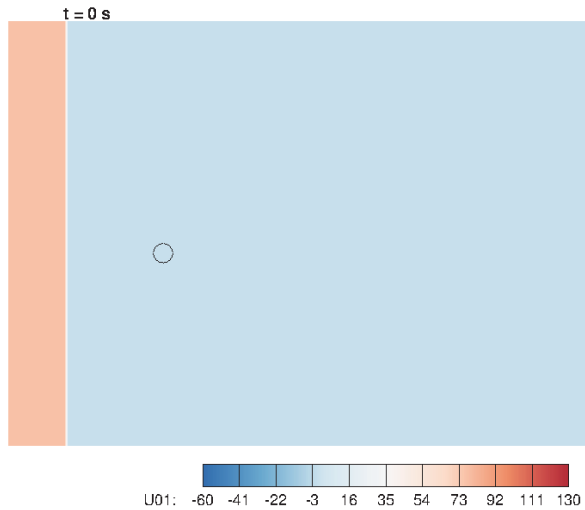


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## Water cross-section

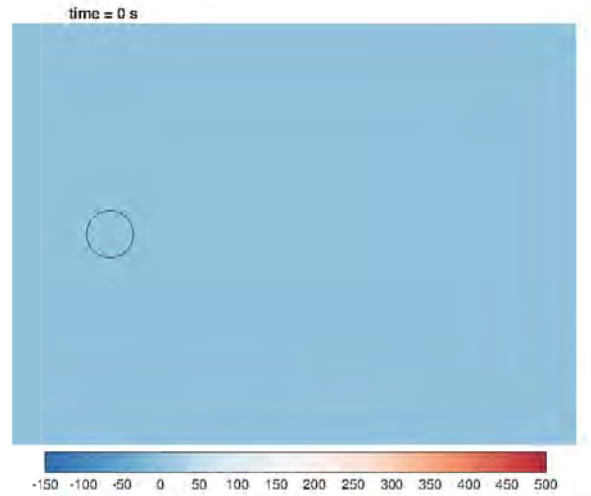
$$U_g = 77 \text{ m/s}$$
$$M = 1.14$$
$$We_g = 106$$



## Aluminum cross-section



$$U_g = 302 \text{ m/s}$$
$$M = 1.64$$
$$We_g = 174$$



Axial vel. [m/s]

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## Path Forward

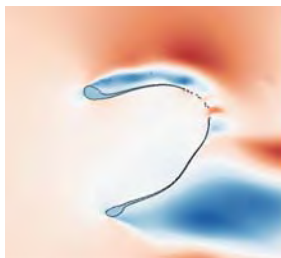


*Continue investigation of break-up morphology for materials*

$$We_g = 6.7$$

$$Oh = 0.00022$$

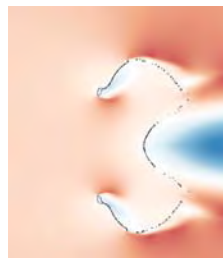
$$Re_g = 5,200$$



$$We_g = 106$$

$$Oh = 0.0030$$

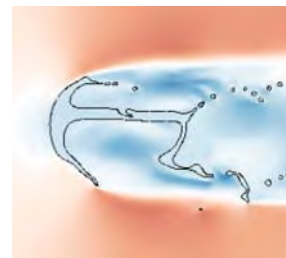
$$Re_g = 5,200$$



$$We_g = 174$$

$$Oh = 0.0022$$

$$Re_g = 20,000$$



- The state of the “skin” of aluminum oxide is difficult to predict; models relies on sparse data
- Switch to galinstan (non toxic alloy, in liquid phase at room conditions) and 3D

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# Cross-flow of molten metal alloy (galinstan)

*Shock tube tests isolate the effects of large surface tension and density ratio*



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## Thank you!



The support by Sandia National Laboratories via the Laboratory Directed Research and Development program is gratefully acknowledged. Sandia National Laboratories is a multi-program laboratory managed and operated by Sandia Corporation, a wholly owned subsidiary of Lockheed Martin Corporation, for the U. S. Department of Energy's National Nuclear Security Administration under contract DE-AC04-94AL85000.

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# BACKUP

## II. Extension to compressible flow

- 1 Advect and update volume fractions and moments for each material,  $m$

## II. Extension to compressible flow

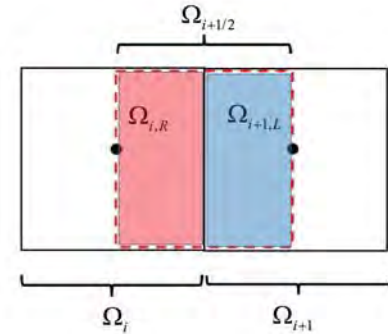
6 Solve for  $P^{n+1}$

$$[1] \quad P^{n+1} - \rho^{n+1} (c^2)^a \Delta t^2 \nabla \cdot \left( \frac{\nabla P^{n+1}}{\rho^{n+1}} \right) = p^a - \rho^{n+1} (c^2)^a \Delta t \nabla \cdot \mathbf{u}^a$$

using the mass-weighted interpolation

$$\mathbf{u}_{i+1/2}^a = \frac{\mathbf{u}_i^a \rho_{i,R} |\Omega_{i,R}| + \mathbf{u}_{i+1}^a \rho_{i+1,L} |\Omega_{i+1,L}|}{\rho_{i+1/2} |\Omega_{i+1/2}|}$$

$$\rho_{i+1/2} = \frac{\rho_{i,R} |\Omega_{i,R}| + \rho_{i+1,L} |\Omega_{i+1,L}|}{|\Omega_{i+1/2}|}$$



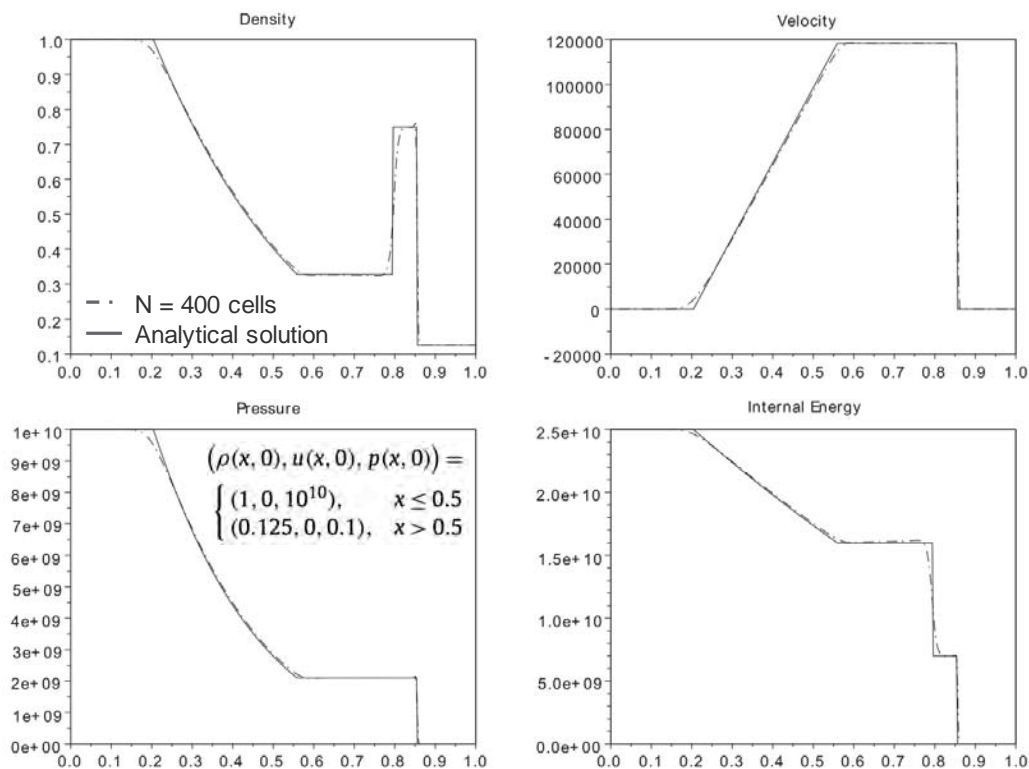
**Note:** the standard incompressible pressure projection method is asymptotically preserved in the limit of infinite sound speed. Each material can be treated as compressible or incompressible

[1] N. Kwatra, J. Su, J.T. Grétarsson, R. Fedkiw, J. Comput. Phys. (2009)

Jemison, Sussman, Arienti, JCP 279 (2014)

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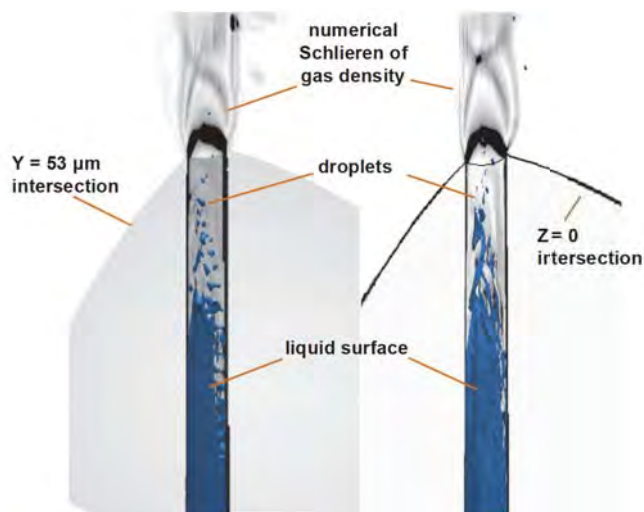
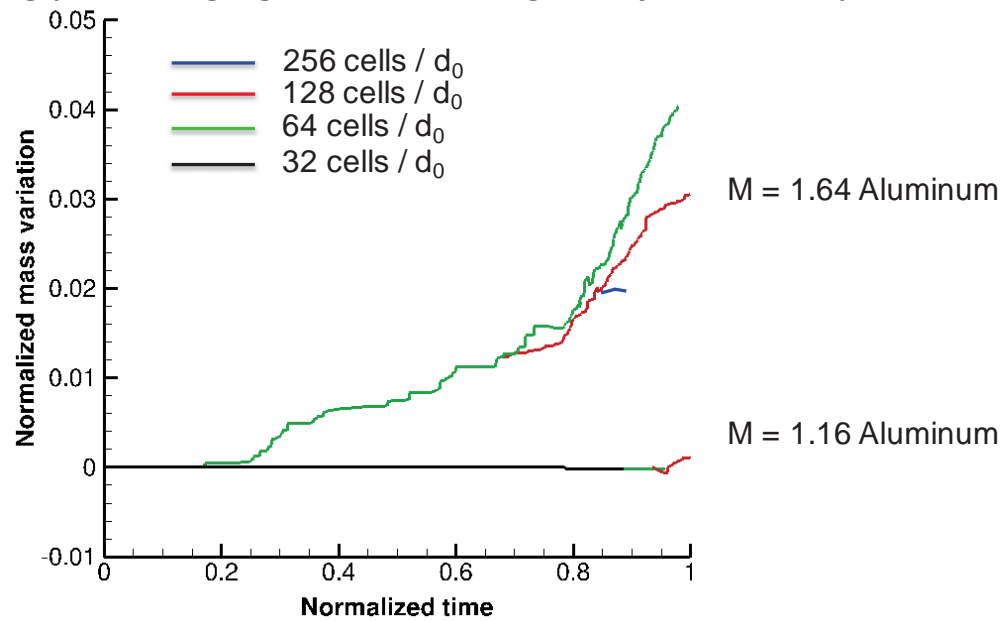
## Single phase test: shock tube



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# Mass conservation properties

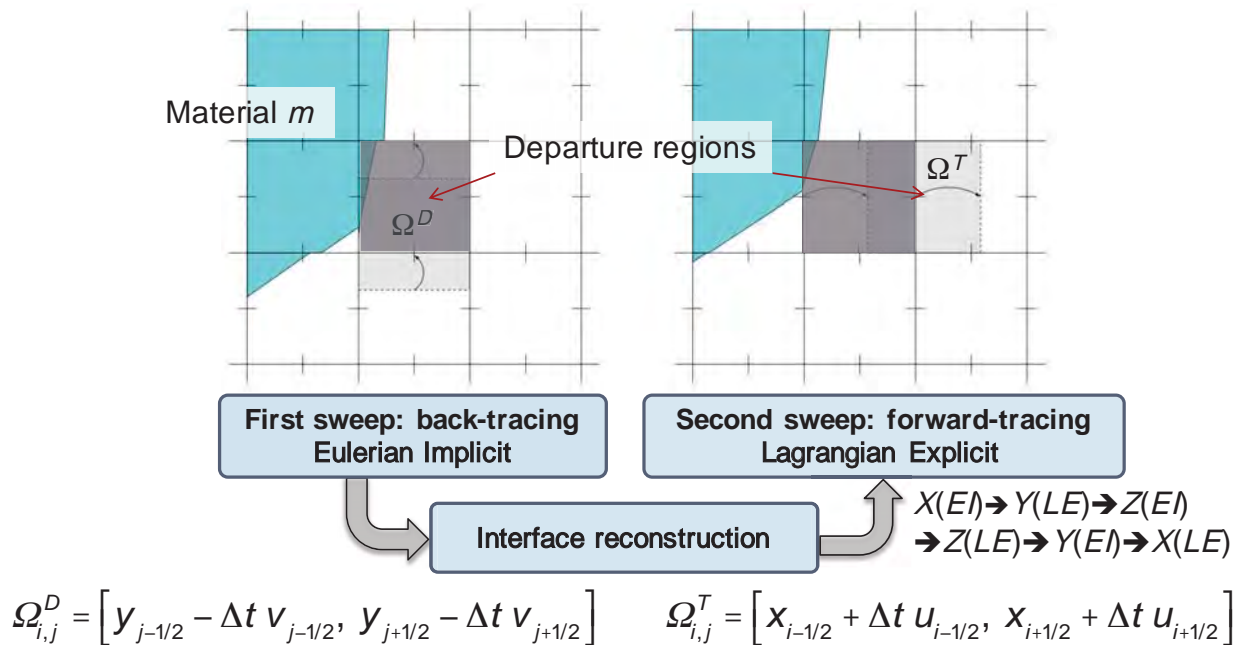
*AMR maintains mass variation under control, but this becomes increasingly challenging with increasing crossflow velocity*





# Directionally split advection for 2D/3D

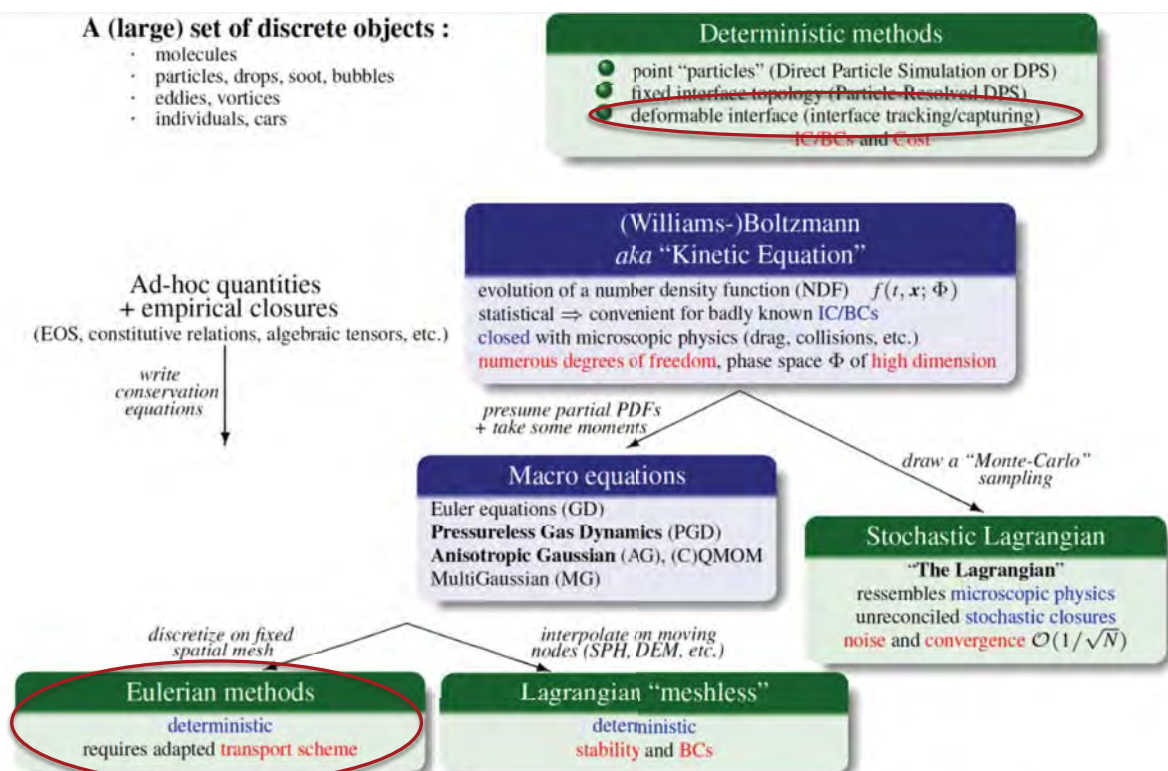
## Eulerian-Implicit/Lagrangian-Explicit (EI-LE)



Jemison, Sussman and Arienti, J. Comp. Physics 2013

31

# A "genealogy" of multiphase models



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# Point-particle modeling for two-way-coupled problems:

## Challenges, verification, and physics-based improvements

Jeremy Horwitz, Ali Mani  
Mohammad Mehrabadi, Shankar Subramaniam

### Multiphase Physics Deep-Dive

October 6-7, 2016



*Acknowledgements: NSF, DOE*



## Outline

- Modeling and Numerical Challenges
- Origin of the point-particle equations
- Verification ( $\tilde{u}$ -undisturbed fluid velocity)
  - Settling particle
  - Consequences for energetics
- What is the appropriate drag force model?
  - Finite Reynolds number effects
- Application to turbulent flow

# Challenges

- Parameter Space:  $\left(\frac{d_p}{\eta}, Re_p, St_\eta, \frac{\rho_p}{\rho_f}, \Phi_m, \Phi_v, Fr \dots\right)$
- Polydispersity/Shape Anisotropy
- Lack of Continuum
  - Euler-Lagrange (point-particle) model
  - Data transfer
  - Interpretation of particle statistics
- Particle Equation of Motion
  - Analytical and empirical correlations
- Verification (Two-way coupled)
  - Particle settling velocity/conversion of potential to kinetic energy
- Validation
  - Comparison with Fully Resolved Simulation/Experiment

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# Point-Particle origins

Anderson & Jackson (1967), more recently in Capecelatro & Desjardins (2012):

-Explicit filtering of fluid equations

$$\frac{\partial}{\partial t} \bar{\rho}_f \tilde{u}_i + \frac{\partial}{\partial x_j} \bar{\rho}_f \tilde{u}_i \tilde{u}_j = -\frac{\partial \bar{p}}{\partial x_i} + \frac{\partial \bar{\tau}_{ij}}{\partial x_j} + \frac{\partial \bar{R}_{ij}}{\partial x_j} + \bar{F}_{g,i} - \frac{1}{V} \sum_k^{N_p} F_d^k \bar{P} \{ \delta(x_i - x_i^k) \}$$

-volume fraction effects explicitly accounted for

Saffman (1973) “Point-Particle equations”

-Represent particles by a truncated multipole expansion

$$\frac{\partial}{\partial t} \rho_f u_i + \frac{\partial}{\partial x_j} \rho_f u_j u_i = -\frac{\partial p}{\partial x_i} + \mu \frac{\partial^2 u_i}{\partial x_j \partial x_j} + F_{g,i} - \frac{1}{V} \sum_k^{N_p} F_d^k P \{ \delta(x_i - x_i^k) \}$$

-volume fraction effects neglected, suitable for dilute flows

## Particle Equations

For each of  $N_p$  particles:

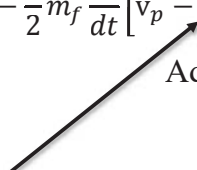
$$\frac{dx_p}{dt} = v_p$$

$$m_p \frac{dv_p}{dt} = (\rho_p - \rho_f)g + \int_S \sigma_{ij} n_j dS$$

Stokes (1850), Boussinesque (1885), Basset (1888), Faxen (1926), Stimson & Jeffrey (1926), Schiller & Nauman (193?), Oseen (1910), Proudman & Pearson (1957), Saffman (1965), Goldman, Cox, Brenner, (1967), Batchelor (1972), Batchelor & Green (1972), Maxey & Riley (1982), Lovelanti & Brady (1993), Mei (1994), Parmar, Haselbacher, Balachandar (2012)

Maxey-Riley Equation ( $Re_p \ll 1$ ):

$$m_p \frac{dv_p}{dt} = \underbrace{(\rho_p - \rho_f)g}_{\text{gravity}} + \underbrace{m_f \frac{D\tilde{u}_p}{Dt}}_{\text{Fluid accel.}} - \underbrace{\frac{1}{2} m_f \frac{d}{dt} \left[ v_p - \tilde{u}_p - \frac{1}{10} a^2 \nabla^2 \tilde{u}_p \right]}_{\text{Added Mass}} - \underbrace{6\pi a \mu \left( v_p - \tilde{u}_p - \frac{1}{6} a^2 \nabla^2 \tilde{u}_p \right)}_{\text{Stokes Drag}} - \underbrace{6\pi a^2 \mu \int_0^t \frac{d}{d\tau} \left[ v_p - \tilde{u}_p - \frac{1}{6} a^2 \nabla^2 \tilde{u}_p \right] d\tau}_{\text{History}} \frac{1}{[\pi \nu (t - \tau)]^{1/2}}$$



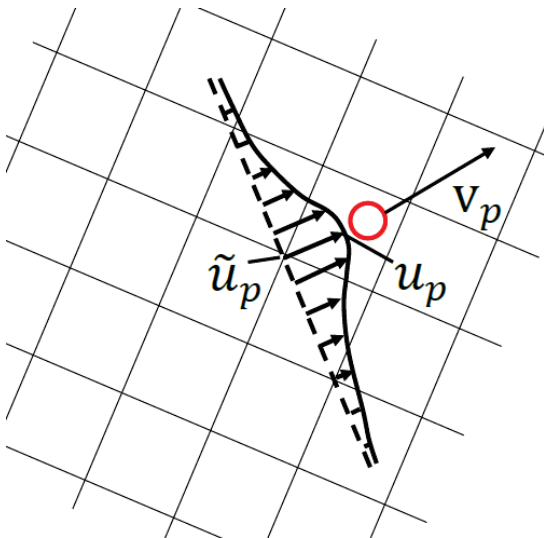
“Undisturbed”  
Fluid Velocity

# Stokes drag

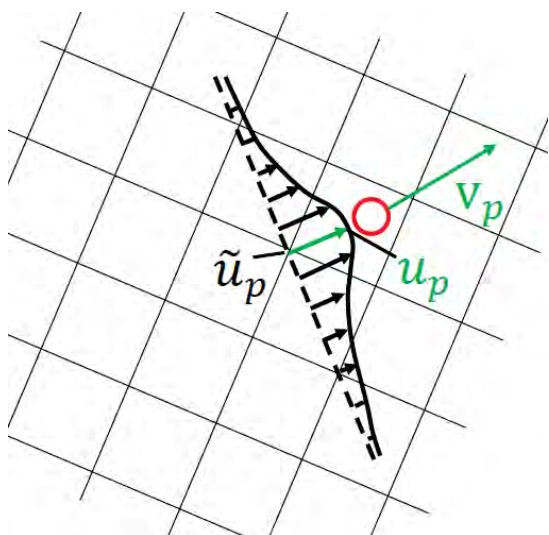
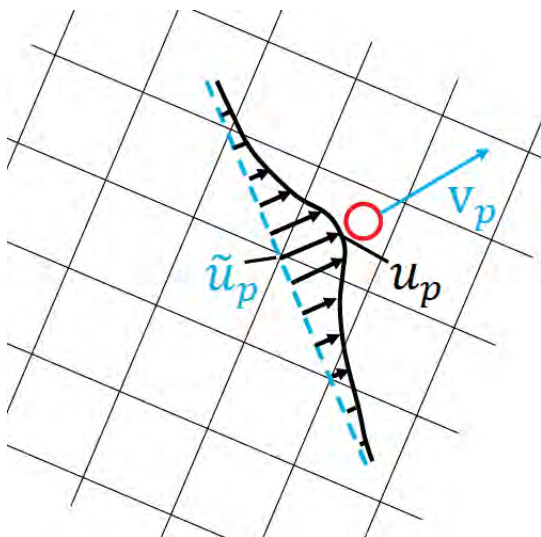
$$F_d = 3\pi\mu d_p(\tilde{u}_p - v_p)$$

Parameter Space:

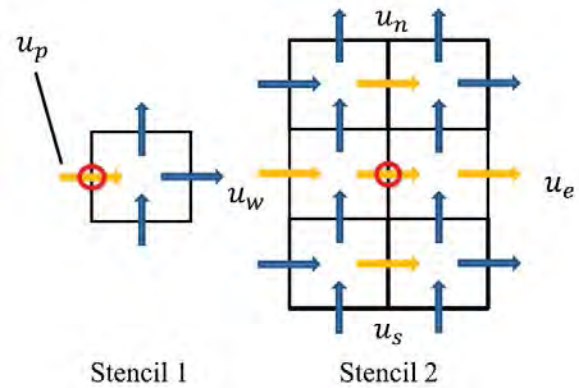
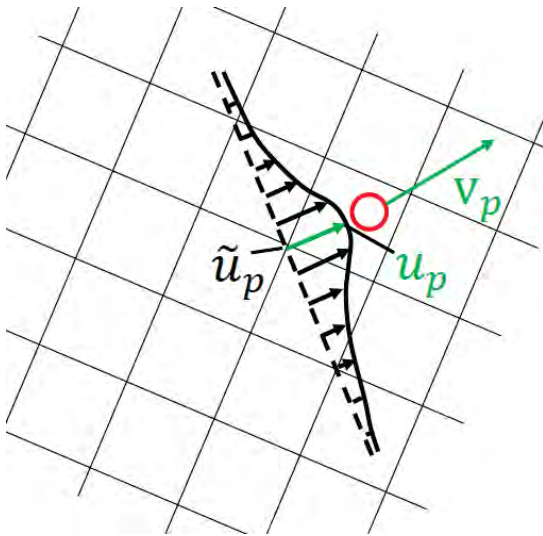
$$\left(\frac{d_p}{\eta} \leq 1, Re_p \leq 1, \frac{\rho_p}{\rho_f} \gg 1, \Phi_v \ll 1\right)$$







## Proposed Correction method

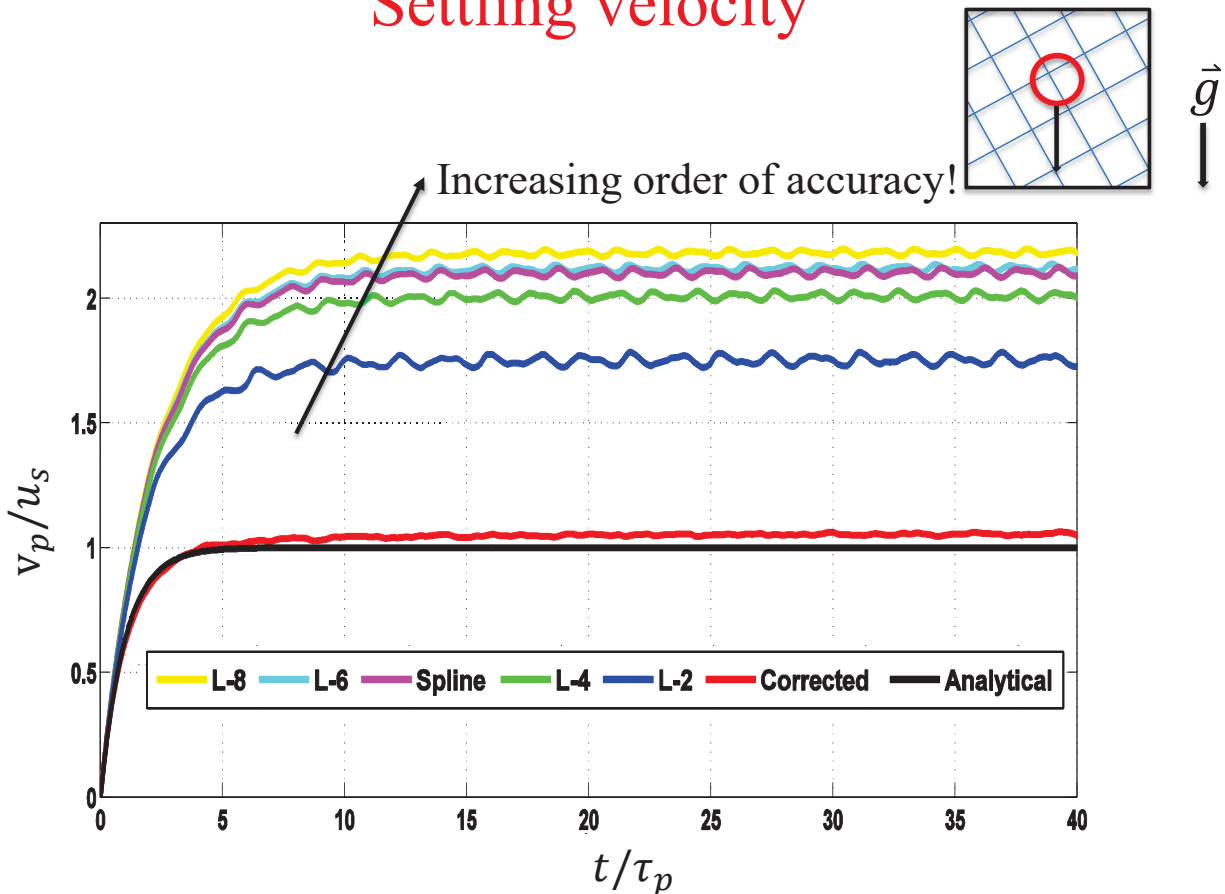


- Stencil 1:  $\tilde{u}_p \approx u_p$
- Stencil 2:  $\tilde{u}_p \approx u_p + C dx^2 \nabla^2 u_p$   

$$= (1 - 4C) \times u_p + C \times (u_n + u_s + u_e + u_w)$$
- Conservative
- Has been extended to general particle location in three dimensions

Horwitz J.A.K., Mani A., JCP, (2016)

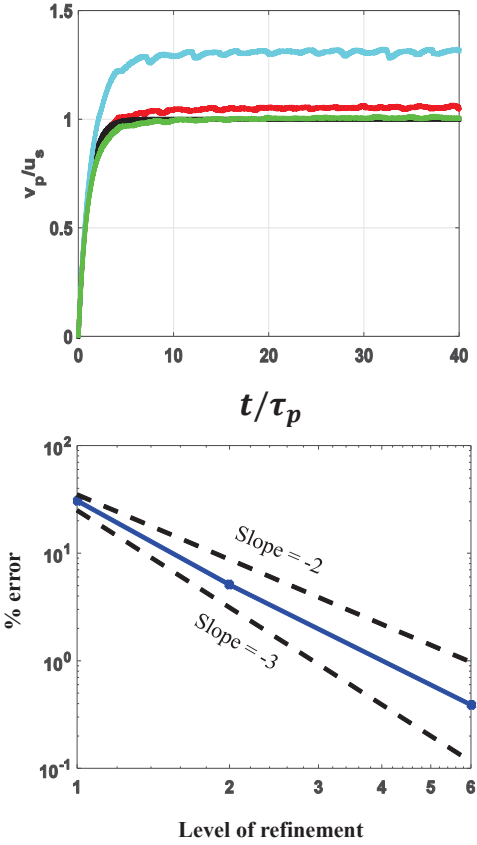
## Settling velocity



# C-refinement

Layer 1				Layer 2			
$C_{141}$	$C_{241}$	$C_{341}$	$C_{441}$	$C_{142}$	$C_{242}$	$C_{342}$	$C_{442}$
0.340	0.384	0.423	0.440	0.402	0.457	0.506	0.527
$C_{131}$	$C_{231}$	$C_{331}$	$C_{431}$	$C_{132}$	$C_{232}$	$C_{332}$	$C_{432}$
0.320	0.360	0.397	0.413	0.376	0.426	0.472	0.491
$C_{121}$	$C_{221}$	$C_{321}$	$C_{421}$	$C_{122}$	$C_{222}$	$C_{322}$	$C_{422}$
0.281	0.315	0.346	0.359	0.325	0.366	0.404	0.420
$C_{111}$	$C_{211}$	$C_{311}$	$C_{411}$	$C_{112}$	$C_{212}$	$C_{312}$	$C_{412}$
0.248	0.275	0.300	0.312	0.281	0.315	0.346	0.359
Layer 3				Layer 4			
$C_{143}$	$C_{243}$	$C_{343}$	$C_{443}$	$C_{144}$	$C_{244}$	$C_{344}$	$C_{444}$
0.478	0.545	0.604	0.629	0.518	0.591	0.655	0.681
$C_{133}$	$C_{233}$	$C_{333}$	$C_{433}$	$C_{134}$	$C_{234}$	$C_{334}$	$C_{434}$
0.443	0.504	0.559	0.582	0.478	0.545	0.604	0.629
$C_{123}$	$C_{223}$	$C_{323}$	$C_{423}$	$C_{124}$	$C_{224}$	$C_{324}$	$C_{424}$
0.376	0.426	0.472	0.491	0.402	0.457	0.506	0.527
$C_{113}$	$C_{213}$	$C_{313}$	$C_{413}$	$C_{114}$	$C_{214}$	$C_{314}$	$C_{414}$
0.320	0.360	0.397	0.413	0.340	0.384	0.423	0.440

Table B.1: Numerical values of  $C$  field coefficients for third level of fineness.



# Energetics

$$(1) \frac{dv_i}{dt} = (\tilde{u}_i - v_i)/\tau_p + (1 - \rho_f/\rho_p)g_i$$

**Particle Momentum**

$$(2) \frac{dk_p}{dt} = m_p v_i (\tilde{u}_i - v_i)/\tau_p + m_p (1 - \rho_f/\rho_p)g_i v_i$$

**Particle Kinetic Energy**

$$(3) \frac{\partial}{\partial t} \rho_f u_i + \frac{\partial}{\partial x_j} \rho_f u_j u_i = -\frac{\partial p}{\partial x_i} + \mu \frac{\partial^2 u_i}{\partial x_j \partial x_j} + F_{g,i} - \frac{1}{V} \sum_k^{N_p} F_d^k P\{\delta(x_i - x_i^k)\}$$

**Fluid Momentum**

$$(4) \frac{dk_f}{dt} = W_v - \epsilon_{pp}$$

**Fluid Kinetic Energy**

$$(5) W_v = \int \mu u_i \frac{\partial^2 u_i}{\partial x_j \partial x_j} dV = \int \left[ \mu \nabla^2 k - \epsilon + \mu \frac{\partial u_i}{\partial x_j} \frac{\partial u_j}{\partial x_i} \right] dV$$

**Viscous dissipation**

$$(6) \epsilon_{pp} = \int \frac{1}{V_{cell}} u_i F_{d,i} P\{\delta(x_i - x_i')\} dV = u_i m_p (\tilde{u}_i - v_i)/\tau_p$$

**Additional dissipation**

# Energetics

$$(1) \frac{dv_i}{dt} = (\tilde{u}_i - v_i)/\tau_p + (1 - \rho_f/\rho_p)g_i$$

**Particle Momentum**

$$(2) \frac{dk_p}{dt} = m_p v_i (\tilde{u}_i - v_i)/\tau_p + m_p (1 - \rho_f/\rho_p)g_i v_i$$

**Particle Kinetic Energy**

$$(3) \frac{\partial}{\partial t} \rho_f u_i + \frac{\partial}{\partial x_j} \rho_f u_j u_i = -\frac{\partial p}{\partial x_i} + \mu \frac{\partial^2 u_i}{\partial x_j \partial x_j} + F_{g,i} - \frac{1}{V} \sum_k^{N_p} F_d^k P\{\delta(x_i - x_i^k)\}$$

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**Additional dissipation**

$$(7) \frac{dk_f}{dt} + \frac{dk_p}{dt} = m_p \underbrace{\frac{(v_i - u_i)(\tilde{u}_i - v_i)}{\tau_p}}_{\text{Heat generation}} + \underbrace{W_v + m_p (1 - \rho_f/\rho_p)g_i v_i}_{\text{Work In}} \quad \text{Total Energy equation}$$

Change in KE

Heat generation

Work In

# Energetics

$$(\tilde{u} \rightarrow 0): Q_g = -m_p \frac{v_p^2}{\tau_p} = -3\pi\mu d_p v_p^2$$

$$(7) \frac{dk_f}{dt} + \frac{dk_p}{dt} = m_p \underbrace{\frac{(v_i - u_i)(\tilde{u}_i - v_i)}{\tau_p}}_{\text{Heat generation}} + \underbrace{W_v + m_p (1 - \rho_f/\rho_p)g_i v_i}_{\text{Work In}} \quad \text{Total Energy equation}$$

Change in KE

Heat generation

Work In

## What has been achieved...

### Developed two-way coupling correction for point-particles

- Stokes limit
- provided an interpretation of dissipation, how to post-process
- predicting  $\tilde{u}$  implies correct work rate
- applicable to 2<sup>nd</sup> order finite difference, uniform grid
- dilute gas-solid flows

## Still requiring more exploration...

- Non-uniform grid (channel flow)
- Finite Reynolds number
- Heat transfer
- Extension to full Maxey Riley equation, Saffman Lift, etc.
- Turbulent flows
- Multiple particle interaction (Stimson & Jeffrey 1926 and Batchelor & Green 1972)

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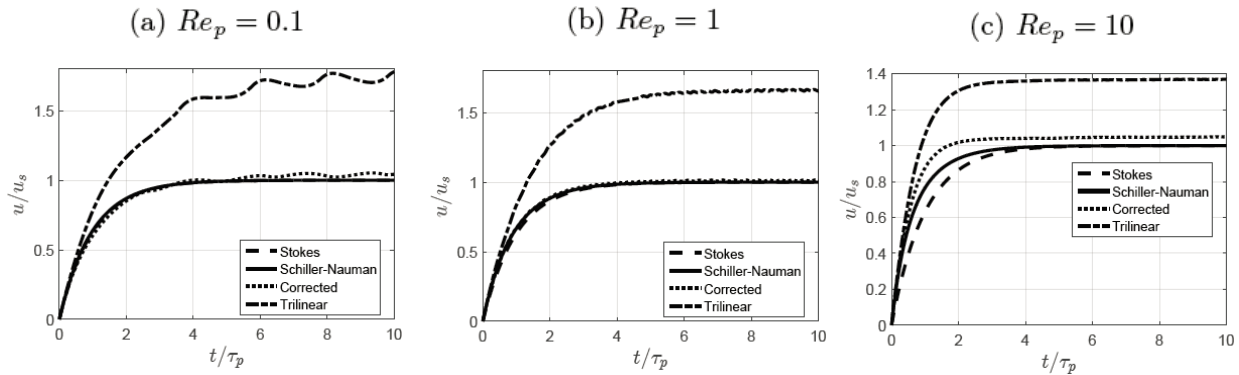
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# Nonlinear Drag

$$F_d = 3\pi\mu d_p(\tilde{u} - v_p)(1 + 0.15Re_p^{0.687}), \quad Re_p < 800, [+5\%, -4\%]$$

(Clift et al. 1978)



$$Re_p = |\tilde{u} - v_p|d_p/\nu$$

## Application to Turbulent Flow

### Particle-Resolved Simulations (Tennetti et al. 2010)

Fluid:

- Spatial: Pseudo-spectral solver
- Time-stepping: Adams-Bashforth (convective), Crank-Nicholson (Viscous)
- Immersed Boundary (satisfy no-slip/no-penetration)
- $N^3 = 1152^3$

Particles:

- Force found by integrating the stress over particle surface
- Elastic soft-sphere collisions (no contribution to energetics) (Cundall & Strack 1979)

### Point-particle Simulations (Pouransari et al. 2015)

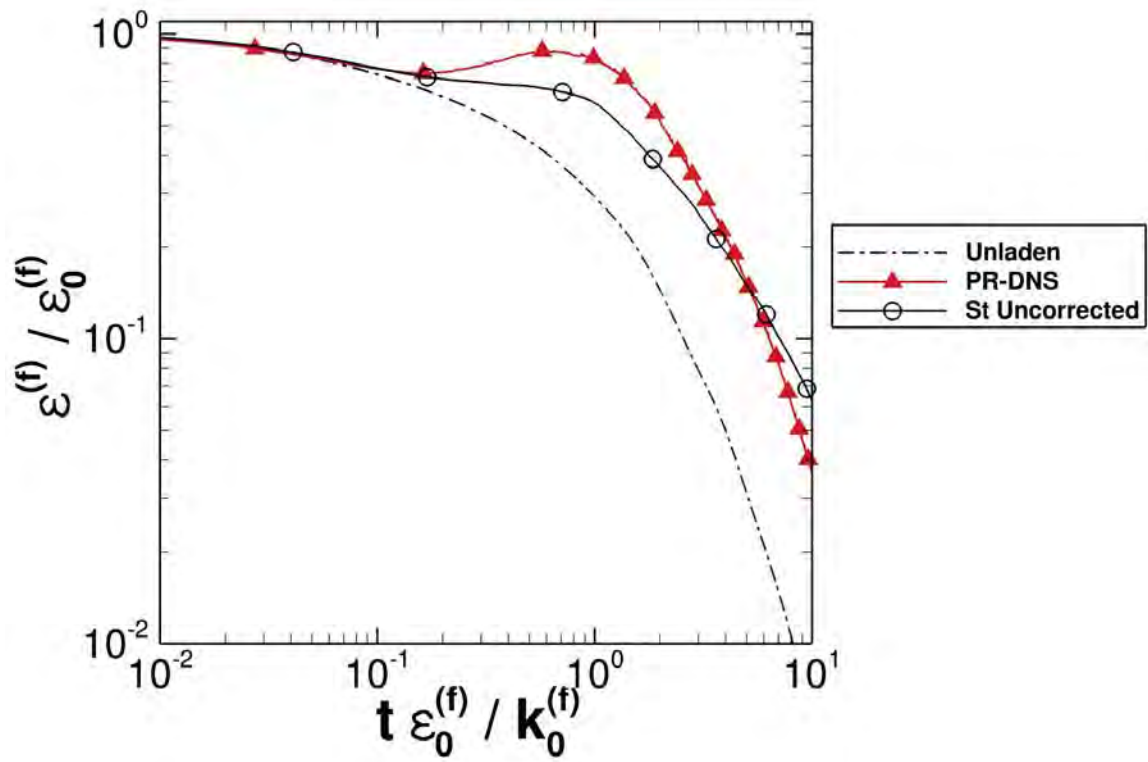
- Spatial: 2<sup>nd</sup> order staggered finite difference
- Time-stepping: RK4
- point (compact-support) forces
- $N^3 = 96^3$

Particles:

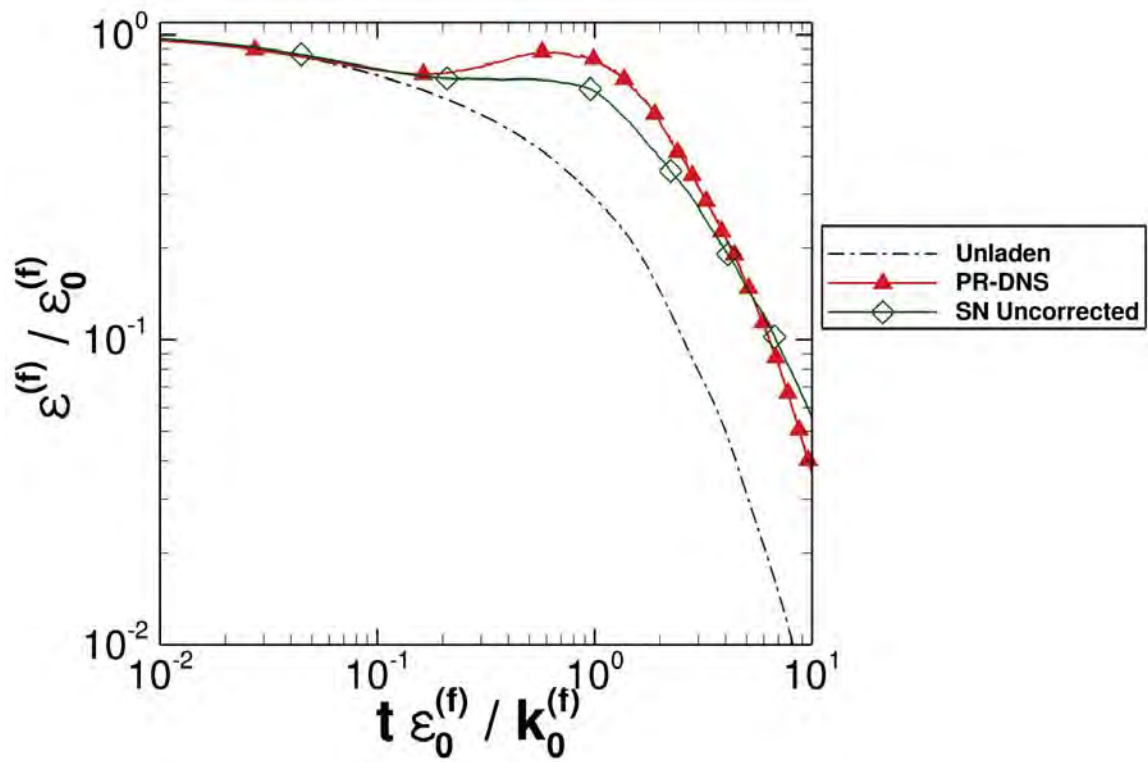
- Force model (Stokes and Schiller) (Horwitz & Mani 2016 a, b)
- collisions neglected

$$\Phi_m = 1.8, \phi_v = 0.001, \frac{d_p}{\eta} = 1, \frac{\rho_p}{\rho_f} = 1800, R_{\lambda,o} \approx 27, N_p = 1689$$

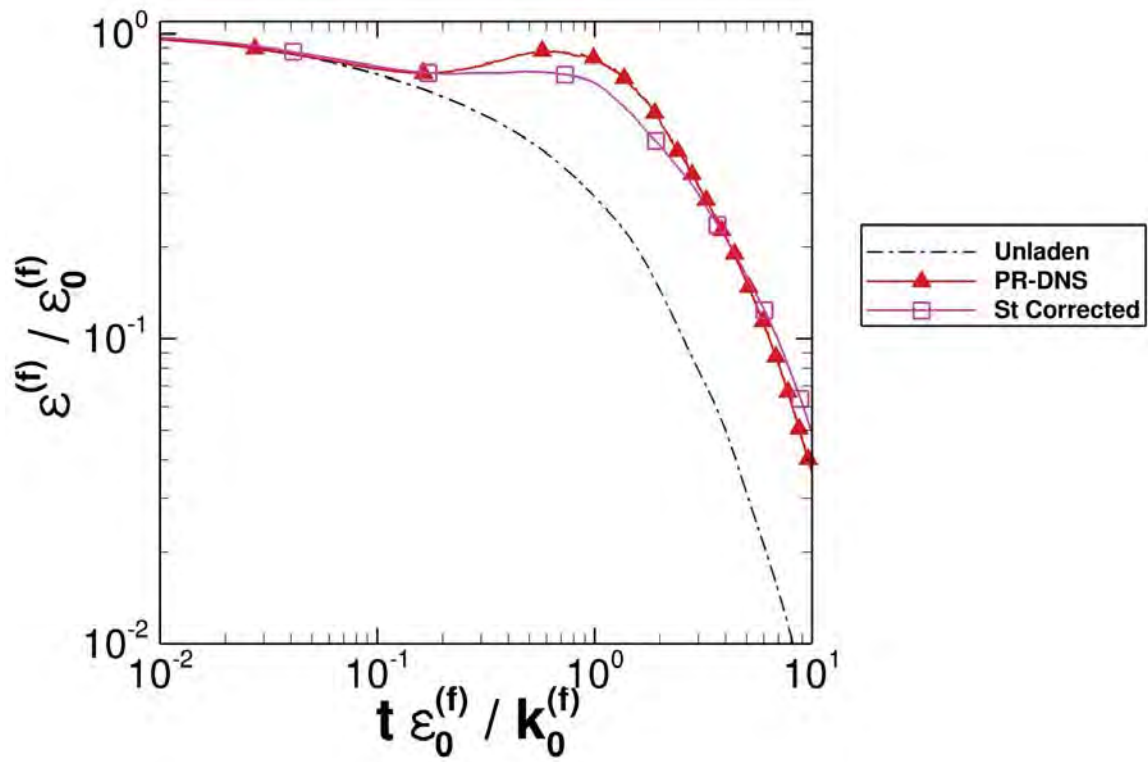
## Fluid Dissipation



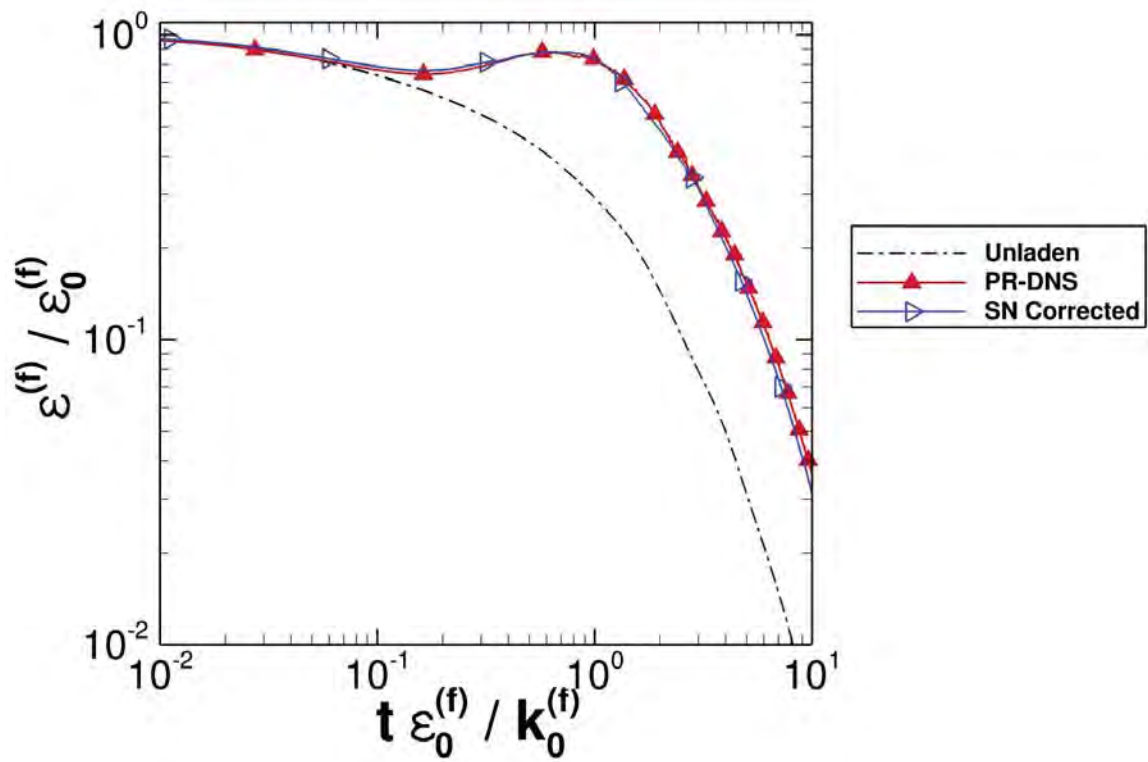
## Fluid Dissipation



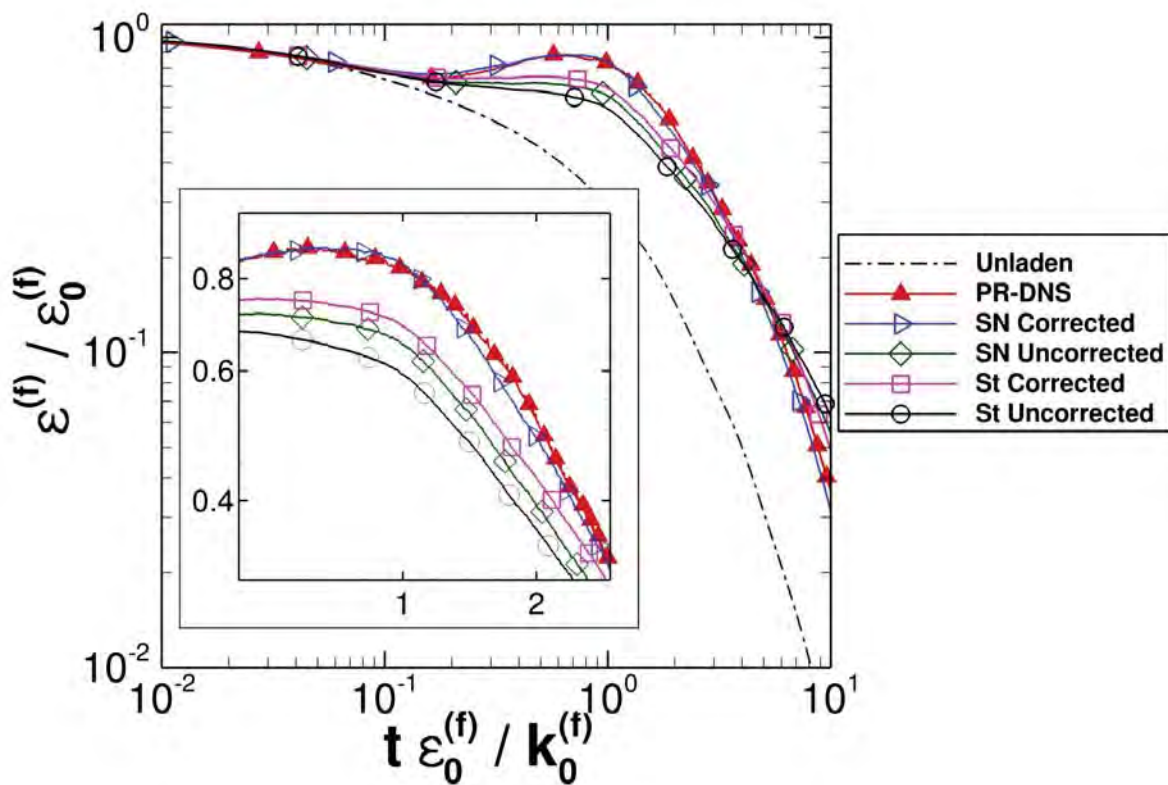
## Fluid Dissipation



## Fluid Dissipation



# Fluid Dissipation



*Stokes Uncorrected* → *Schiller – Nauman Uncorrected* →  
*Stokes Corrected* → *Schiller – Nauman Corrected* → *Particle – Resolved*

## What has been achieved...

Developed two-way coupling correction for point-particles

- Stokes limit
- provided an interpretation of dissipation, how to post-process
- predicting  $\tilde{u}$  implies correct work rate
- applicable to 2<sup>nd</sup> order finite difference, uniform grid
- dilute gas-solid flows

## Still requiring more exploration...

- Non-uniform grid (channel flow)
- Finite Reynolds number
- Heat transfer
- Extension to full Maxey Riley equation, Saffman Lift, etc.
- Turbulent flows
- Multiple particle interaction (Stimson & Jeffrey 1926 and Batchelor & Green 1972)

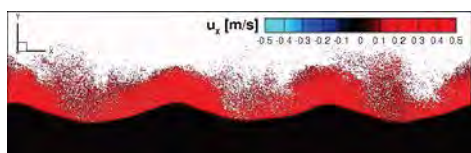
Questions?

# Quantifying and Modeling the Force Variation within Random Arrays of Spheres

*G. Akiki, T. L. Jackson and S. Balachandar*  
*Center for Compressible Multiphase Turbulence*  
*University of Florida*

Multiphase Physics Deep-Dive Meeting  
 St. Petersburg, Florida, Oct. 6-7, 2016

## Point-Particle Model

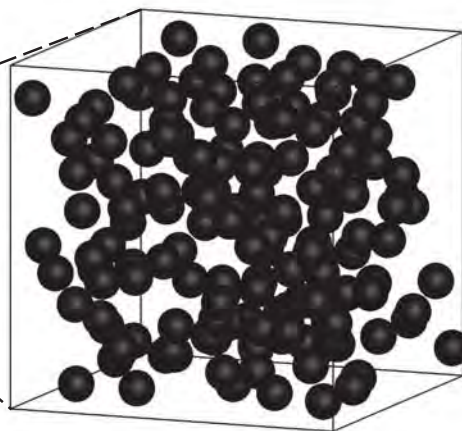
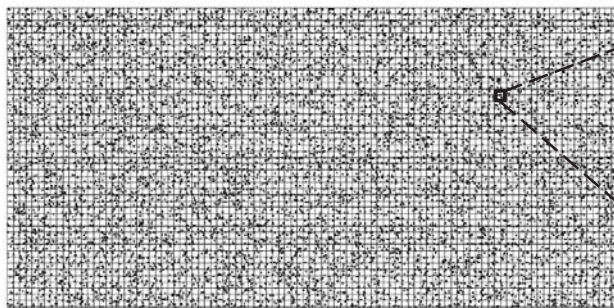


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Closure Models obtained at the microscale:

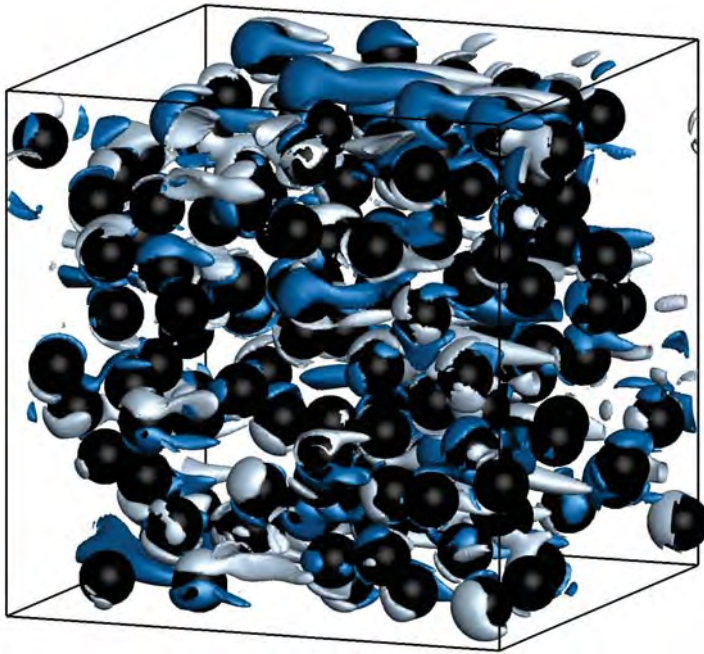
- ✓ Mean cell velocity
- ✓ Mean cell volume fraction
- Mean drag model:  $F(\phi, Re)$

- Exact location of particles ?





## Questions to Investigate



- How good are the current mean drag models?
- How significant is the variation of the forces from one particle to another within the same array?
- How significant are lateral forces?
- What governs those variations?
- Can we model it?

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## Outline

- Motivation
- Variation of Hydrodynamic Forces in Arrays of Spheres
- Pairwise Interaction Extended Point-particle (PIEP) Model
- PIEP Model in Sedimentation Problem
- Summary and Future Work

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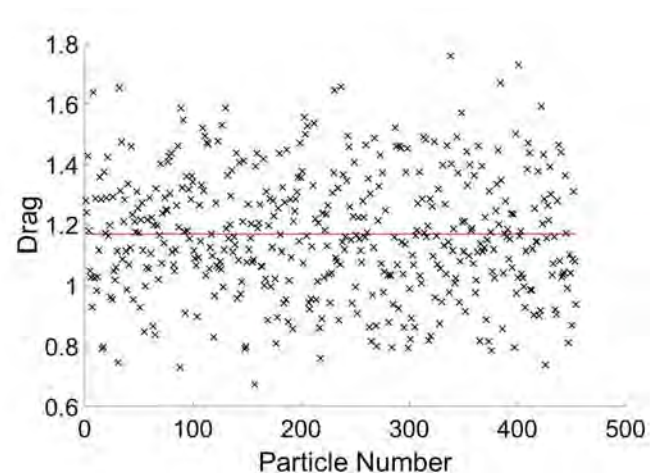
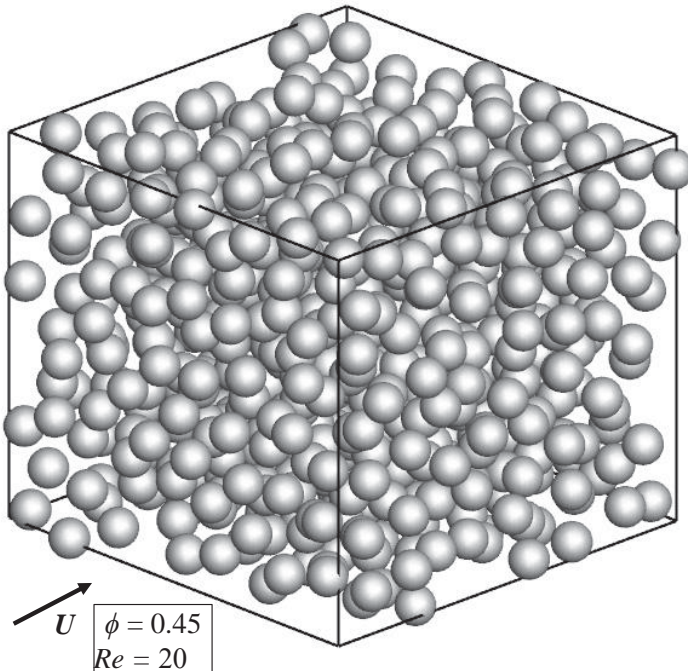
# Outline

- Motivation
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## Force Variations

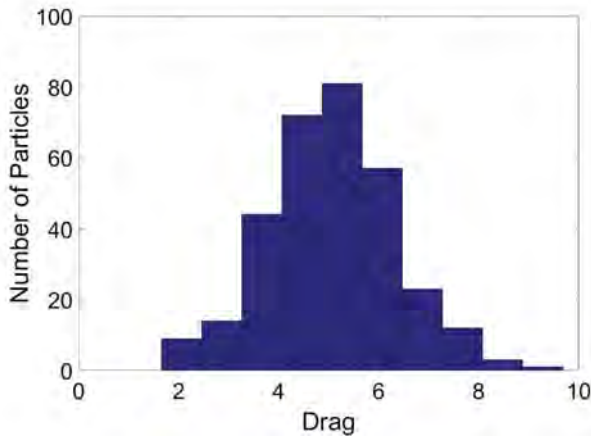
- Fully-Resolved DNS simulations of fixed random arrays  $0.1 \leq \phi \leq 0.45$   
 $2 \leq Re \leq 620$



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## Quantifying Drag Variations

Case	Mean Drag $F_d$	$\sigma$	$\sigma/F_d$	Skewness	Kurtosis
$\phi = 0.1, Re = 170$	4.69	1.26	26.95%	-0.07	3.25
$\phi = 0.2, Re = 87$	2.48	0.61	24.71%	-0.17	2.94
$\phi = 0.4, Re = 20$	1.17	0.20	17.31%	0.22	2.81

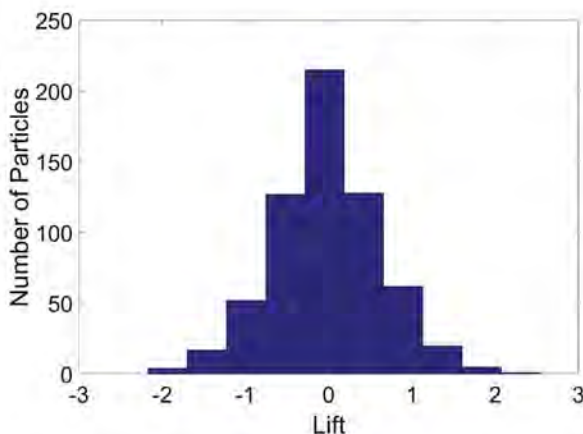


- Normal Distribution for all cases
- Standard deviation/mean drag decreases with volume fraction.
- Standard deviation/mean drag is mildly affected by  $Re$ .

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## Quantifying Lift Variations

Case	Mean Lift	$\sigma$	$\sigma/F_d$	Skewness	Kurtosis
$\phi = 0.1, Re = 170$	$-8.4 \times 10^{-4}$	0.66	14.15%	0.21	3.84
$\phi = 0.2, Re = 87$	$2.2 \times 10^{-3}$	0.33	13.19%	-0.02	3.60
$\phi = 0.4, Re = 20$	$3.9 \times 10^{-3}$	0.18	15.16%	-0.02	3.59



- Slightly higher Kurtosis, but still closer to normal distribution.
- Standard deviation/mean drag does not significantly vary with volume fraction nor  $Re$

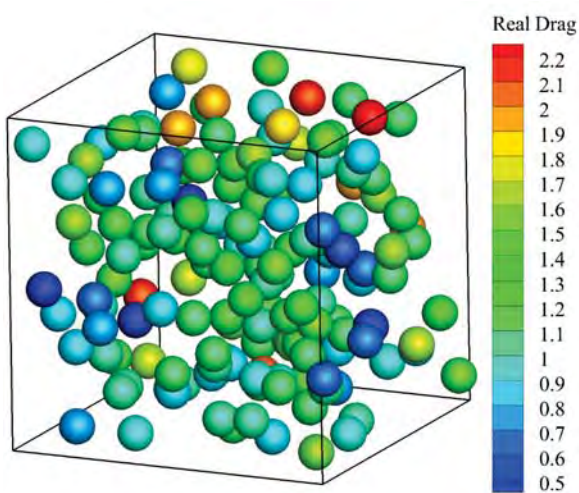
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# Outline

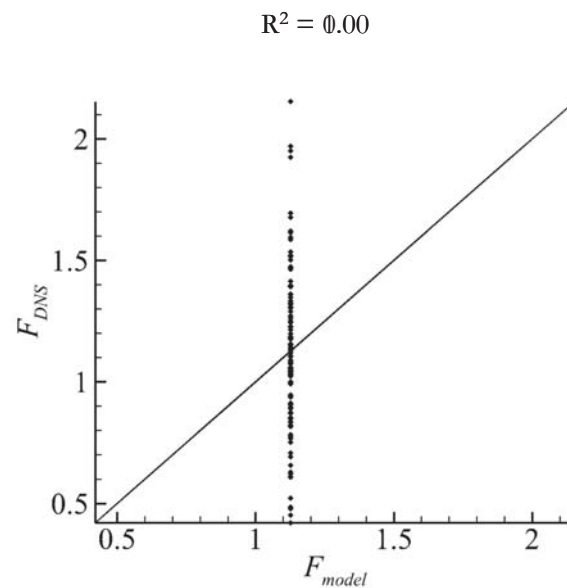
- Motivation
- Variation of Hydrodynamic Forces in Arrays of Spheres
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## Goal of Model



$$R^2 = 1 - \frac{\sum_{n=1}^{N_p} [F_{DNS}(n) - F_{PI}(n)]^2}{\sum_{n=1}^{N_p} [F_{DNS}(n) - \langle F_{DNS} \rangle]^2}$$



If the model is a fully-resolved DNS simulation!

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## Pairwise Interaction

Force on a single sphere in a steady non-uniform flow at finite  $Re$ :

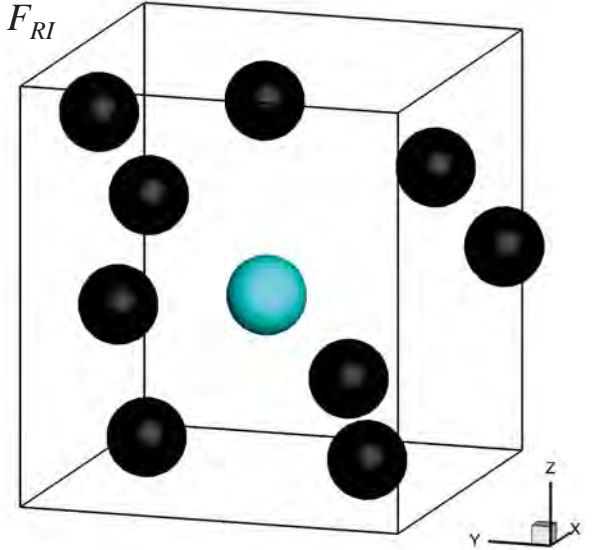
$$\mathbf{F} = \underbrace{3\pi\mu d \mathbf{u}_{un}}_{F_{QS}} (1 + 0.15 Re^{0.687}) + \underbrace{m_f (1 + C_m)}_{F_{II}} \frac{D\mathbf{u}_{un}}{Dt} + \underbrace{m_f C_{IL} \mathbf{u}_{un} \times \boldsymbol{\omega}_{un}}_{F_{RI}}$$

Undisturbed Velocity:

$$\mathbf{u}_{un,i}(\mathbf{x}_i) = \mathbf{u}_{macro}(\mathbf{x}_i) + \tilde{\mathbf{u}}_{\nabla N \rightarrow i}(\mathbf{x}_i)$$

Pairwise Interaction assumption:

$$\mathbf{u}_{un,i}(\mathbf{x}_i) \approx \mathbf{u}_{macro}(\mathbf{x}_i) + \sum_{j=1}^{N-1} \tilde{\mathbf{u}}_{j \rightarrow i}(\mathbf{x}_i)$$



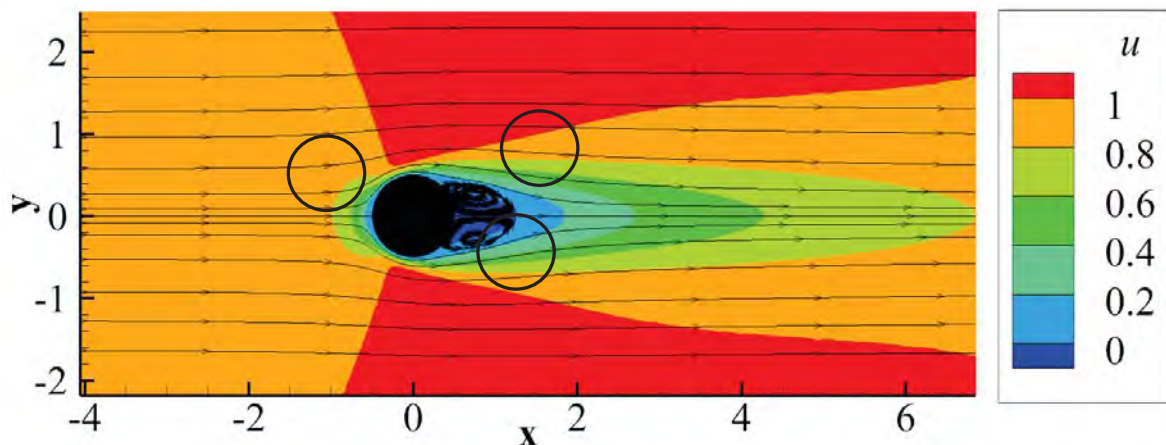
## Effect on Neighbor

$$\mathbf{F}_{QS} = 3\pi\mu \overline{(\mathbf{u}_{un}(\mathbf{x}))}^S d \left[ 1 + 0.15 (Re)^{0.687} \right]$$

$$\mathbf{F}_{II} = (1 + C_m) \rho \iiint_{|\mathbf{x} - \mathbf{x}_i| \leq d/2} \frac{D\mathbf{u}_{un}(\mathbf{x})}{Dt} d\mathbf{x}$$

$$\mathbf{F}_{RI} \approx C_{IL} \rho \iiint_{|\mathbf{x} - \mathbf{x}_i| \leq d/2} [\mathbf{u}_{un}(\mathbf{x}) \times \boldsymbol{\omega}_{un}(\mathbf{x})] d\mathbf{x}$$

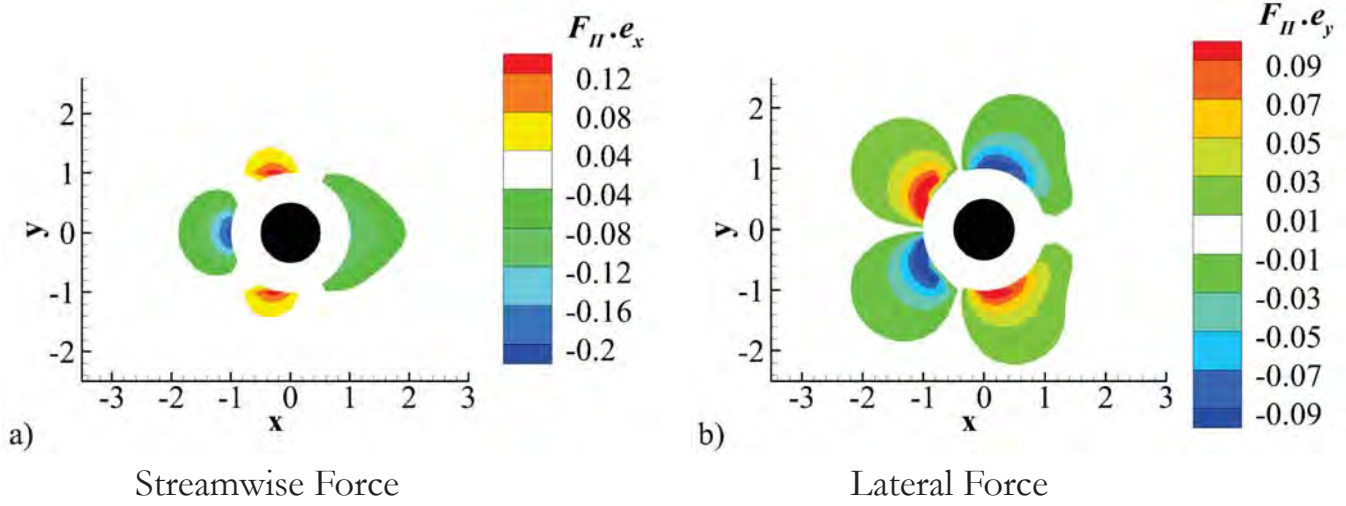
We use the Generalized Faxén's theorem to account for non-uniformity.





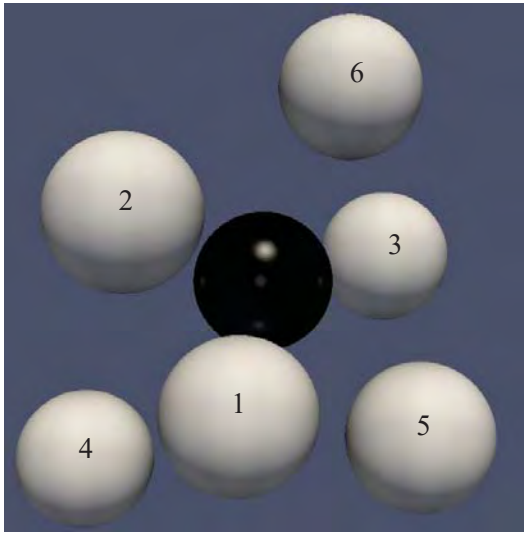
## Irrotational Inviscid Force Mappings

$$\mathbf{F}_{II} = (1 + C_m) \rho \iint_{|x-x_i|=d/2} [\boldsymbol{\tau} - p\mathbf{I}(\mathbf{x})] \cdot \mathbf{n} dA$$



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## Pairwise Interaction Extended Point-Particle (PIEP) Model



$$\mathbf{F}_{QS,PI}(i) = \frac{3\pi}{Re} \left[ \{1, 0\} + \sum_{j=1}^P \overline{(\mathbf{u}_{un}(\mathbf{x}_i - \mathbf{x}_j))}_{map}^S - \{1, 0\} \right] \left[ 1 + 0.15 (Re_{surf}(i))^{0.687} \right]$$

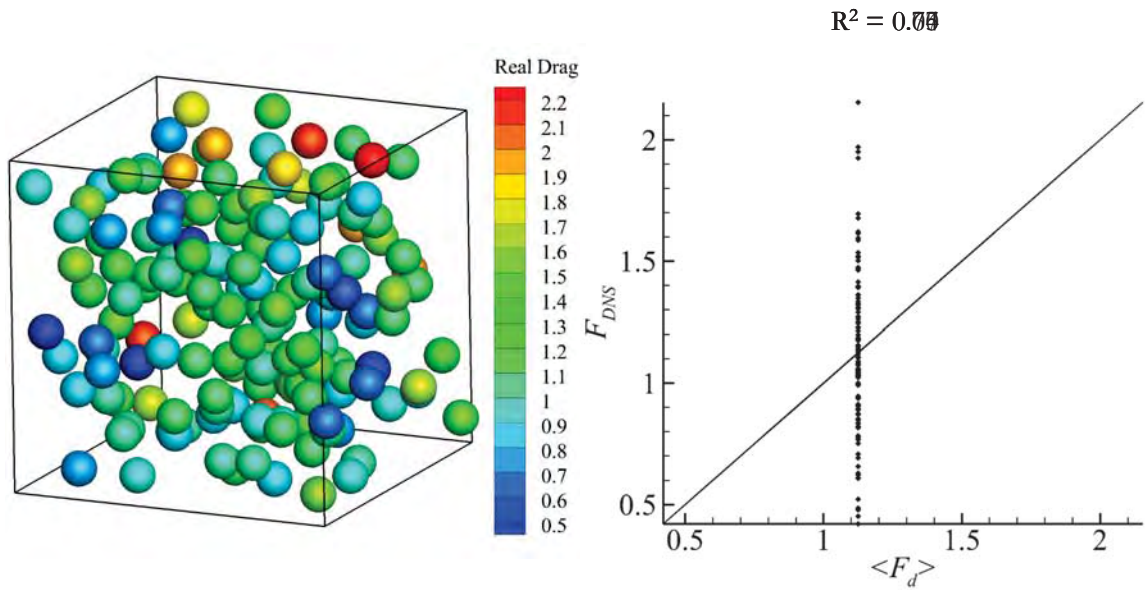
$$\mathbf{F}_{II,PI}(i) = \sum_{j=1}^P \mathbf{F}_{II,map}(\mathbf{x}_i - \mathbf{x}_j)$$

$$\mathbf{F}_{RI,PI}(i) = \sum_{j=1}^P \mathbf{F}_{RI,map}(\mathbf{x}_i - \mathbf{x}_j)$$

$$\mathbf{F}_{PIEP} = \mathbf{F}_{QS,PI} + \mathbf{F}_{II,PI} + \mathbf{F}_{RI,PI}$$

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## PIEP vs DNS for drag

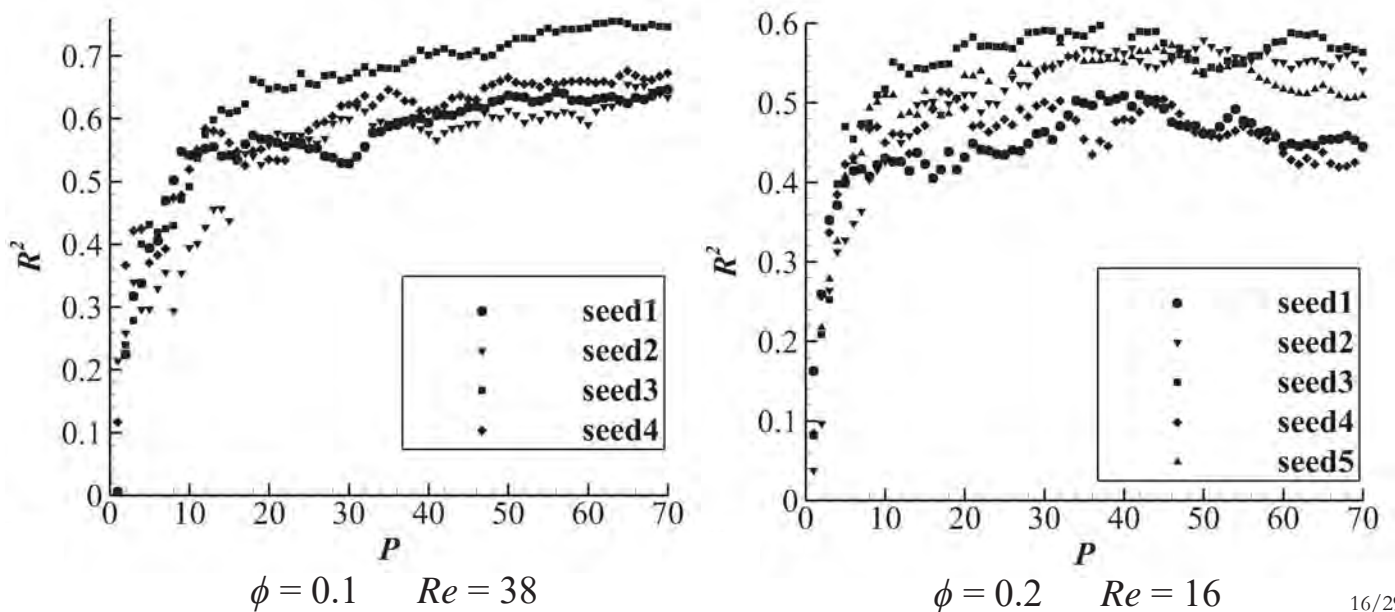


With Quasi-Steady, Finite-Volume and Rotational Force Correction

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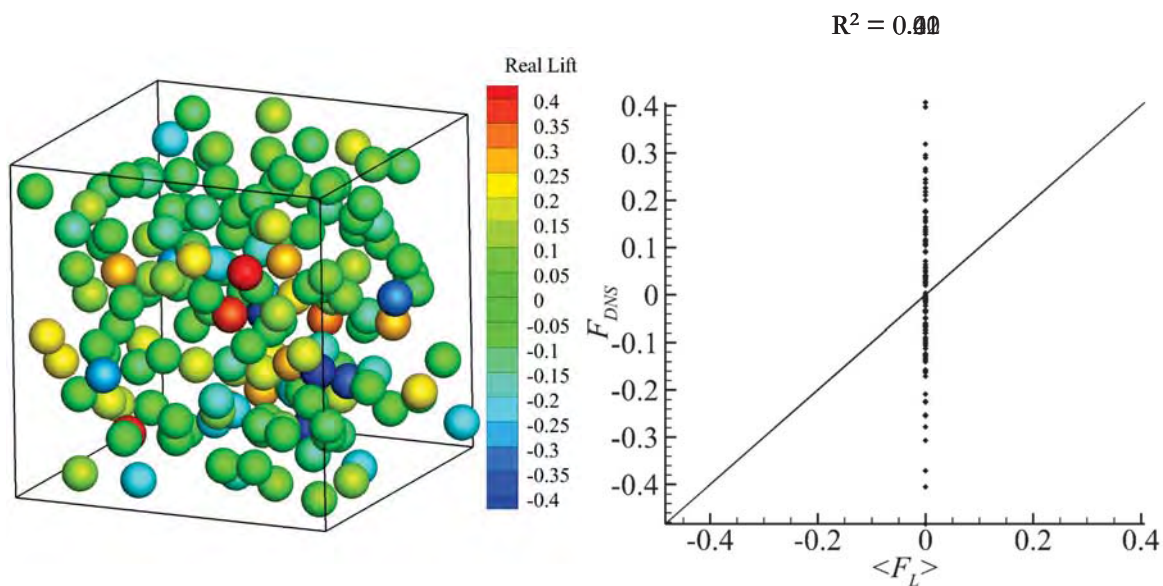
## PIEP Using Limited Neighbors

$P$  is the number of neighbors included in the model



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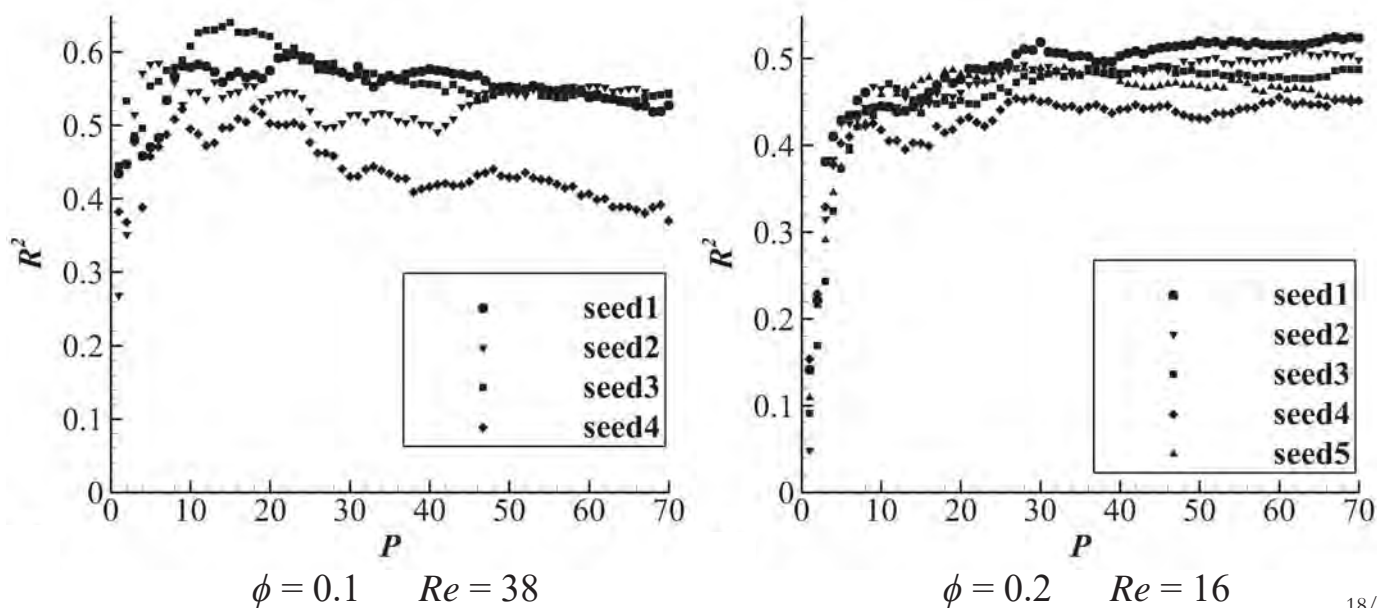
## PIEP vs DNS for lift



With Quasi-Steady, Finite-Volume, and Rotational-Induced Force Correction

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## PIEP Using Limited Neighbors (lift)



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# Outline

- Motivation
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## Using PIEP for Sedimentation

Drafting-Kissing-Tumbling test case

Equations of Motion:

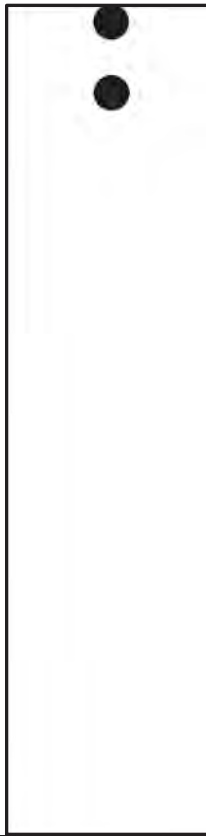
$$\begin{cases} \frac{d\tilde{\mathbf{x}}_i}{d\tilde{t}} = \tilde{\mathbf{v}}_i \\ \tilde{m}_i \frac{d\tilde{\mathbf{v}}_i}{d\tilde{t}} = (\tilde{m}_i - \tilde{m}_{fi}) \tilde{\mathbf{g}} + \tilde{\mathbf{F}}_{hyd,i} + \tilde{\mathbf{F}}_{lu,i} + \tilde{\mathbf{F}}_{col,i} \\ \tilde{I}_i \frac{d\tilde{\boldsymbol{\Omega}}_i}{d\tilde{t}} = \tilde{\mathbf{T}}_{hyd,i} + \tilde{\mathbf{T}}_{lu,i} + \tilde{\mathbf{T}}_{col,i} \end{cases}$$

$F_{hyd}$  is obtained using PIEP to account for the position of the second sphere.

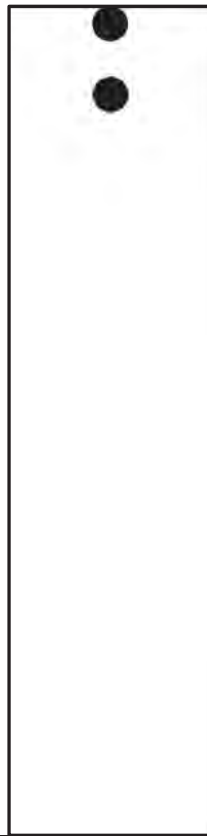


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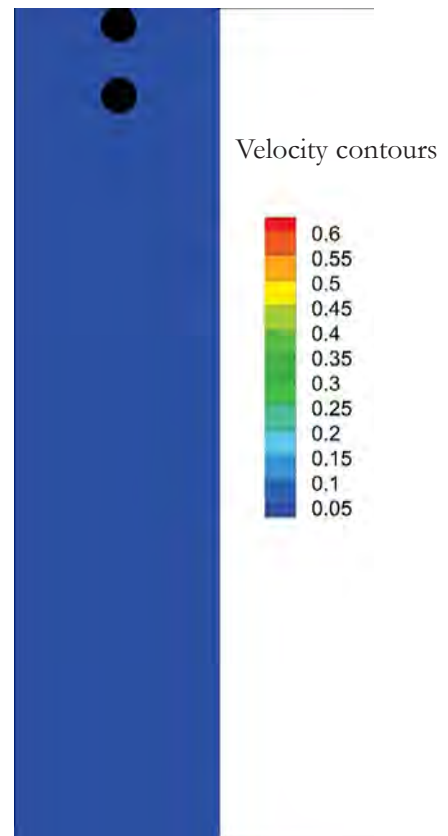
DEM w/ Standard Drag Model



DEM w/ PIEP Model



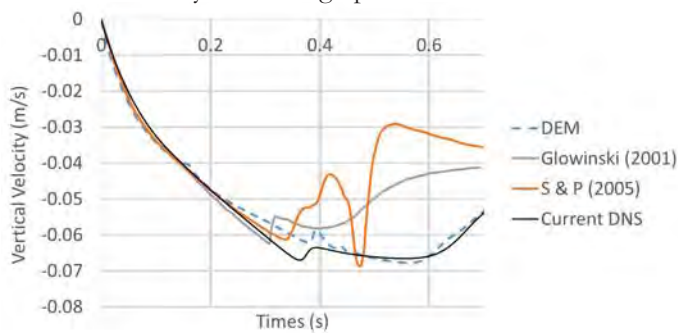
DNS



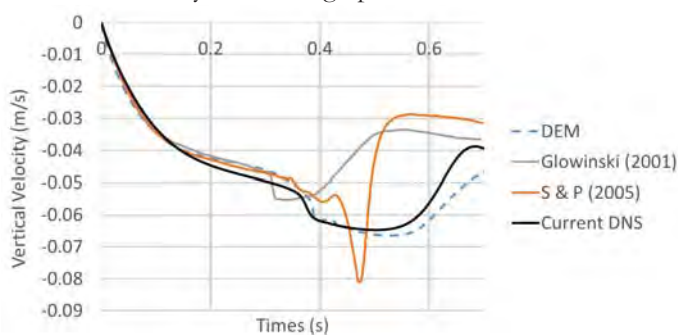
21/29

## Vertical Velocity Comparison

Velocity of Trailing Sphere



Velocity of Leading Sphere



### Parameters:

$$\begin{aligned}\frac{\rho_p}{\rho_f} &= 1.14 \\ \nu &= 1 \times 10^{-6} \text{ m}^2/\text{s} \\ d &= 1/600 \text{ m} \\ Re_{p(\max)} &\approx 112\end{aligned}$$

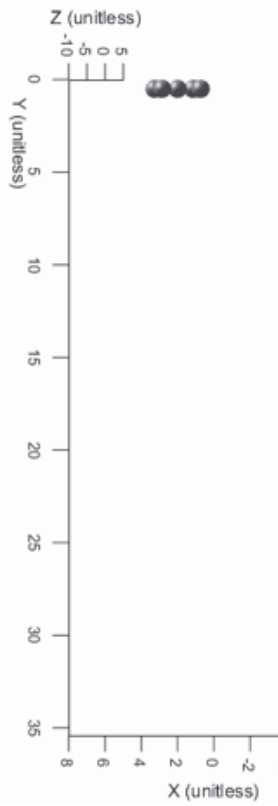
### Other studies running the same case using DNS:

- Glowsinki, JCP 2001
- Sharma & Patankar, JCP 2005

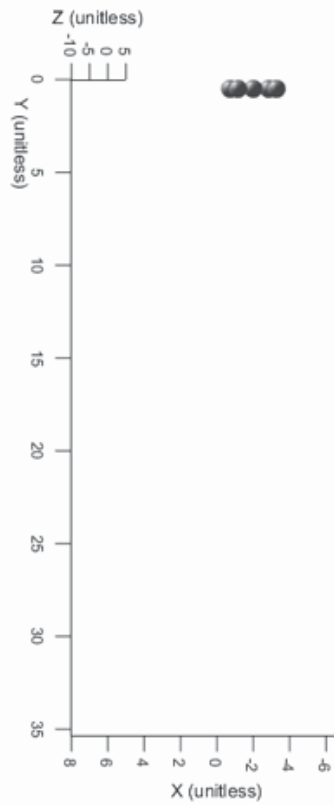
22/29



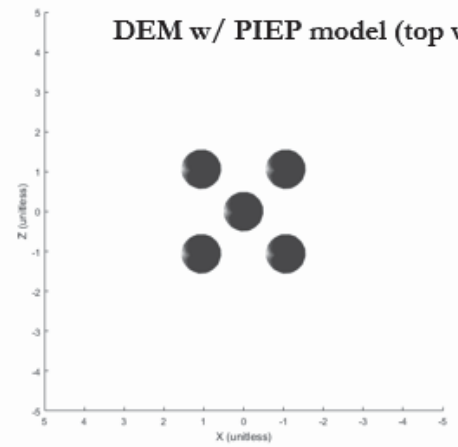
DEM w/ PIEP model



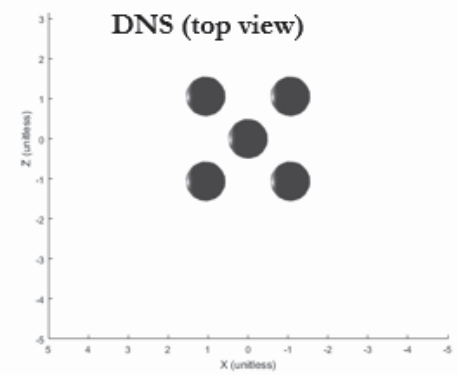
DNS



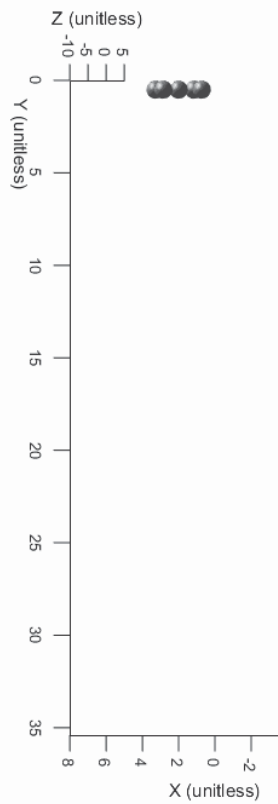
DEM w/ PIEP model (top view)



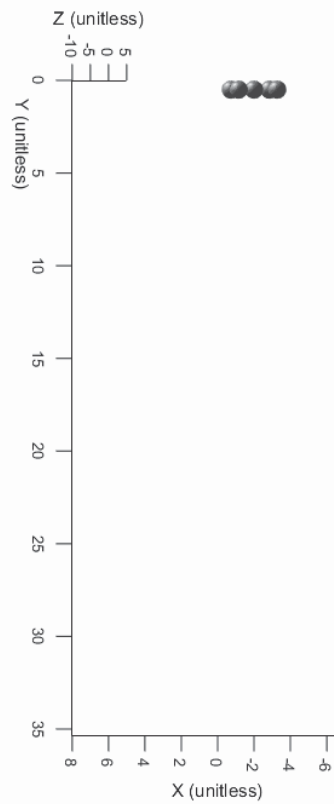
DNS (top view)



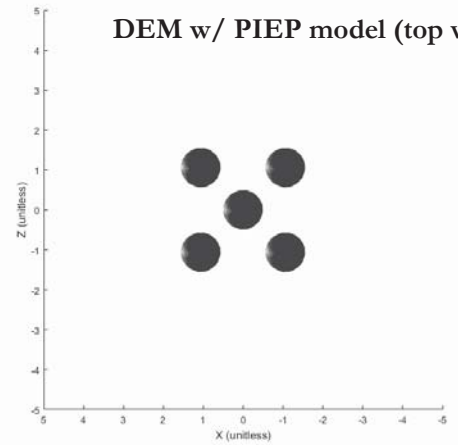
DEM w/ PIEP model



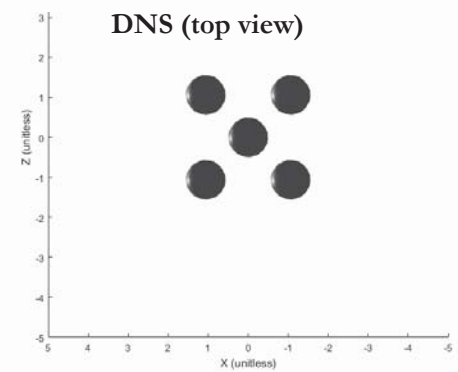
DNS



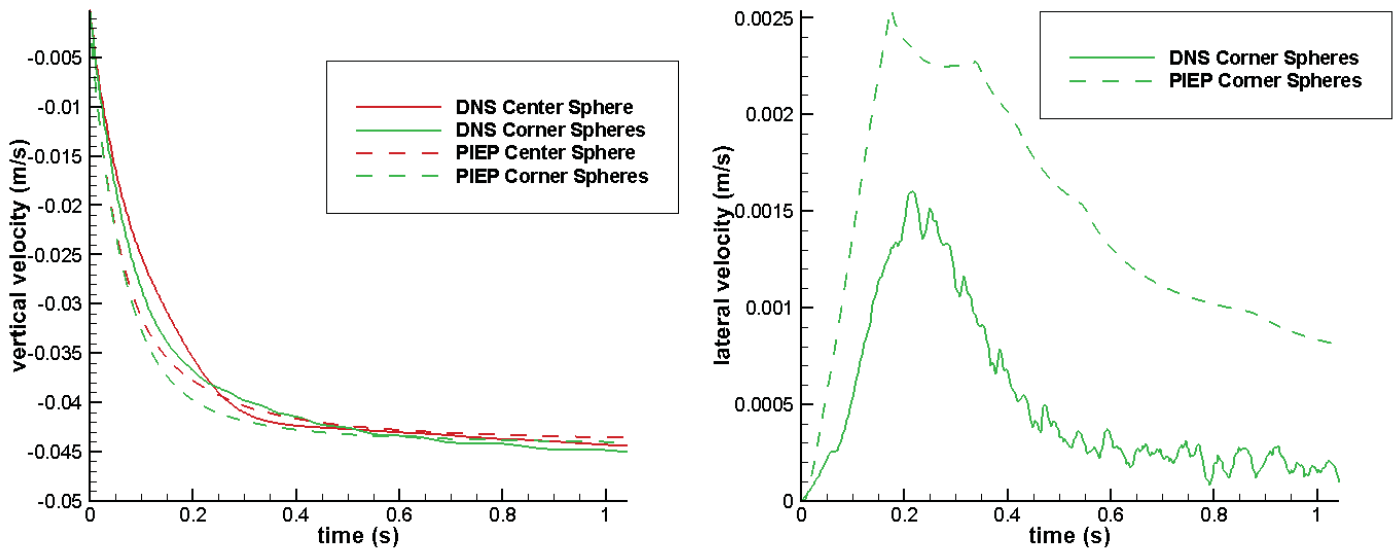
DEM w/ PIEP model (top view)



DNS (top view)



## Velocity Comparison



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## Summary and Future Work

- Quantification of the force variations inside arrays of spherical particles.
- Quantified the drag and lateral force variations within random arrays of spheres.
  - Standard deviation of the drag distribution is very significant with respect to the mean (~17%-27%) and decreases with the volume fraction.
  - Standard deviation of the lateral force distribution is approximately 14% of the mean drag.
- We developed a model (PIEP) which can approximate the drag and lateral force on each particle using the relative locations of few neighboring particles.

### Future Work:

- Implementing PIEP model in Eulerian-Lagrangian simulations and compare statistics with classical point-particle models.
- Using PIEP model to explore sedimentation problems with a large number of particles.

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## Acknowledgment

---

This work was supported by the U.S. Department of Energy, National Nuclear Security Administration, Advanced Simulation and Computing Program, as a Cooperative Agreement to the University of Florida under the Predictive Science Academic Alliance Program, under Contract No. DE-NA0002378.

This work was done in part at Los Alamos National Lab, during a visit hosted by Dr. Marianne Francois.

Thank you  
Questions?

# RECENT INSIGHTS ON TURBULENCE MODELING OF STRONGLY-COUPLED PARTICLE-LADEN FLOWS

## MULTIPHASE PHYSICS DEEP-DIVE

JESSE CAPECELATRO<sup>1,2</sup>

<sup>1</sup>DEPARTMENT OF MECHANICAL ENGINEERING, UNIVERSITY OF MICHIGAN

<sup>2</sup>CETNER FOR EXASCALE SIMULATION OF PLASMA COUPLED COMBUSTION  
(XPACC), UNIVERSITY OF ILLINOIS



MichiganEngineering

Multiphase Physics Deep-Dive |

Introduction

Modeling Aspects

VF-EL Framework

Results & Discussion

Conclusions

### XPACC @ Illinois

- Positions
  - postdoc (2014—2015)
  - research scientist (2015—2016)
- Advisors: Jonathan Freund, Daniel Bodony



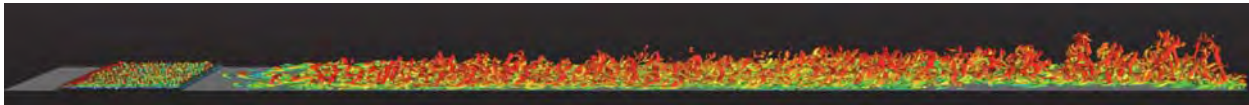
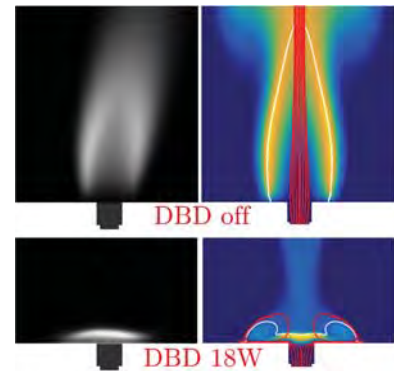
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Multiphase Physics Deep-Dive | 2

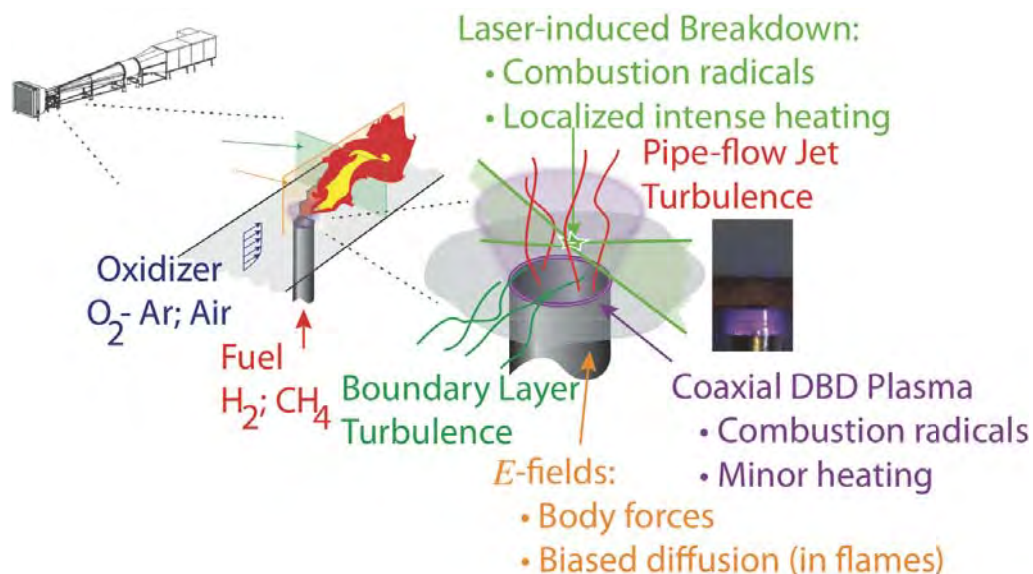
## XPACC @ Illinois: Overarching problem

- **Combustion** is our energy workhorse and will remain so in the foreseeable future
- Truly **predictive** simulations will accelerate fundamental advances in the use of **plasmas** to advance combustion technology



- The computer science **advances** that enable such massive-scale predictive simulations will have impact across engineering and science

## XPACC @ Illinois: Target application



- Predict the ignition threshold of a jet in crossflow via thermal discharge and dielectric-barrier-discharge (DBD) plasmas
- Canonical combustion flow with new 'knobs' for mediating ignition



## Particle work in XPACC

- No physical particles, per se
- Developing an integrated particle-based discretization for plasma kinetics (PIC)
- Two research projects
  - Adjoint-based sensitivity of PIC methods
  - High-order/high-resolution scalable PIC
- Contact [jbfreund@illinois.edu](mailto:jbfreund@illinois.edu) for more info



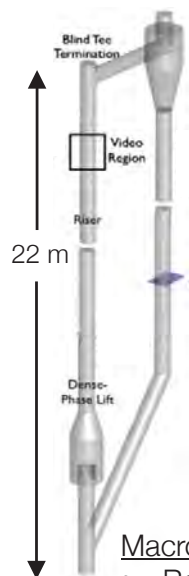
## On to Predictive Modeling of Turbulent Particle-Laden flows

### Collaborators:

- Olivier Desjardins, Cornell University
- Rodney Fox, Iowa State University



## Multi-scale nature of strongly-coupled gas-solid flows

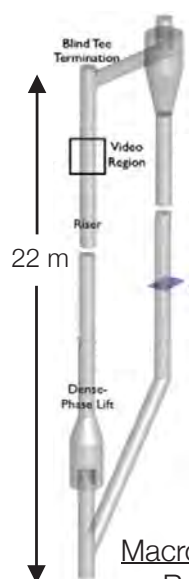


### Macroscale

- Reactor geometry:  $O(10)$  m
- Number of particles:  $O(10^9)$



## Multi-scale nature of strongly-coupled gas-solid flows



### Mesoscale

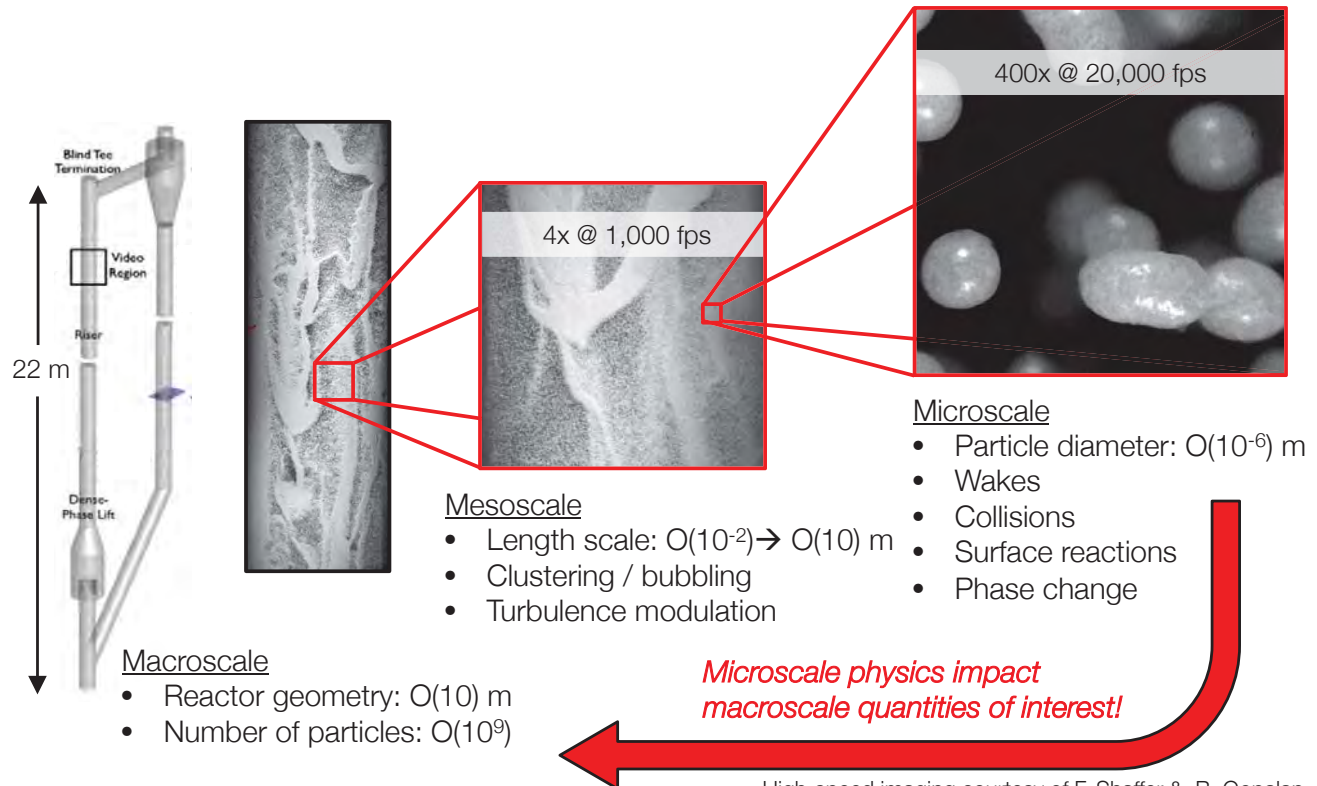
- Length scale:  $O(10^{-2}) \rightarrow O(10)$  m
- Clustering / bubbling
- Turbulence modulation

### Macroscale

- Reactor geometry:  $O(10)$  m
- Number of particles:  $O(10^9)$



# Multi-scale nature of strongly-coupled gas-solid flows



High-speed imaging courtesy of F. Shaffer & B. Gopalan  
D.O.E. National Energy Technology Lab



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## Multi-scale nature of strongly-coupled gas-solid flows

### Key objectives

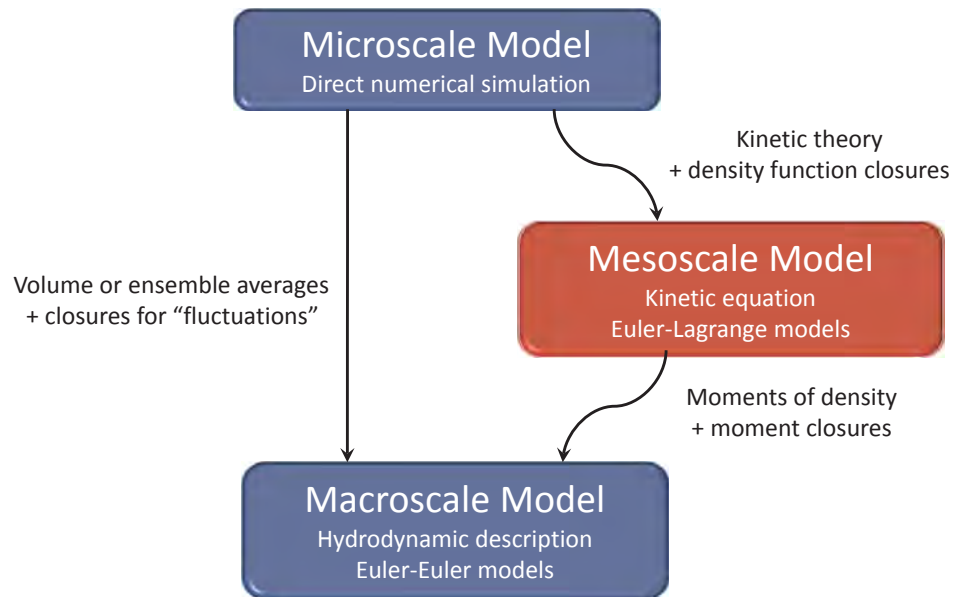
1. Develop accurate and tractable methods that capture detailed mesoscale features
2. Extract useful data from simulations to inform reduced-order model closure
3. Develop a predictive turbulence model valid across granular regimes



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Multiphase Physics Deep-Dive | 12

# Towards a macroscopic turbulence model<sup>1</sup>



*Mesoscale model incorporated more microscale physics in closures!*

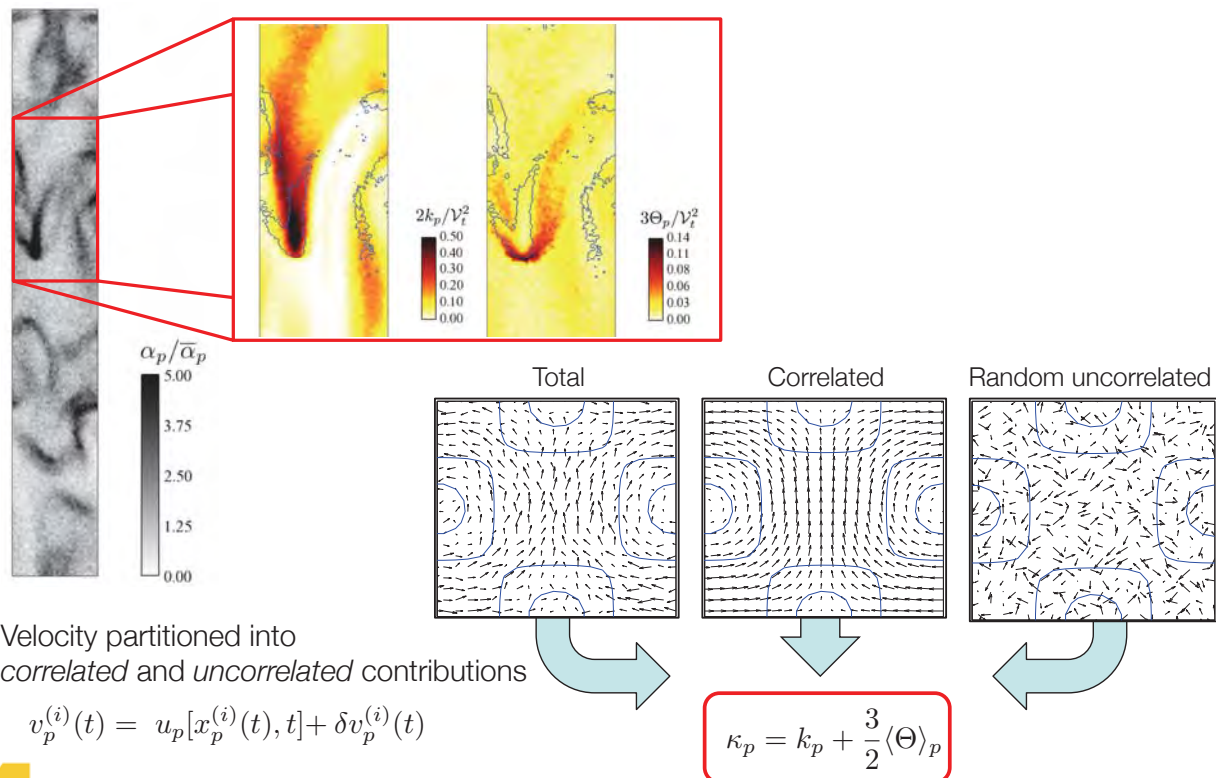
1. R.O. Fox (2012), *Annu. Rev. Fluid Mech.*



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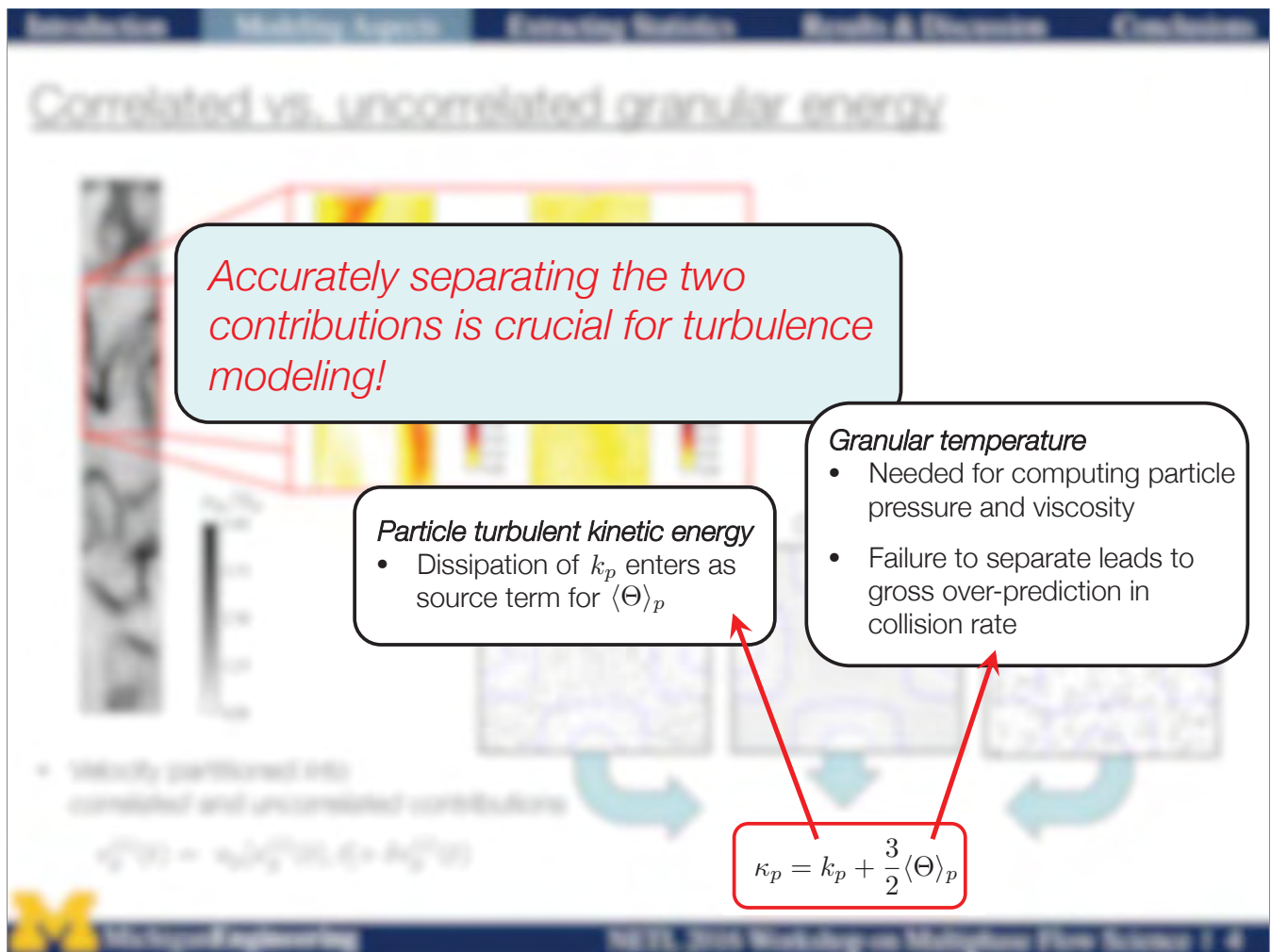
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# Correlated vs. uncorrelated granular energy



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## Exact Reynolds-average equations<sup>1</sup>

- Homogeneous gravity-driven flow: Fluid-phase Reynolds stress

$$\begin{aligned}
 \frac{1}{2} \frac{\partial \langle u_{f,1}''^2 \rangle_f}{\partial t} = & \frac{1}{\rho_f} \left( \left\langle p_f \frac{\partial u_{f,1}'''}{\partial x_1} \right\rangle - \left\langle \sigma_{f,1i} \frac{\partial u_{f,1}'''}{\partial x_i} \right\rangle \right) & PS/VD \\
 & + \frac{\varphi}{\tau_p} \left( \langle u_{f,1}''' u_{p,1}'' \rangle_p - \langle u_{f,1}''^2 \rangle_p \right) & DE \\
 & + \frac{\varphi}{\tau_p} \langle u_{f,1}''' \rangle_p \langle u_{p,1} \rangle_p & DP \\
 & + \frac{\varphi}{\rho_p} \left( \left\langle u_{f,1}''' \frac{\partial p'_f}{\partial x_1} \right\rangle_p - \left\langle u_{f,1}''' \frac{\partial \sigma'_{f,1i}}{\partial x_i} \right\rangle_p \right) & PE/VE
 \end{aligned}$$

- Fluid-phase TKE is generated by the **drag production (DP)** term, as opposed to in single-phase turbulence where it is produced by mean velocity gradients.
- Fluid velocity seen by the particles:  $\langle u_f \rangle_p = \langle \alpha_p u_f \rangle / \langle \alpha_p \rangle$
- Because clusters arise spontaneously in CIT, and are negatively correlated with the fluid velocity  
 $\langle u_f \rangle_f = \langle \alpha_f u_f \rangle / \langle \alpha_f \rangle \neq \langle u_f \rangle_p$



# Exact Reynolds-average equations<sup>1</sup>

- PA particle-phase Reynolds stress

$$\begin{aligned} \frac{1}{2} \frac{\partial \langle u_{p,1}''^2 \rangle_p}{\partial t} &= \boxed{\left\langle \Theta \frac{\partial u_{p,1}''}{\partial x_1} \right\rangle_p - \left\langle \sigma_{p,1i} \frac{\partial u_{p,1}''}{\partial x_i} \right\rangle_p} && \text{Turbulent dissipation of } k_p \text{ PS/VD} \\ &+ \frac{1}{\tau_p} (\langle u_{f,1}'' u_{p,1}'' \rangle_p - \langle u_{p,1}''^2 \rangle_p) && \text{DE} \\ &+ \frac{1}{\rho_p} \left( \left\langle u_{p,1}'' \frac{\partial \sigma'_{f,1i}}{\partial x_i} \right\rangle_p - \left\langle u_{p,1}'' \frac{\partial p'_f}{\partial x_1} \right\rangle_p \right) && \text{VE/PE} \end{aligned}$$

- Particle-phase stress tensor:

$$\begin{aligned} \frac{1}{2} \frac{\partial \langle P_{11} \rangle_p}{\partial t} &= - \boxed{\left\langle \Theta \frac{\partial u_{p,1}''}{\partial x_1} \right\rangle_p + \left\langle \sigma_{p,1i} \frac{\partial u_{p,1}''}{\partial x_i} \right\rangle_p} && \text{Production of } \langle \Theta \rangle_p \text{ PS/VD} \\ &- \frac{1}{\tau_p} \langle P_{11} \rangle_p && \text{DE} \\ &+ \frac{6}{\sqrt{\pi} d_p} \langle \alpha_p \Theta^{1/2} (\Delta_{11}^* - P_{11}) \rangle_p && \text{CD} \end{aligned}$$

Collisional equilibrium<sup>2</sup>

$$\Delta_{11}^* = \frac{1}{4} (1 + e)^2 \Theta + \frac{1}{4} (1 - e)^2 P_{11}$$

1. J. Capecelatro, O. Desjardins, R.O. Fox (Under review), *J. Fluid Mech.*
2. A. Passalacqua et al. (2011), *Comm in Comp. Phys.*



## Volume-Filtered Euler-Lagrange Framework



## Volume-filtered Description

- Objective: formulate equations for fluid-particle flows that allow  $\Delta x > d_p$
- Introduce a local volume filter based on convolution product with kernel<sup>1</sup>  $G(r)$ 
  - Accurate solution requires  $\Delta x \ll \delta_f$

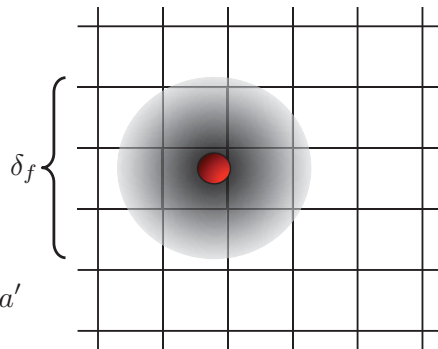
- Define local volume fraction

$$\alpha_f(\mathbf{x}, t) = \int_{V_f} G(|\mathbf{x} - \mathbf{y}|) d\mathbf{y}$$

- Define a filtered variable  $\bar{a}$  from a point variable  $a$

$$\alpha_f \bar{a}(\mathbf{x}, t) = \int_{V_f} a(\mathbf{y}, t) G(|\mathbf{x} - \mathbf{y}|) d\mathbf{y} \quad a = \bar{a} + a'$$

- Closure for sub-filter terms depend on choice of  $\delta_f$



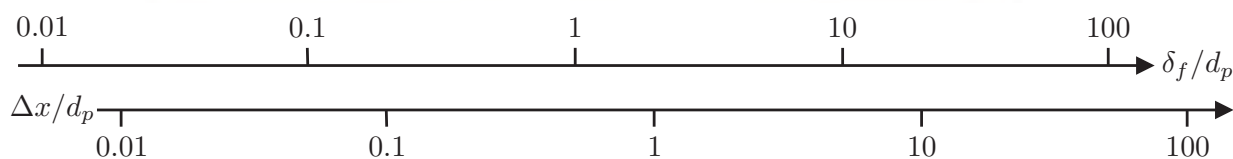
<sup>1</sup>Anderson & Jackson (1967)



## Volume-filtered Description

- What is an appropriate choice for  $\delta_f$ ?

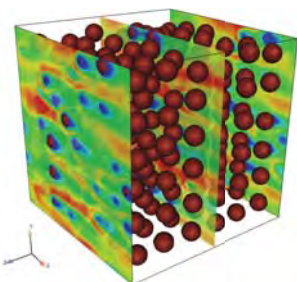
$$\int_{V_f} \nabla \cdot \tau G(|\mathbf{x} - \mathbf{y}|) d\mathbf{y} = \nabla \cdot (\alpha_f \bar{\tau}) - \sum_{i=1}^{N_p} \int_{S_p} \mathbf{n} \cdot \tau G(|\mathbf{x} - \mathbf{y}|) d\mathbf{y}$$



Particle-resolved DNS

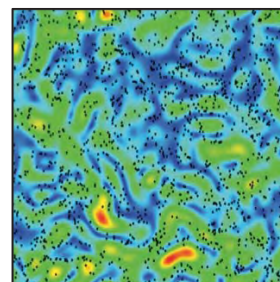
Partially resolved

Point particle



Relevant to strongly-coupled flows

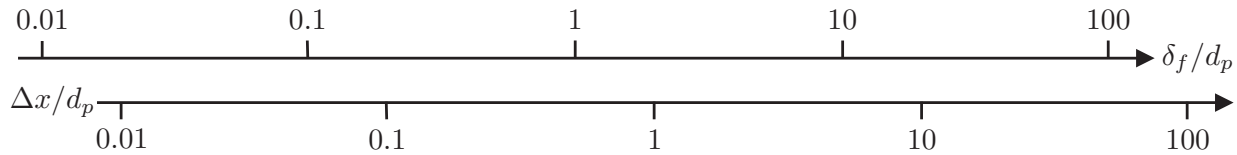
*Requires an efficient implementation to distribute data to the mesh*



## Volume-filtered Description

- What is an appropriate choice for  $\delta_f$ ?

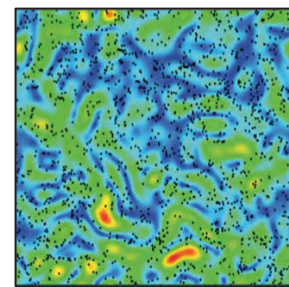
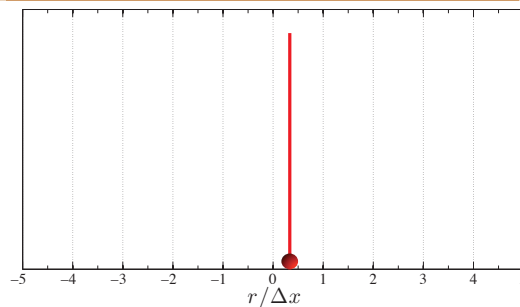
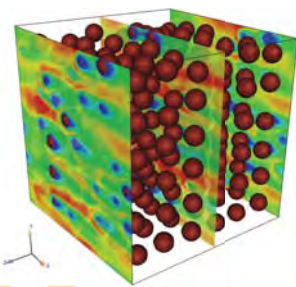
$$\int_{V_f} \nabla \cdot \tau G(|\mathbf{x} - \mathbf{y}|) d\mathbf{y} = \nabla \cdot (\alpha_f \bar{\tau}) - \sum_{i=1}^{N_p} \int_{S_p} \mathbf{n} \cdot \tau G(|\mathbf{x} - \mathbf{y}|) d\mathbf{y}$$



Particle-resolved DNS

Partially resolved

Point particle



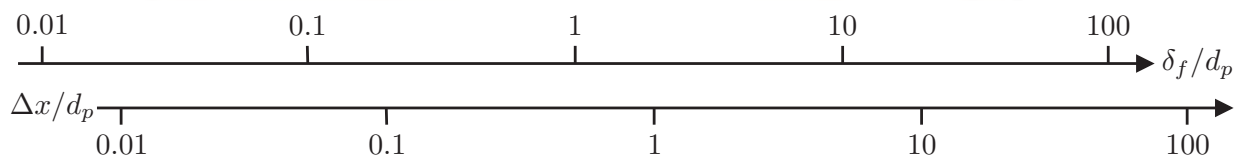
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## Volume-filtered Description

- What is an appropriate choice for  $\delta_f$ ?

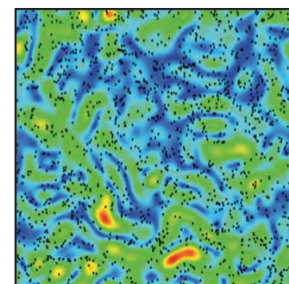
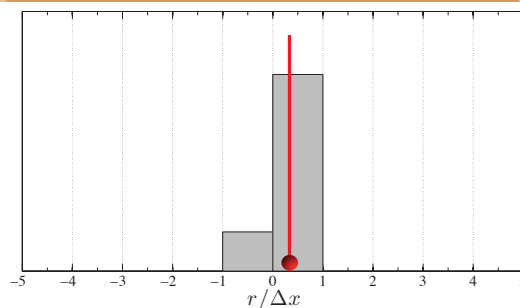
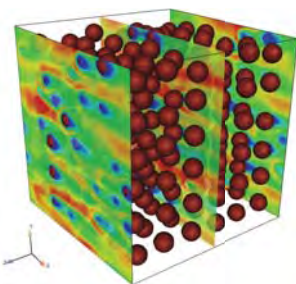
$$\int_{V_f} \nabla \cdot \tau G(|\mathbf{x} - \mathbf{y}|) d\mathbf{y} = \nabla \cdot (\alpha_f \bar{\tau}) - \sum_{i=1}^{N_p} \int_{S_p} \mathbf{n} \cdot \tau G(|\mathbf{x} - \mathbf{y}|) d\mathbf{y}$$



Particle-resolved DNS

Partially resolved

Point particle



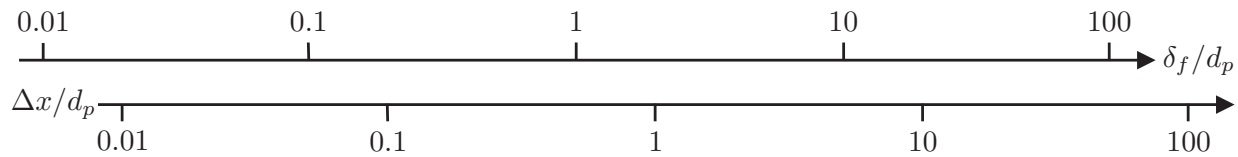
MichiganEngineering

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# Volume-filtered Description

- What is an appropriate choice for  $\delta_f$ ?

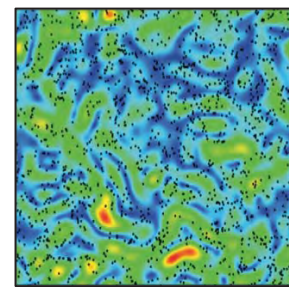
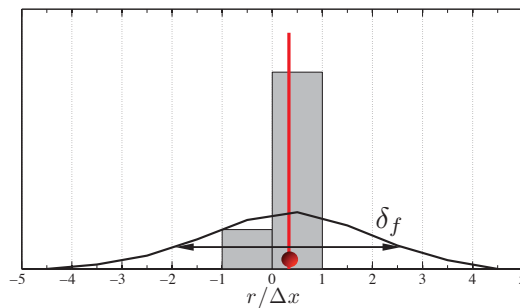
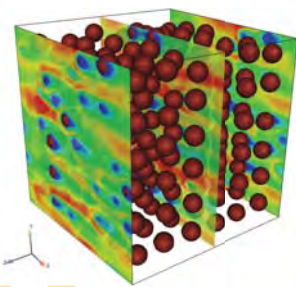
$$\int_{V_f} \nabla \cdot \tau G(|\mathbf{x} - \mathbf{y}|) d\mathbf{y} = \nabla \cdot (\alpha_f \bar{\tau}) - \sum_{i=1}^{N_p} \int_{S_p} \mathbf{n} \cdot \tau G(|\mathbf{x} - \mathbf{y}|) d\mathbf{y}$$



Particle-resolved DNS

Partially resolved

Point particle



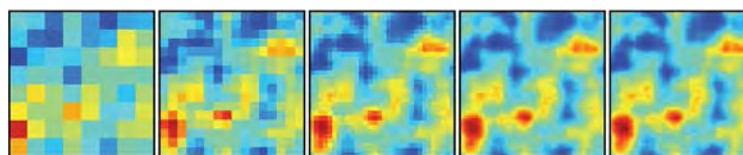
MichiganEngineering

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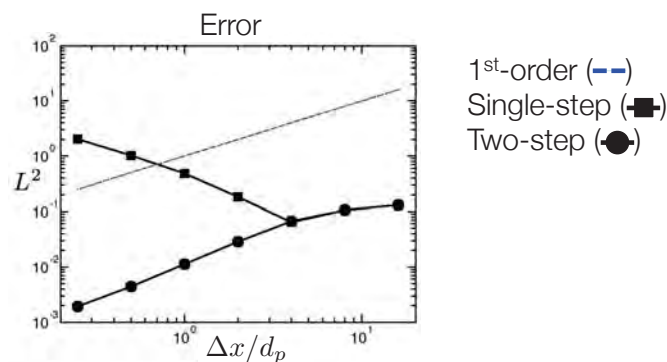
## Volume-Filtered Description

- What is an appropriate choice for  $\delta_f$ ?

Filter width  $\delta_f$  is independent of the mesh size<sup>1</sup>



$\Delta x = 4d_p$     $\Delta x = 2d_p$     $\Delta x = d_p$     $\Delta x = d_p/2$     $\Delta x = d_p/4$



1. J. Capecelatro and O. Desjardins (2013), *J. Comp. Phys.*



## Volume-filtered equations of motion

- Implemented in a low Mach, variable density solver<sup>1</sup>

$$\frac{\partial \alpha_f \rho_f}{\partial t} + \nabla \cdot (\alpha_f \rho_f \bar{\mathbf{u}}_f) = 0$$

$$\frac{\partial \alpha_f \rho_f \bar{\mathbf{u}}_f}{\partial t} + \nabla \cdot (\alpha_f \rho_f \bar{\mathbf{u}}_f \otimes \bar{\mathbf{u}}_f) = \nabla \cdot \bar{\boldsymbol{\tau}} - \rho_p \alpha_p \bar{\mathbf{A}} + \alpha_f \rho_f \mathbf{g}$$

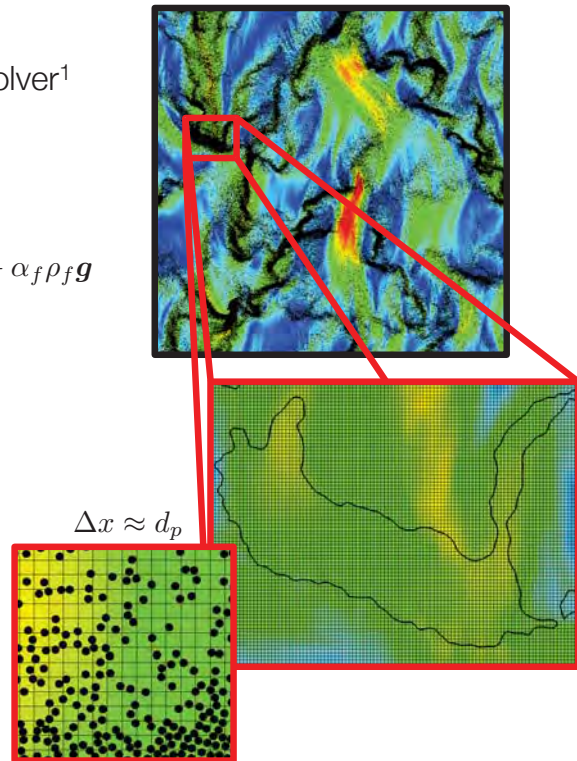
$$\bar{\boldsymbol{\tau}} = -\bar{p} \mathbf{I} + (\mu + \mu^*) \left[ \nabla \bar{\mathbf{u}}_f + \nabla \bar{\mathbf{u}}_f^T - \frac{2}{3} (\nabla \cdot \bar{\mathbf{u}}_f) \mathbf{I} \right]$$

- Lagrangian particle tracking<sup>2</sup>

$$\frac{d\mathbf{x}_p^{(i)}}{dt} = \mathbf{v}_p^{(i)}$$

$$\frac{d\mathbf{v}_p^{(i)}}{dt} = \bar{\mathbf{A}}^{(i)} + \bar{\mathbf{F}}_c^{(i)} + \mathbf{g}$$

$$I_p \frac{d\omega_p}{dt} = \sum_j \frac{d_p}{2} \mathbf{n} \times \bar{f}_{t,i \rightarrow j}^{\text{col}}$$



1. O. Desjardins, G. Blanquart, G. Balarac, H. Pitsch (2008), *J. Comp. Phys.*
2. J. Capecelatro and O. Desjardins (2013), *J. Comp. Phys.*



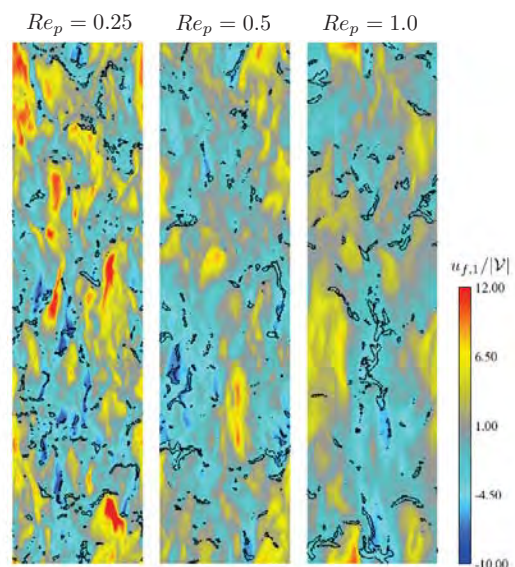
## Fully-developed CIT configuration<sup>1</sup>

Physical parameters				
$d_p$	Particle diameter	0.09 mm		
$\rho_p$	Particle density	1000 kg m <sup>-3</sup>		
$\rho_f$	Fluid density	1 kg m <sup>-3</sup>		
$\nu_f$	Fluid kinematic viscosity	1.8 × 10 <sup>-5</sup> m <sup>2</sup> s <sup>-1</sup>		
$e$	Coefficient of restitution	0.9		
$N_p$	Number of particles	55 × 10 <sup>6</sup>		
Non-dimensional parameters				
$\langle \alpha_p \rangle$	Mean particle volume fraction	0.01		
$\varphi$	Mean mass loading	10.1		
$Re_p$	Particle Reynolds number	0.25	0.5	1.0
$Re_f$	Fluid Reynolds number	3.5	13.9	55.5
$L_x/\mathcal{L}$	Normalized domain length	256	128	64
Dimensional parameters				
$\tau_p$	Drag time	0.025 s		
$\mathcal{V}$	Cluster velocity	0.05	0.10	0.20 m s <sup>-1</sup>
$\mathcal{L}$	Cluster length	1.25	2.50	5.00 mm

Table 1: Fluid-particle parameters used in CIT simulations.

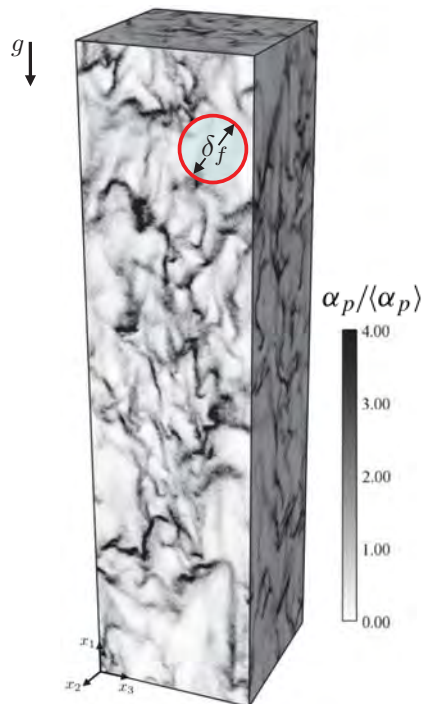
Mesh: 2048 × 512 × 512 (3584 $d_p$  × 896 $d_p$  × 896 $d_p$ )

Particles:  $N_p = 55 \times 10^6$

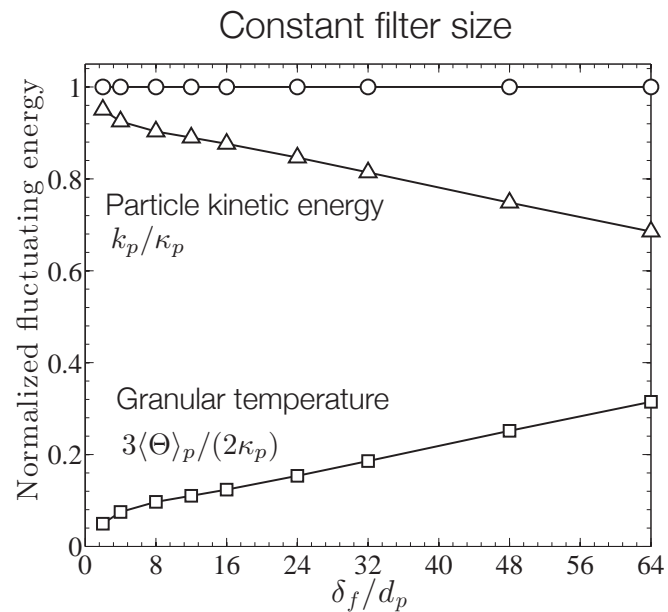




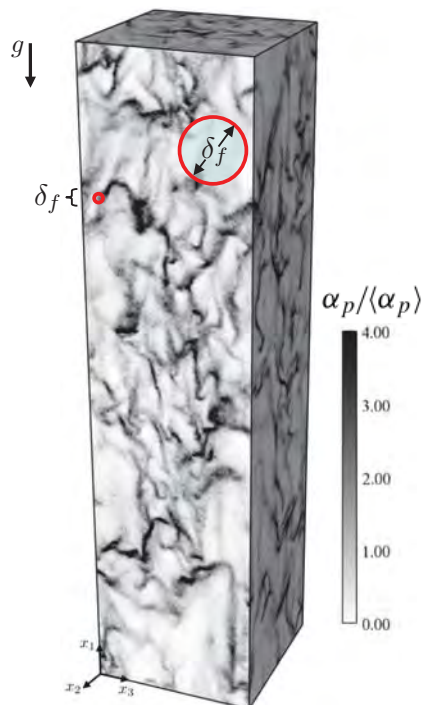
## Extracting multiphase statistics<sup>1</sup>



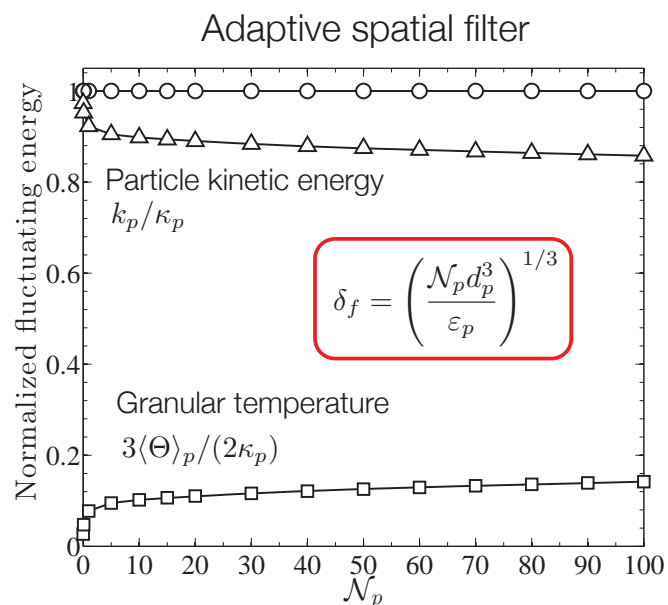
- How to extract statistics from Euler-Lagrange data?



## Extracting multiphase statistics<sup>1</sup>

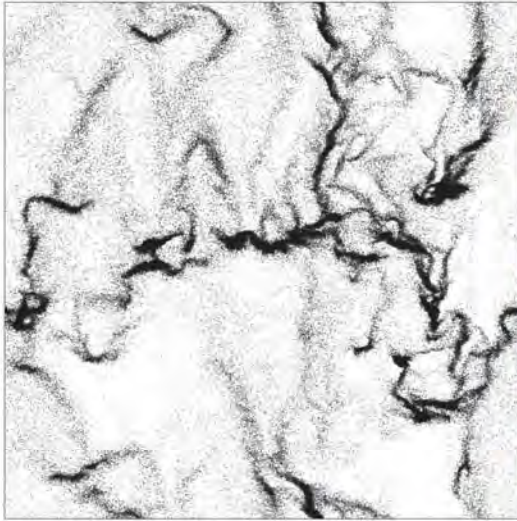


- How to extract statistics from Euler-Lagrange data?

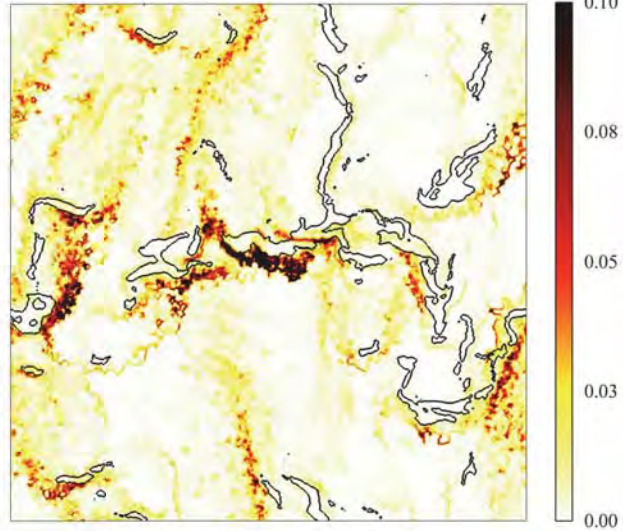


## Local instantaneous granular temperature (2D example)

Particle position



Granular temperature



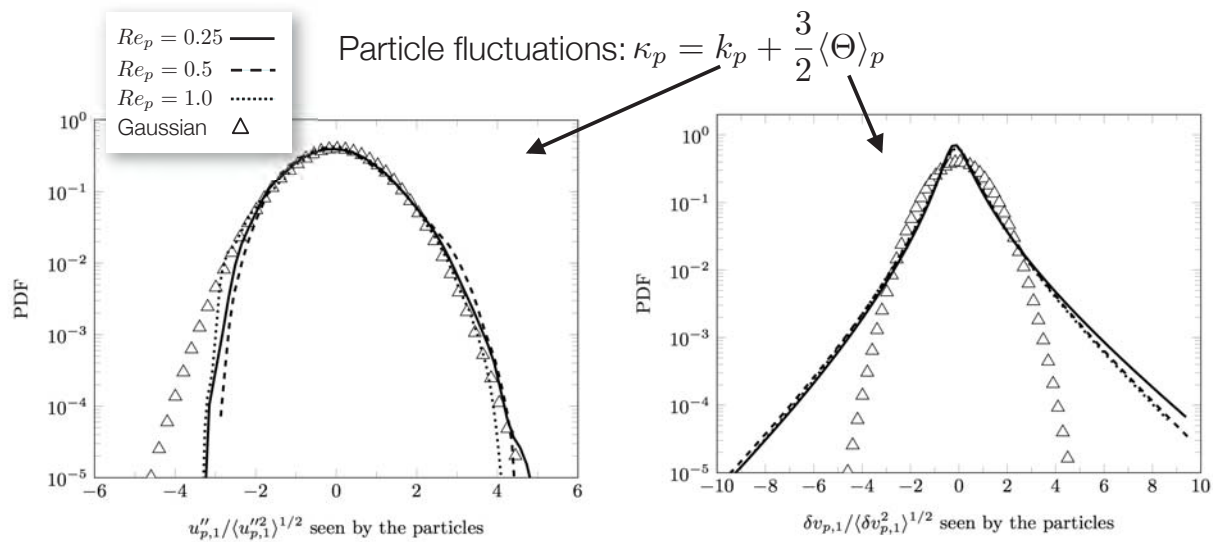
- Minimum agitation within clusters
- Maximum granular temperature located at the upstream boundary of clusters



## Results & Discussion



## Particle behavior in homogeneous CIT

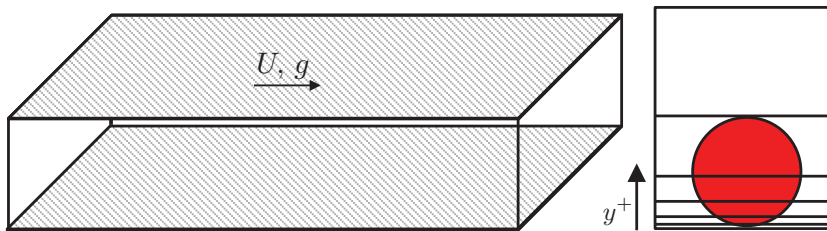


- Spatially-correlated and uncorrelated velocity behave fundamentally different!



## CIT in wall-bounded flows<sup>1</sup>

- Two-step filter permits grid stretching near the wall

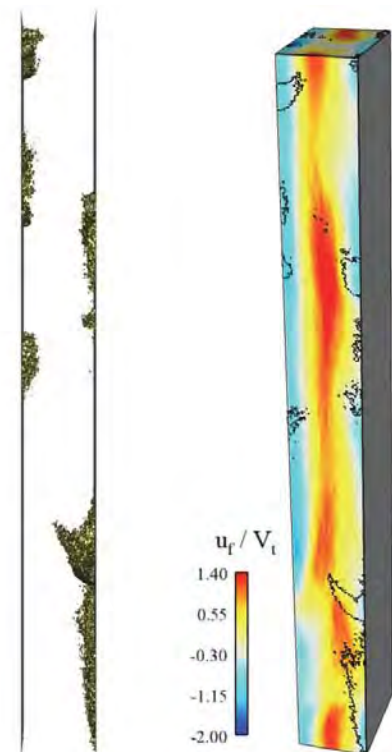


### Physical parameters

$d_p$	Particle diameter	0.20	mm
$\rho_p$	Particle density	2000	kg/m <sup>3</sup>
$\rho_g$	Gas density	1	kg/m <sup>3</sup>
$\nu_g$	Gas kinematic viscosity	$1.8 \times 10^{-5}$	m <sup>2</sup> /s
$W$	Channel width	0.05	m
$g$	Gravity magnitude	3.645	m/s <sup>2</sup>
$V_t$	Terminal velocity	0.9	m/s
$\mathcal{L}$	Cluster length	0.22	m

### Non-dimensional parameters

$\bar{\alpha}_p$	Average particle volume fraction	0.01
$\bar{\varphi}$	Average mass loading	20.2
$e, e_w$	Coefficient of restitution	0.9
$Re_p$	Particle Reynolds number	10
$Re_g$	Fluid Reynolds number	11,000
$Re_w$	Channel Reynolds number	2500

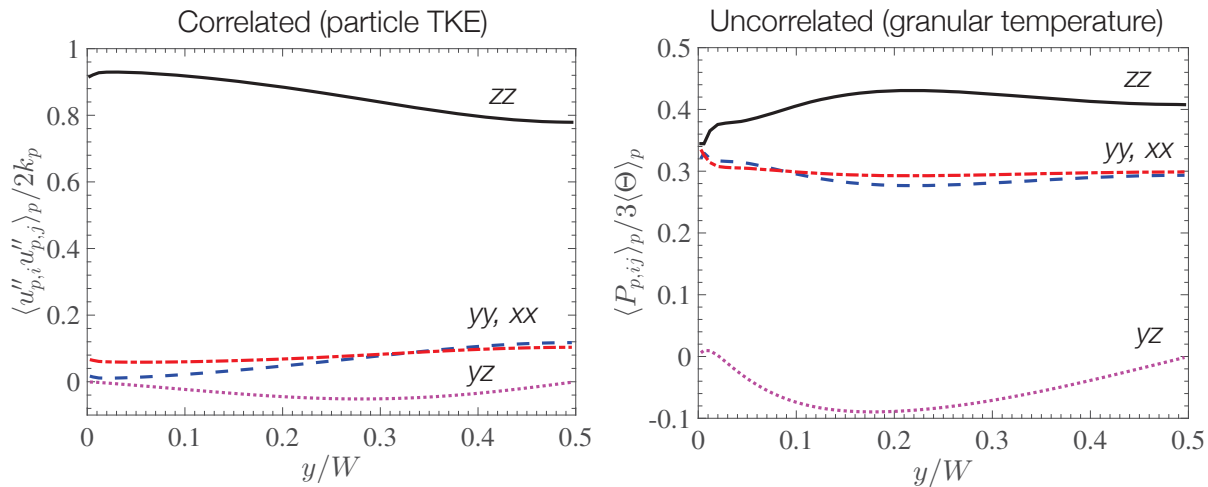


1. J. Capecelatro *et al.* (2016), *Phys. Fluids*.



## Considerations for turbulence modeling

- Decomposition of correlated / uncorrelated components is needed to get BC correct



- Boundary conditions for components  $\kappa_p = k_p + \frac{3}{2}\langle \Theta \rangle_p$ 
  - Wall-normal correlated energy:  $k_p = 0$
  - Wall-normal uncorrelated energy:  $\langle \Theta \rangle_p \neq 0$



## Summary & conclusions (numerics)

- Volume-filtered Euler-Lagrange approach is an efficient method to handle large number of particles without requiring  $\Delta x \gg d_p$
- Microscale models (e.g., drag) are agnostic to volume filter
  - If  $\delta_f/d_p \ll 1$ : drag entirely accounted for in resolved flow
  - If  $\delta_f/d_p \gg 1$ : model must account for entire drag contribution

Logical solution: develop closure models that are a function of  $\delta_f/d_p$

$$\frac{du_p}{dt} = \frac{F}{\tau_p} (u_f - u_p), \quad F = F(\text{Re}_p, \alpha_p, \delta_f/d_p)$$



## Summary & conclusions (modeling)

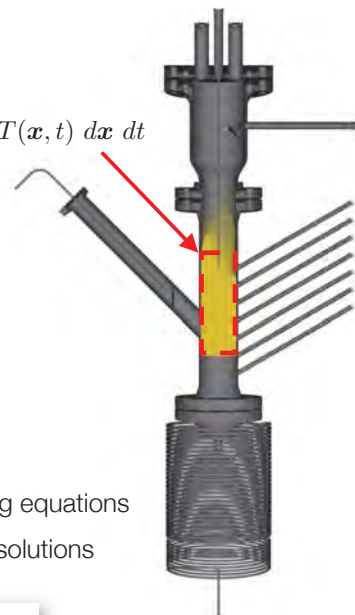
- In high mass loading flows, turbulence arises mainly due to CIT (drag production)
- TKE is produced by fluid velocity fluctuations seen by the particles
- Partitioning the granular energy into spatially correlated (particle-phase TKE) and uncorrelated (granular temperature) components is crucial
- Predictive models need to account for anisotropy in wall-bounded flows
- How best to incorporate effects of heat transfer and chemical kinetics in a reduce-order model for strongly-coupled flows?



## Looking forward: from prediction to design

- Up to this point we have focused on *predictive* models in order to compute a *quantity of interest*  $\mathcal{J} = \mathcal{J}(\mathbf{Q}, \mathbf{f})$ 
  - $\mathbf{Q}$ : flow solution, e.g.  $\mathbf{Q} = [\alpha_f, \rho_f, \mathbf{u}_f, \dots]^T$
  - $\mathbf{f}$ : design / modeling parameters
- We want to *optimize*  $\mathcal{J}$  or measure its *sensitivity*
  - Find optimal reactor geometry to maximize  $\mathcal{J}$
  - Determine which modeling parameters  $\mathcal{J}$  is most sensitive to
- Brute force: guess  $\mathbf{f}$ , calculate  $\mathbf{Q}$ , calculate  $\mathcal{J}$ , repeat,...
  - Option a: hire a graduate student to do this
  - Option b: formulate a variational problem constrained by the governing equations
    - Adjoint system that provides sensitivity without requiring repeated solutions

$$\mathcal{J} = \int_{t_1}^{t_2} \int_{\Omega} T(\mathbf{x}, t) d\mathbf{x} dt$$



$$\frac{\delta \mathcal{J}}{\delta \mathbf{f}} = \int_{\Omega} \int_{t_1}^{t_2} \mathbf{Q}^\dagger \frac{\partial \mathcal{N}}{\partial \mathbf{f}} d\mathbf{x} dt \quad -\frac{\partial \mathbf{Q}^\dagger}{\partial t} = \mathcal{R}^\dagger [\mathbf{Q}, \mathbf{Q}^\dagger](\mathbf{x}, t)$$

**Adjoint-based methods can accelerate the design of multiphase systems!**





Questions?



Backup slides



# Exact Reynolds-average equations<sup>1</sup>

- Homogeneous gravity-driven flow: Fluid-phase Reynolds stress

$$\begin{aligned}
 \frac{1}{2} \frac{\partial \langle u_{f,1}''^2 \rangle_f}{\partial t} = & \frac{1}{\rho_f} \left( \left\langle p_f \frac{\partial u_{f,1}'''}{\partial x_1} \right\rangle - \left\langle \sigma_{f,1i} \frac{\partial u_{f,1}'''}{\partial x_i} \right\rangle \right) & PS/VD \\
 & + \frac{\varphi}{\tau_p} \left( \langle u_{f,1}''' u_{p,1}'' \rangle_p - \langle u_{f,1}''^2 \rangle_p \right) & DE \\
 & + \frac{\varphi}{\tau_p} \langle u_{f,1}''' \rangle_p \langle u_{p,1} \rangle_p & DP \\
 & + \frac{\varphi}{\rho_p} \left( \left\langle u_{f,1}''' \frac{\partial p'_f}{\partial x_1} \right\rangle_p - \left\langle u_{f,1}''' \frac{\partial \sigma'_{f,1i}}{\partial x_i} \right\rangle_p \right) & PE/VE
 \end{aligned}$$

- Fluid-phase TKE is generated by the **drag production (DP)** term, as opposed to in single-phase turbulence where it is produced by mean velocity gradients.
- Fluid velocity seen by the particles:  $\langle u_f \rangle_p = \langle \alpha_p u_f \rangle / \langle \alpha_p \rangle$
- Because clusters arise spontaneously in CIT, and are negatively correlated with the fluid velocity  $\langle u_f \rangle_f = \langle \alpha_f u_f \rangle / \langle \alpha_f \rangle \neq \langle u_f \rangle_p$



# Exact Reynolds-average equations<sup>1</sup>

- PA particle-phase Reynolds stress

$$\begin{aligned}
 \frac{1}{2} \frac{\partial \langle u_{p,1}''^2 \rangle_p}{\partial t} = & \left\langle \Theta \frac{\partial u_{p,1}''}{\partial x_1} \right\rangle_p - \left\langle \sigma_{p,1i} \frac{\partial u_{p,1}''}{\partial x_i} \right\rangle_p & PS/VD & \text{Turbulent dissipation of } k_p \\
 & + \frac{1}{\tau_p} \left( \langle u_{f,1}''' u_{p,1}'' \rangle_p - \langle u_{p,1}''^2 \rangle_p \right) & DE \\
 & + \frac{1}{\rho_p} \left( \left\langle u_{p,1}'' \frac{\partial \sigma'_{f,1i}}{\partial x_i} \right\rangle_p - \left\langle u_{p,1}'' \frac{\partial p'_f}{\partial x_1} \right\rangle_p \right) & VE/PE
 \end{aligned}$$

- Particle-phase stress tensor:

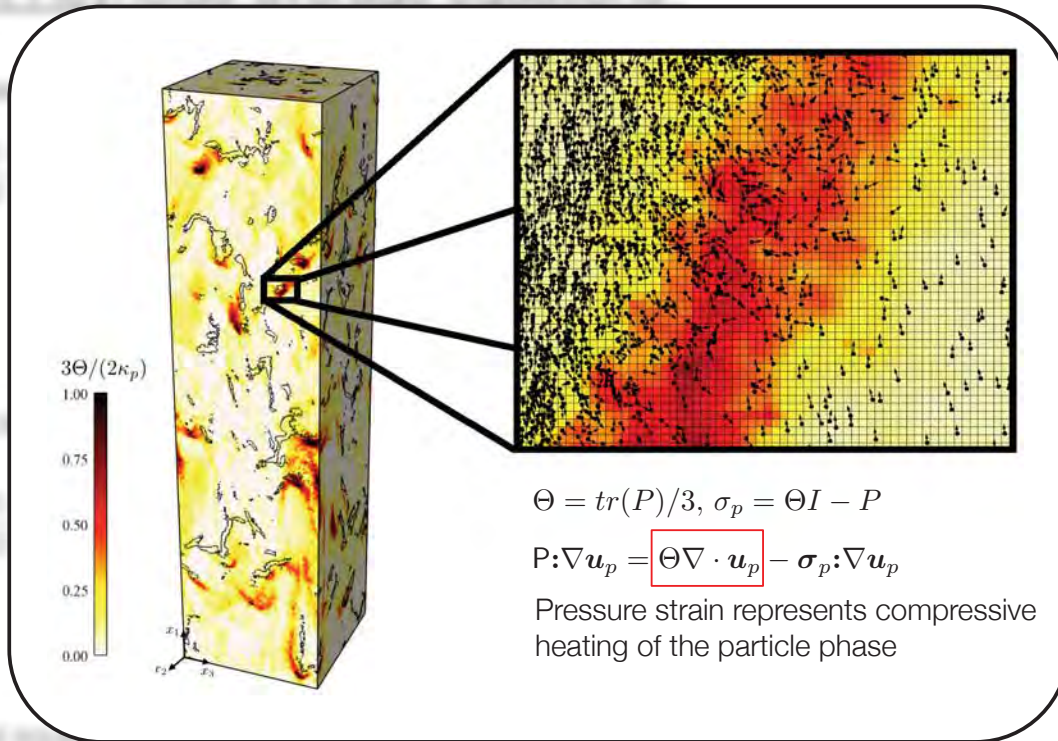
$$\begin{aligned}
 \frac{1}{2} \frac{\partial \langle P_{11} \rangle_p}{\partial t} = & - \left\langle \Theta \frac{\partial u_{p,1}''}{\partial x_1} \right\rangle_p + \left\langle \sigma_{p,1i} \frac{\partial u_{p,1}''}{\partial x_i} \right\rangle_p & PS/VD & \text{Production of } \langle \Theta \rangle_p \\
 & - \frac{1}{\tau_p} \langle P_{11} \rangle_p & DE \\
 & + \frac{6}{\sqrt{\pi} d_p} \langle \alpha_p \Theta^{1/2} (\Delta_{11}^* - P_{11}) \rangle_p & CD
 \end{aligned}$$

Collisional equilibrium<sup>2</sup>

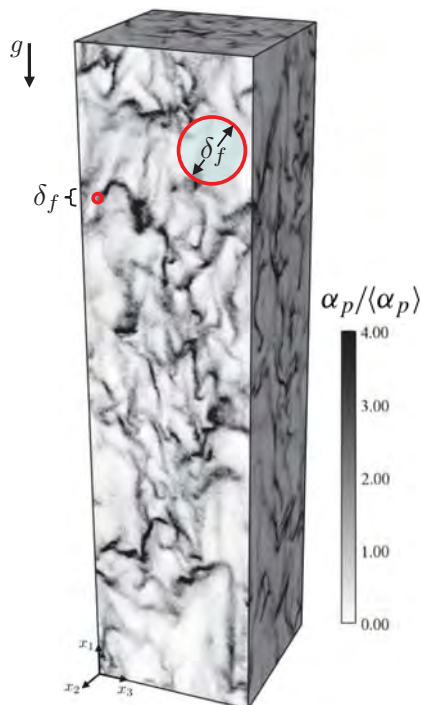
$$\Delta_{11}^* = \frac{1}{4} (1 + e)^2 \Theta + \frac{1}{4} (1 - e)^2 P_{11}$$



## Exact Reynolds-average equations?



## Validating the adaptive filter<sup>1</sup>

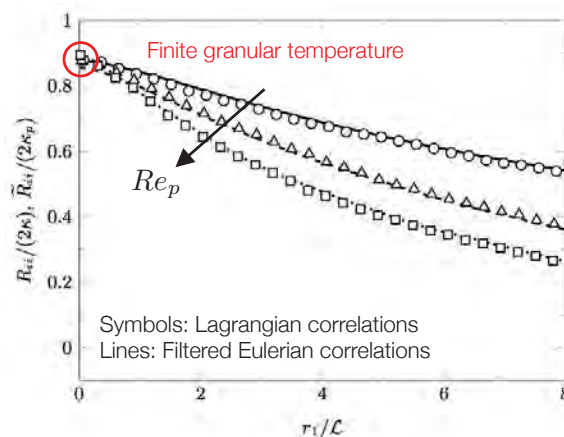


- Verification of the proposed filter

- Lagrangian data projected to Eulerian mesh via

$$\alpha_p \tilde{A}(\mathbf{x}, t) = \sum_{i=1}^{N_p} A^{(i)}(t) G(|\mathbf{x} - \mathbf{x}_p^{(i)}|) V_p$$

- Filter kernel  $G$  will sample  $\mathcal{N}_p$  particles with  $\delta_f = \left( \frac{\mathcal{N}_p d_p^3}{\alpha_p} \right)^{1/3}$
- Comparison with Lagrangian two-point velocity correlations



# Microscale Simulations Of Shock-Particle Interactions

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University of Florida*

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C. Neal, B. Osborne, P. Sridharan, S. Thakur  
J. Zhang (FIT)**



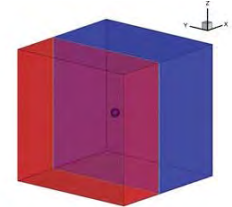
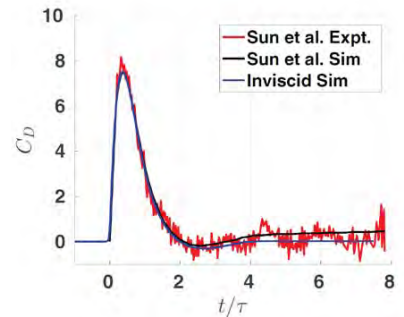
## Goals

- Advance modeling to next generation point-particle force and temperature models for meso/macroscale simulations that take into account neighboring particles
- Perform DNS simulations of shock propagating over a random pack of particles in air

Frost et al. 2012



- Shock-particle  $\tau = d_p/u_s$
- Acoustic  $\tau_a = d_p/c$
- Inviscid  $\tau_i = d_p/u_{ps}$
- Viscous diffusional  $\tau_v = \delta_v^2/\nu_m$
- Particle motion  $\tau_p = \rho_p d_p^2/18\mu_m$

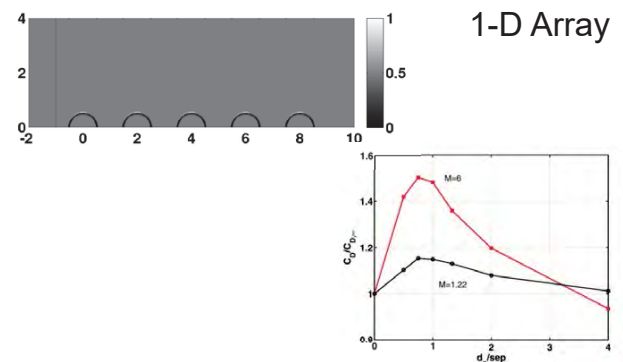
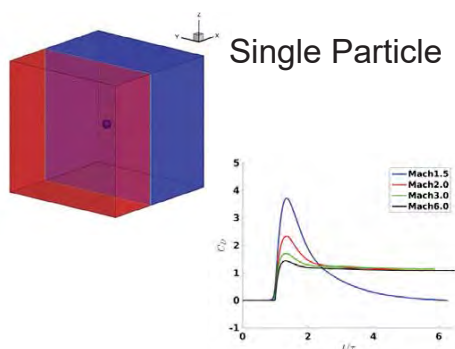


- Ratios:  $\frac{\tau_p}{\tau} \sim \frac{\rho_p}{\rho_m} Re \sim 10^4$   $\frac{\tau_v}{\tau} = \frac{u_s}{u_{ps}} \sim 2$

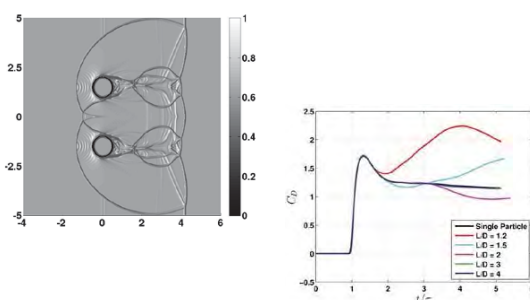
- Inviscid simulations will accurately capture peak and short time evolution of force

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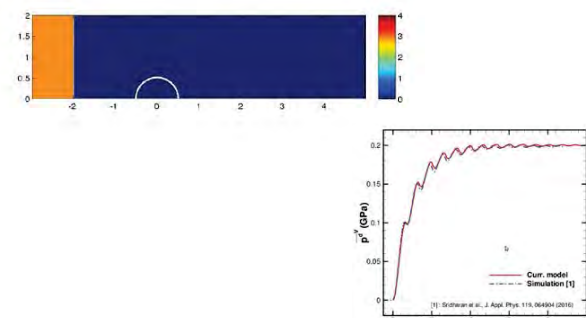
3



### Transverse Array



### Deformable Particle (NM/AI)

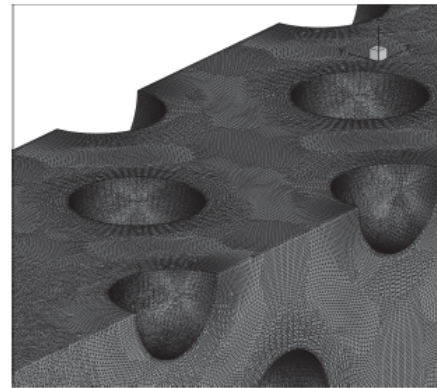
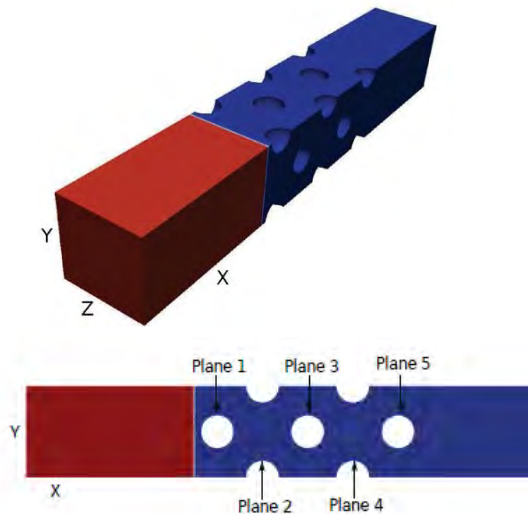


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4



- Shock propagation over a FCC array of spheres



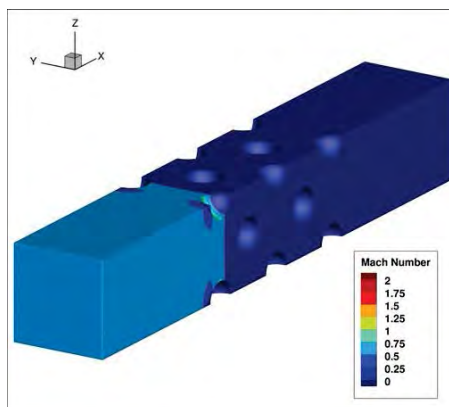
Resolution ~ 30M Cells

	$\phi = 10\%$	$\phi = 20\%$	$\phi = 30\%$	$\phi = 40\%$
$M_s = 1.5$	RUN1	RUN5	RUN9	RUN13
$M_s = 2.0$	RUN2	RUN6	RUN10	RUN14
$M_s = 3.0$	RUN3	RUN7	RUN11	RUN15
$M_s = 6.0$	RUN4	RUN8	RUN12	RUN16

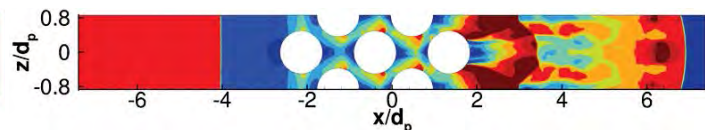
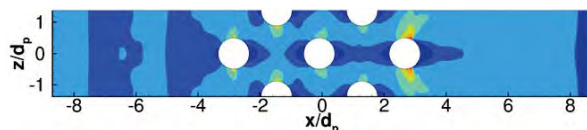
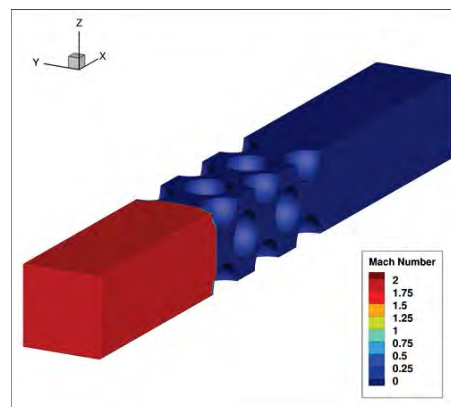
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5

$\phi = 10\%$   
 $M_s = 1.5$



$\phi = 40\%$   
 $M_s = 6.0$



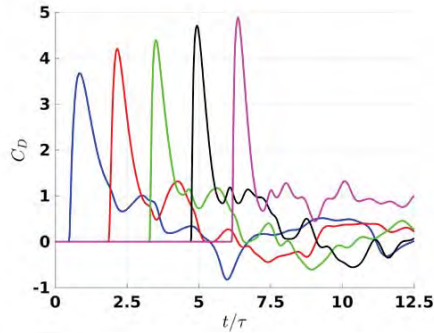
- Even for structured array, ensuing flow is quite complicated
- There is a strong effect of the particles on the flow

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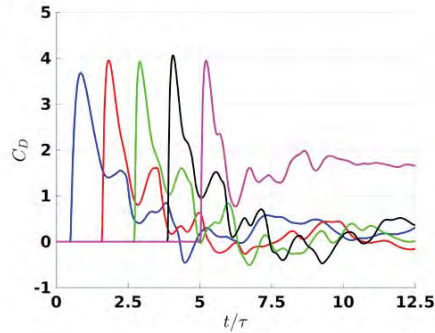
6

# FCC - Effect of Volume-fraction; $M_s = 1.5$

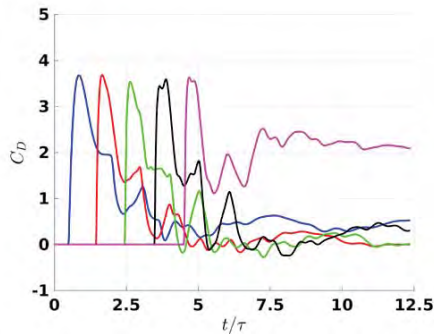
$\phi = 10\%$



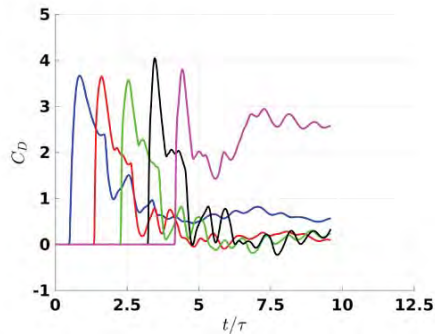
$\phi = 20\%$



$\phi = 30\%$



$\phi = 40\%$



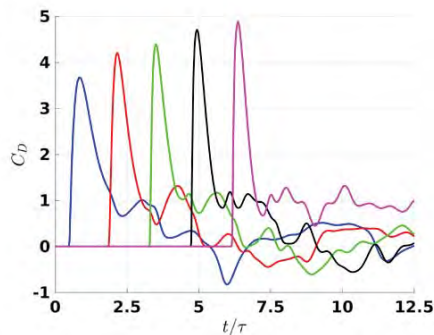
- At higher volume fractions the last plane of particles will move faster, resulting in curtain spreading

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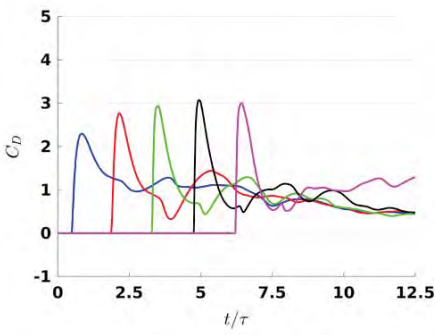
7

# FCC - Effect of Mach number; $\phi = 10\%$

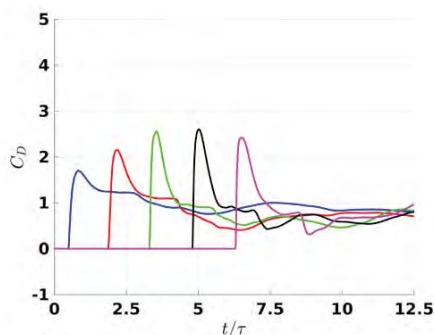
$M_s = 1.5$



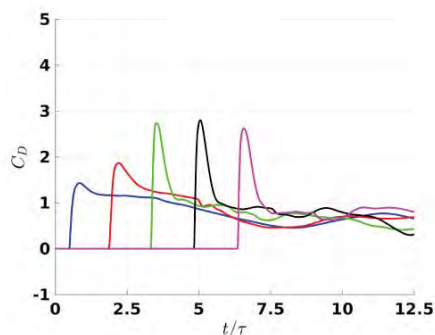
$M_s = 2.0$



$M_s = 3.0$



$M_s = 6.0$



- There is a high variability for peak forces and no clear pattern

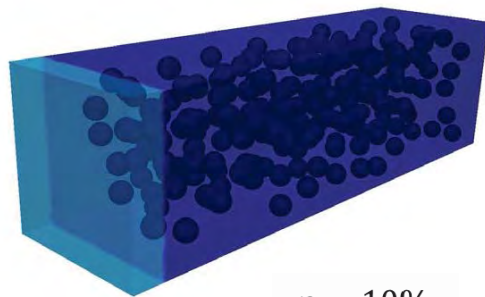
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# Multi-Particle Microscale Simulations

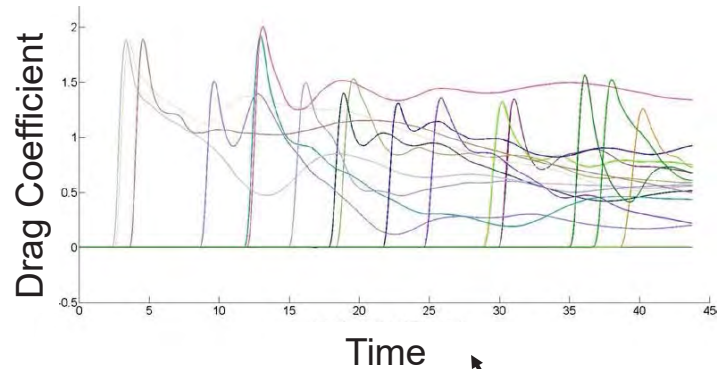
- Simulation of random cluster of particles to extract force history information
- Extracted information is compared with current models to establish areas that need model enhancement
- In collaboration with K. Salari

**Mach 3 shock over 200 particles**



$\phi = 10\%$

**Force histories of 20 particles**



- Current point-particle models do not capture these large particle-to-particle variations

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## Random Pack of Particles; $M_s = 3.0$

$\phi = 10\%$



$\phi = 15\%$



$\phi = 20\%$



$\phi = 25\%$



- Mach 3.0 shock traveling through the particle beds for different volume fractions

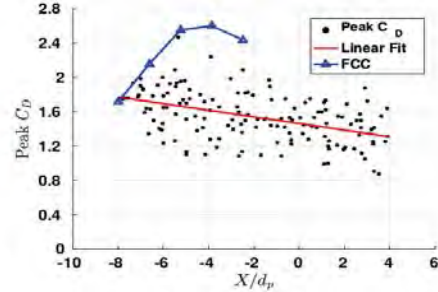
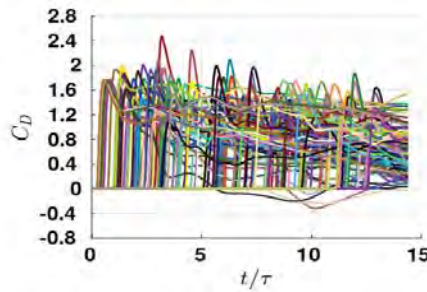
CCMT

10

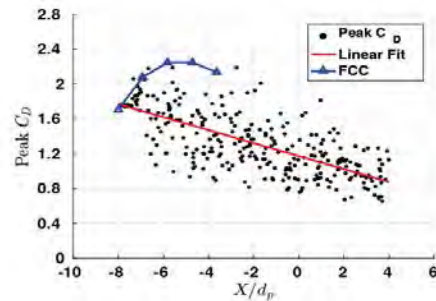
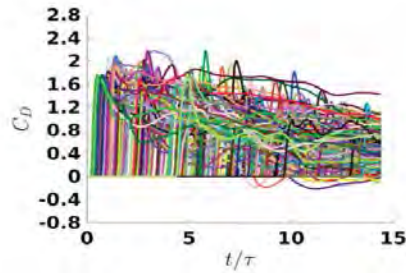


# Streamwise Drag; $M_s = 3.0$

$\varphi = 10\%$



$\varphi = 20\%$

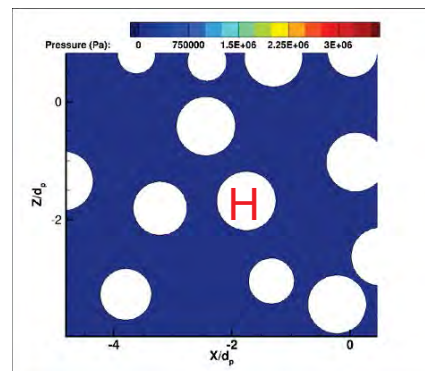
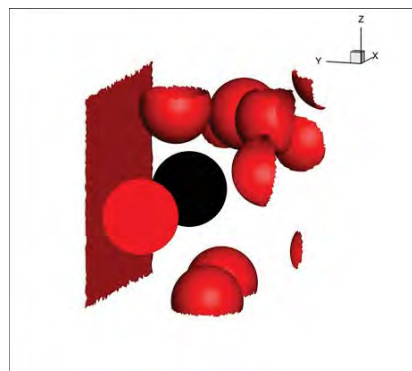
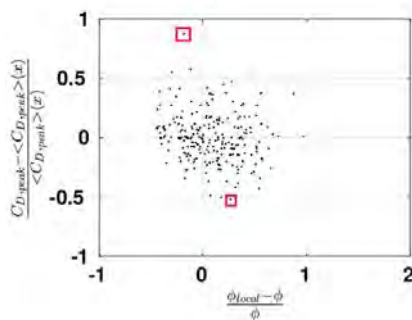
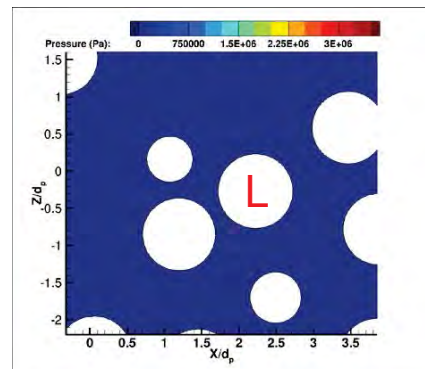
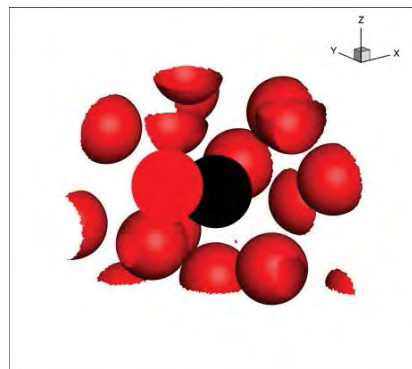
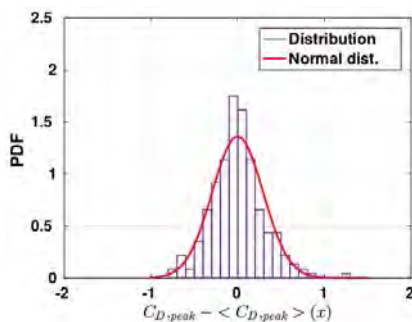


- There is a high variability for the peak drag and a clear downward trend
- Variability in peak drag force is not captured by the current models

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# Streamwise Drag; $M_s = 3.0$ ; $\varphi = 15\%$

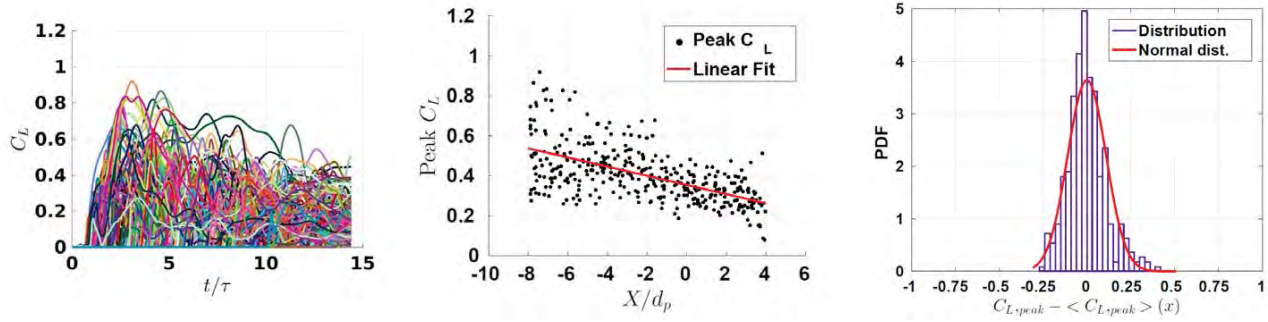


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## Lateral Forces; $\phi = 25\%$ ; $M_s = 3.0$

- Lateral forces are same order as streamwise drag
- Show similar trend
- Current models ignore lateral forces

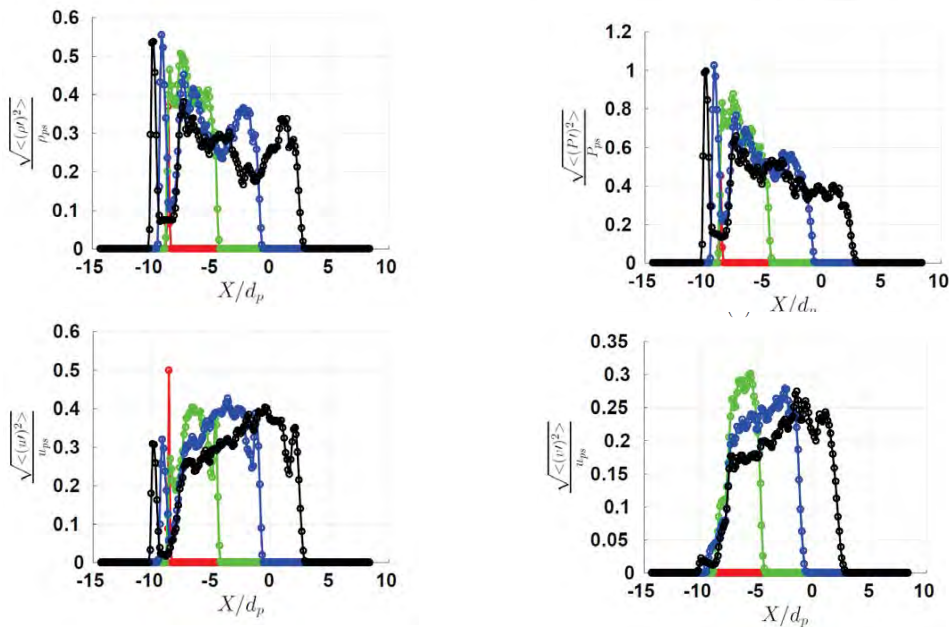


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## Fluctuations (Pseudo-turb); $\phi = 25\%$ ; $M_s = 3.0$

$$u = \langle u \rangle(x, t) + u'(x, y, z, t) \quad \langle F \rangle(x, t) = \frac{1}{A_{slice}} \int_{y_1}^{y_2} \int_{z_1}^{z_2} F(x, y, z, t) dy dz$$

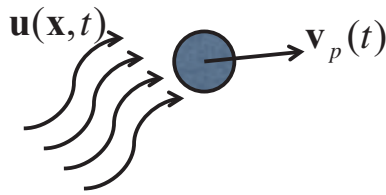


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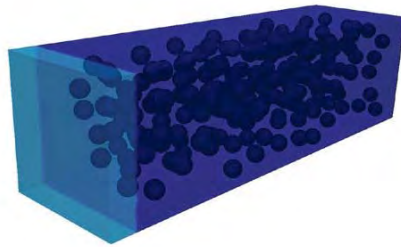


# Point-Particle Model Development



Commonly used models are based on:

- Uniform flow
- Quasi-steady flow
- Isolated particles
- Low Mach number, modest Reynolds number



But reality:

- Highly unsteady and spatially non-uniform flow
- Large Mach and Reynolds numbers
- Closely clustered particles
- Particles can deform
- Real gas & turbulence effects

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# Physics-Based Modeling

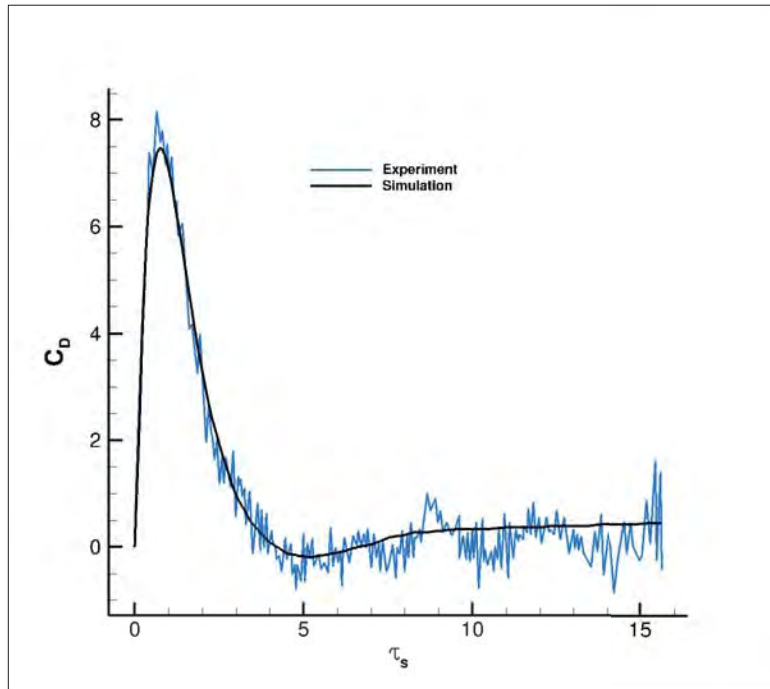
- Isolated particle
  - Rigorous point-particle formulation with empirical extension to finite Re and Mach condition
- Particle-particle interaction (current status)
  - Simple volume fraction correction to point-particle model for predicting average behavior
- Particle-particle interaction (New Development)
  - A novel Pairwise-Interaction Extended Point-particle (PIEP) model for predicting average & fluctuation behavior

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# Isolated Particle – Model Validation

Experiments by Tanno et al (2003) at Tohoku, Japan



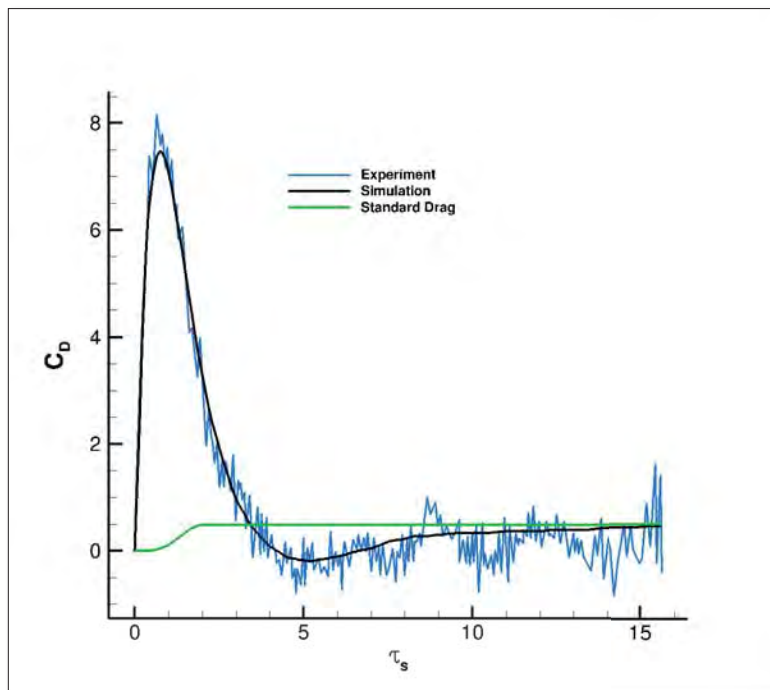
Data:  
Sun et al. (2005)  
 $M_s = 1.22$   
 $d = 8 \text{ mm}$

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# Isolated Particle – Model Validation

Experiments by Tanno et al (2003) at Tohoku, Japan



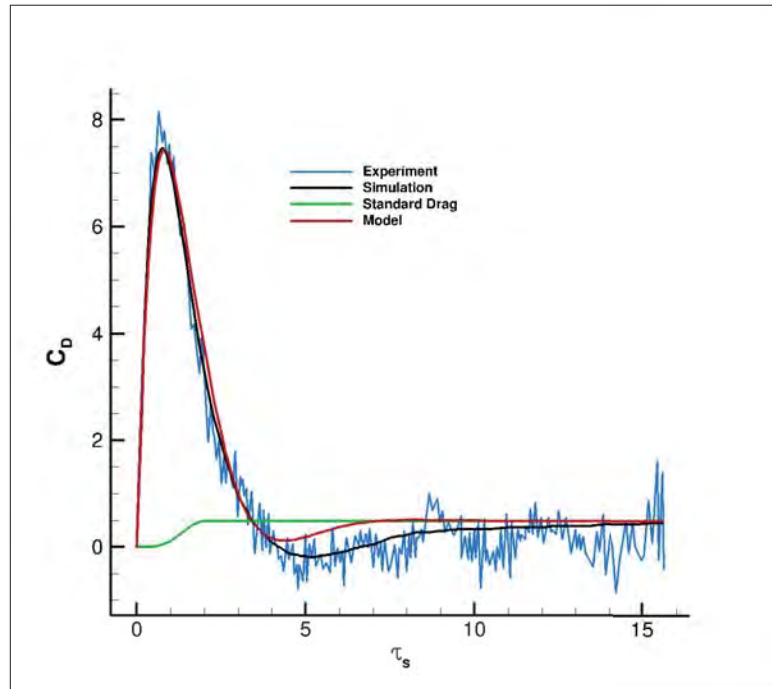
Data:  
Sun et al. (2005)  
 $M_s = 1.22$   
 $d = 8 \text{ mm}$

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# Isolated Particle – Model Validation

## Experiments by Tanno et al (2003) at Tohoku, Japan



Data:  
Sun et al. (2005)  
 $M_s = 1.22$   
 $d = 8 \text{ mm}$

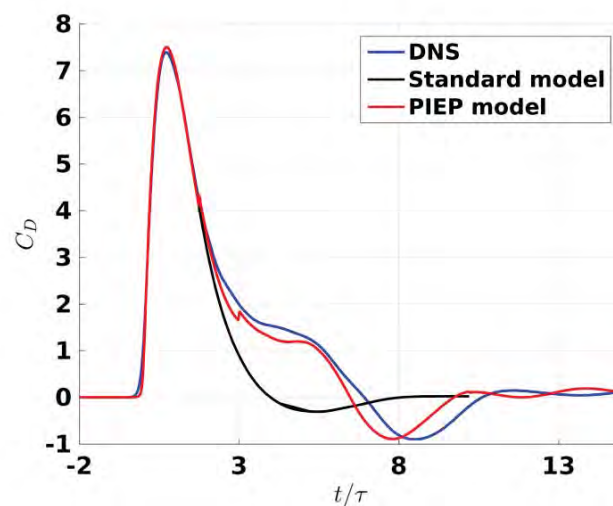
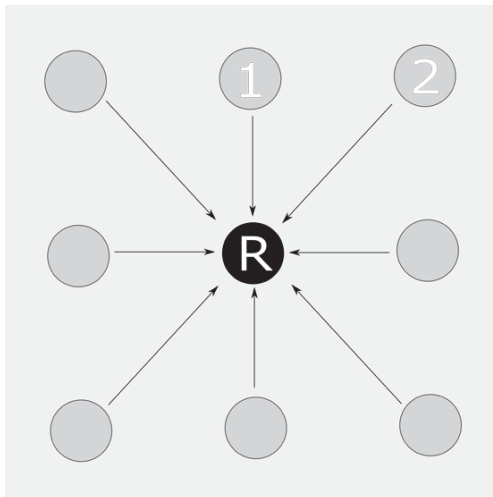
*We have a very good model for a single particle*

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## PIEP – Streamwise Force

- Shock interaction with transverse array of particles

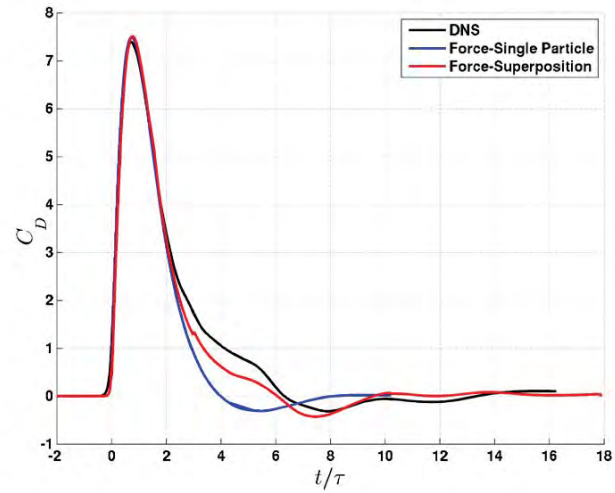
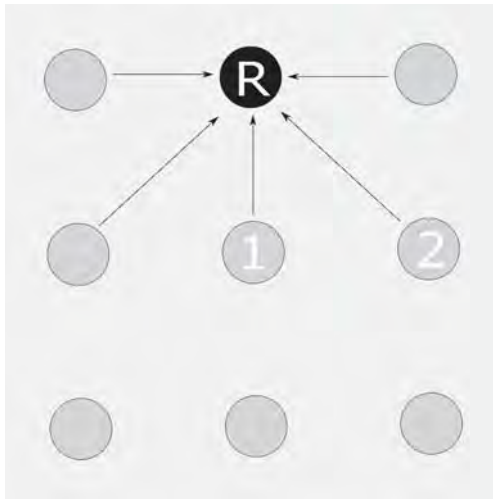


- Force found by superposition
- $\text{Force}_{\text{part-R}} = \text{Force}_{\text{shock}} + 4 * \text{Force}_{\text{diff}}_{\text{part-1}} + 4 * \text{Force}_{\text{diff}}_{\text{part-2}}$

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## PIEP – Streamwise Force

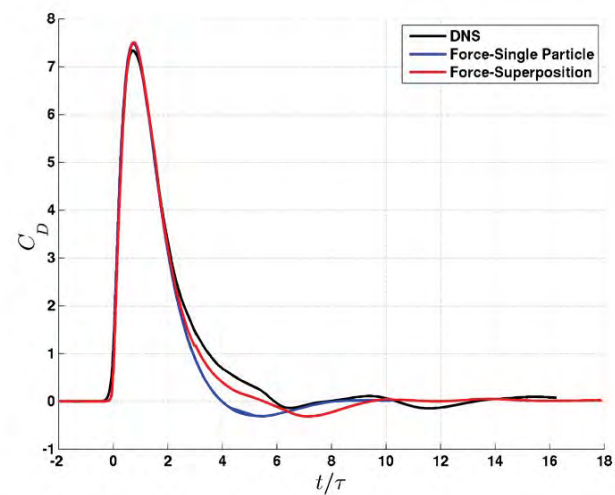
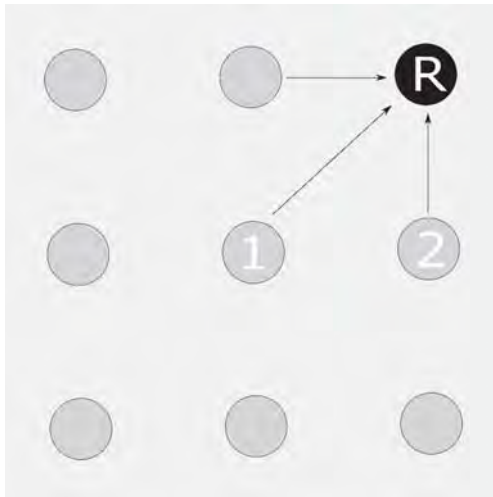


$$\blacksquare \text{ Force}_{\text{part-R}} = \text{Force}_{\text{shock}} + 3 * \text{Force}_{\text{diff}}_{\text{part-1}} + 2 * \text{Force}_{\text{diff}}_{\text{part-2}}$$

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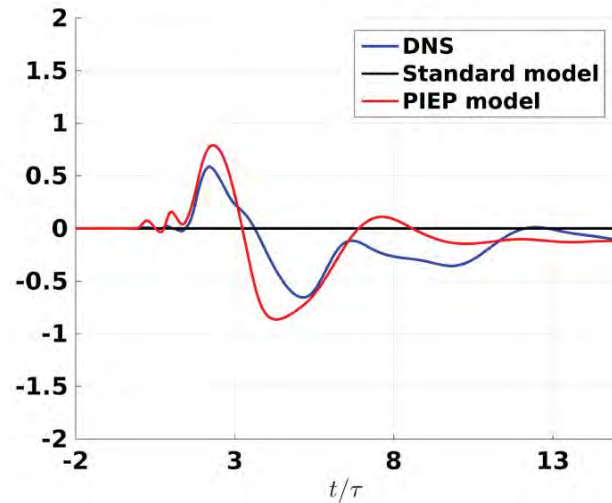
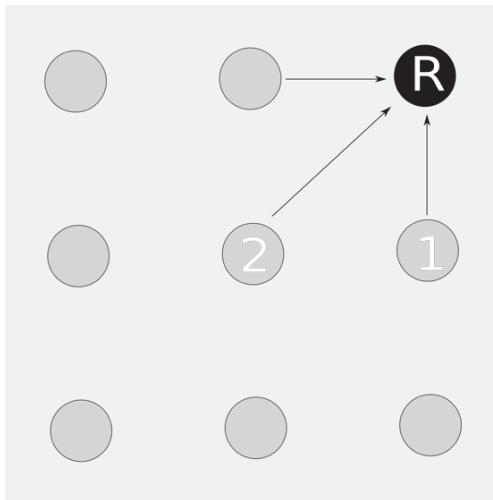
## PIEP – Streamwise Force



$$\blacksquare \text{ Force}_{\text{part-R}} = \text{Force}_{\text{shock}} + 2 * \text{Force}_{\text{diff}}_{\text{part-1}} + 1 * \text{Force}_{\text{diff}}_{\text{part-2}}$$

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$$\text{Force}_{\text{part-R}} = 2 * \text{Force\_diff}_{\text{part-1}} + 1 * \text{Force\_diff}_{\text{part-2}}$$

- Simulations with a large cluster of moving particles
  - Dynamic gridding (immersed boundary method?)
  - How to handle particle-particle interaction?
- Simulations of detonation-induced particle deformation
  - Numerical issue of particle mass preservation
  - Material modeling and EOS
- Upscaling of the microscale results
  - Point-particle force model (effect bow shock & wave drag)
  - Heat transfer model, deformation model, etc
  - PIEP model extension to compressible flows and testing



- Sridharan, P., Jackson, T.L., Zhang, J. and Balachandar, S. (2015). Shock interaction with 1-D array of particles in air. **Journal of Applied Physics**, Vol. 117, 075902.
- Sridharan, P., Jackson, T.L., Zhang, J., Balachandar, S., and Thakur, S. (2016). Shock interaction with deformable particles using a constrained interface reinitialization scheme. **Journal of Applied Physics**, Vol. 119, 064904, 18 pages.
- Mehta, Y., Jackson, T. L., Zhang, J., & Balachandar, S. (2016). Numerical investigation of shock interaction with one-dimensional transverse array of particles in air. **Journal of Applied Physics**, 119(10), 104901.
- Akiki, G., Jackson, T.L., & Balachandar, S. (2016). Force variation within arrays of mono-disperse spherical particles. **Physical Review Fluids**, 1, 044202.
- Mehta, Y., Neal, C., Jackson, T.L., Balachandar, S., Thakur, S. (2016). Shock interaction with three-dimensional face centered cubic array of particles in air. **Physical Review Fluids**, 1, 054202.
- Annamalai, S., Balachandar, S. Sridharan, P. and Jackson, T.L. (2016). A pressure evolution equation for the particulate phase in inhomogeneous compressible disperse multiphase flows. **Physical Review Fluids**, (submitted).
- Akiki, G., Jackson, T.L., & Balachandar, S. (2016). PIEP model for a random array of monodisperse spheres. *Journal of Fluid Mechanics*, under review.
- Neal, C., Mehta, Y., Salari, K., Jackson, T.L., Balachandar, S., Thakur, S. (2016). Shock propagation in air over a random bed of spherical particles. (in preparation).

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***Do you have any questions?***

# Particle dynamics: coal-specific modeling

Sean T. Smith  
University of Utah,  
Particle Deep Dive  
St. Petersburg, FL – Friday, 7<sup>th</sup> of Oct, 2016

ALSTOM

U University for  
CLEAN AND SECURE ENERGY  
THE UNIVERSITY OF UTAH

SC www.sc.utah.edu

Berkeley  
UNIVERSITY OF CALIFORNIA

BYU  
BRIGHAM YOUNG  
UNIVERSITY

NNSA  
National Nuclear Security Administration

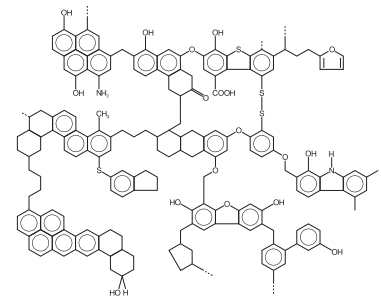
## Coal: properties

- Context: pulverized coal
- $SG \approx 1$  ( $\rho \approx 1000 \text{ kg/m}^3$ )
- $D_p \sim 10\text{--}200 \mu\text{m}$   
Poly-disperse: Rosin-Rammler
- ~10% ash  
Si, Al, Fe oxides
- ~90% organic  
C: 75%, O: 9%, H: 5%,  
S: 1%, trace N & Hg
- Multiple coal types (rank)



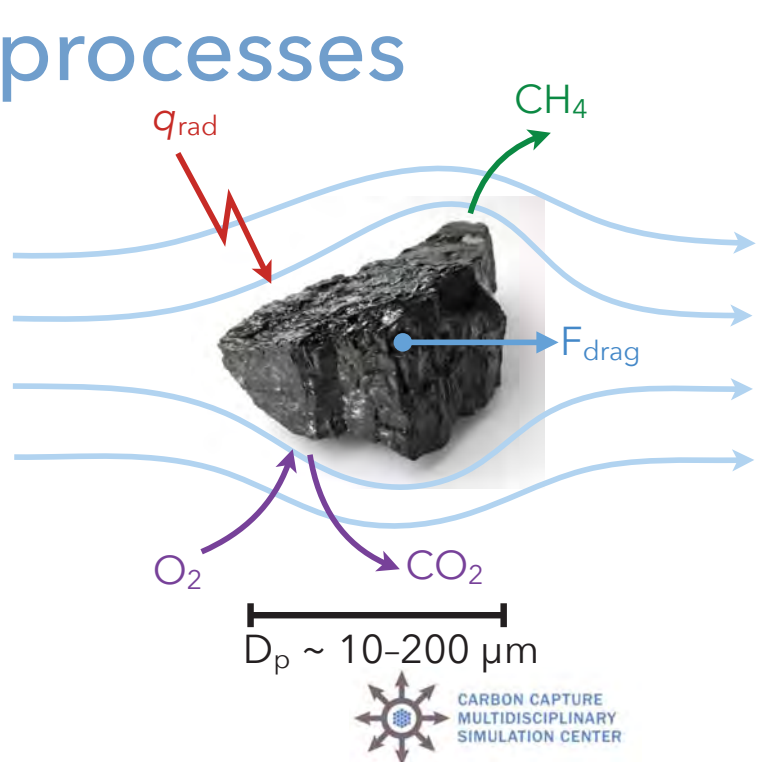
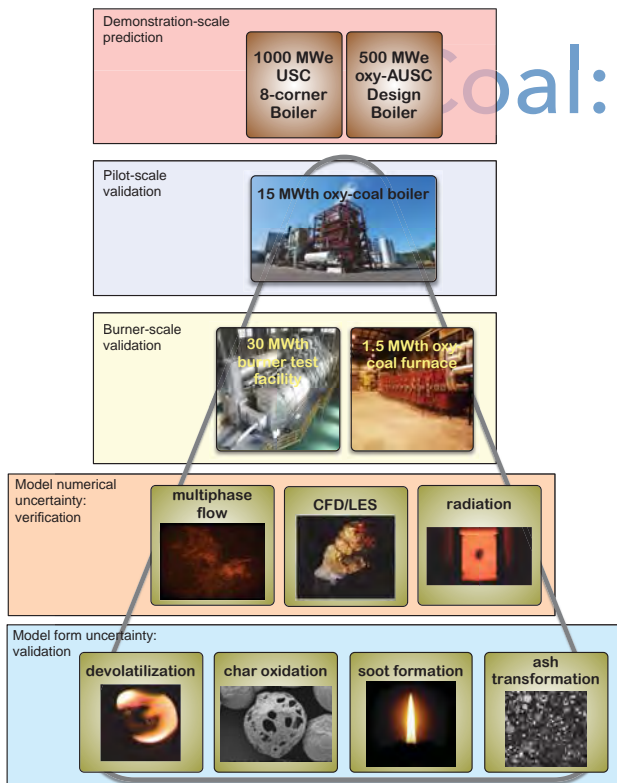
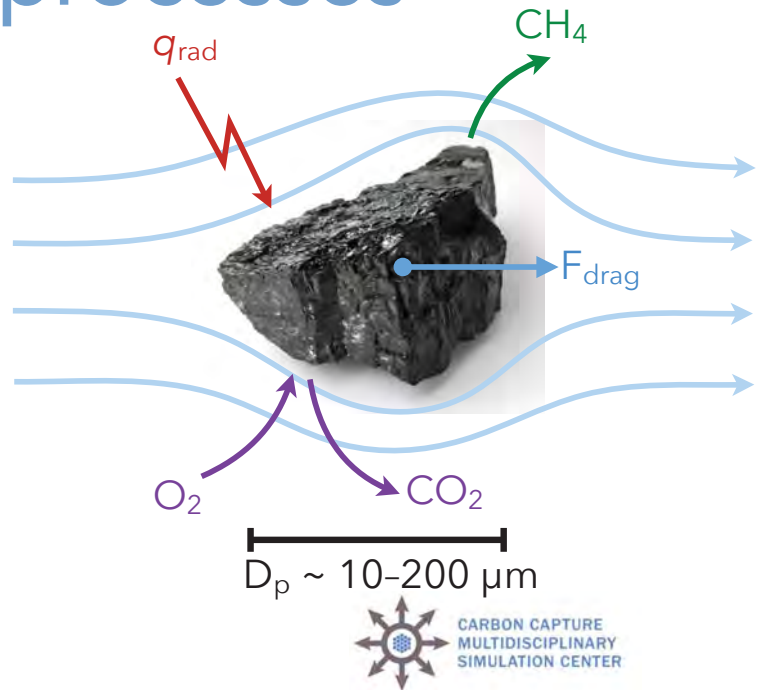
$D_p \sim 10\text{--}200 \mu\text{m}$

● Mineral Ash  
● Organic

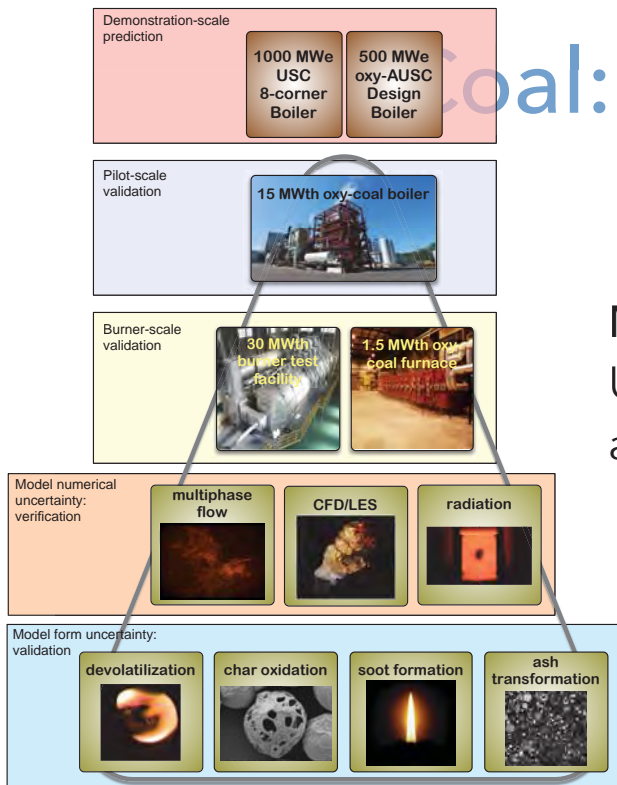


# Coal: processes

- **Heat transfer**  
primarily: radiation (Mie)
- **Devolatilization**  
metaplast, network breakdown,  
VLE & swelling  
 $\tau_{\text{devol}} \approx 10 \text{ ms}$
- **Char oxidation**  
diffusion, reactions,  
porosity & friability  
 $\tau_{\text{char ox}} \approx 1 \text{ sec.}$
- **Drag**  
turbulence, blowing, slip  
broad range on  $St$
- **Thermophoresis**  
and deposition (at cold walls)



# Coal: processes



## Modeling:

Use the model that has been made as simple as possible, but no simpler



## Devolatilization – model form

- Detailed approach: CPD
  - historical gold standard
  - very good at rates (not ultimate yield)
  - computationally demanding

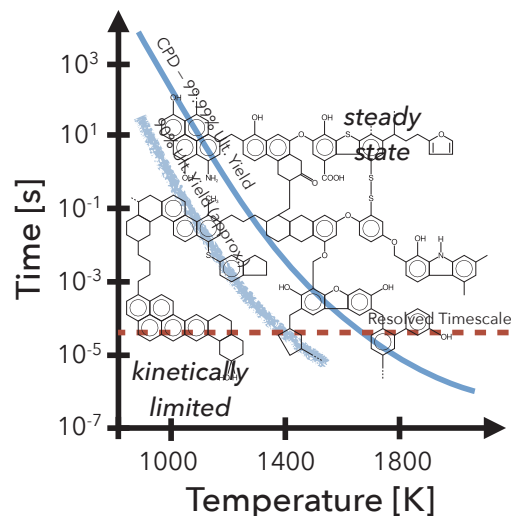
### • Timescale analysis

- BT model (1<sup>st</sup> order w/ yield):

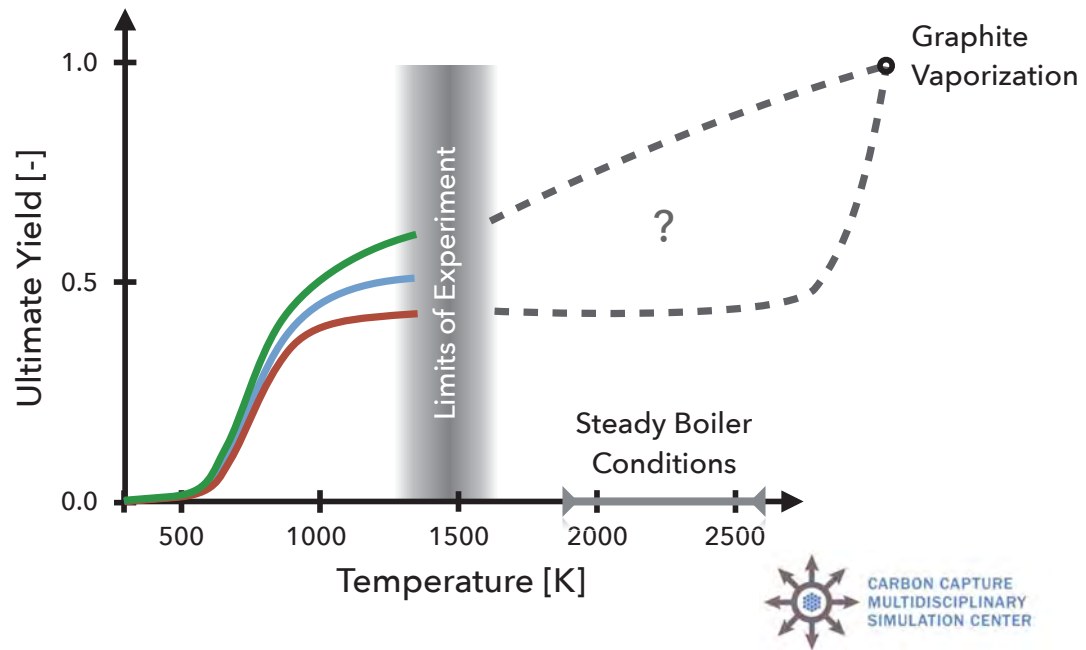
$$\frac{dV}{dt} = A \exp\left(\frac{-E_a}{RT}\right) \max(V_{ult.}(T) - V, 0)$$

$$V_{ult.}(T) = 1 - \exp\left(\frac{-T}{B}\right)$$

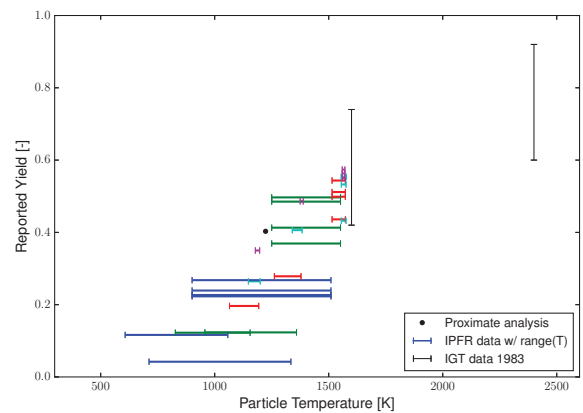
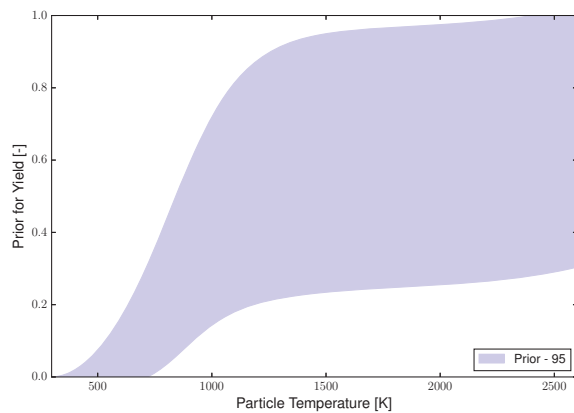
Biagini, E., and Tognotti, L., 2014. *Fuel Process. Technol.*, 126, pp. 513-520.



# Devolatilization – Yield

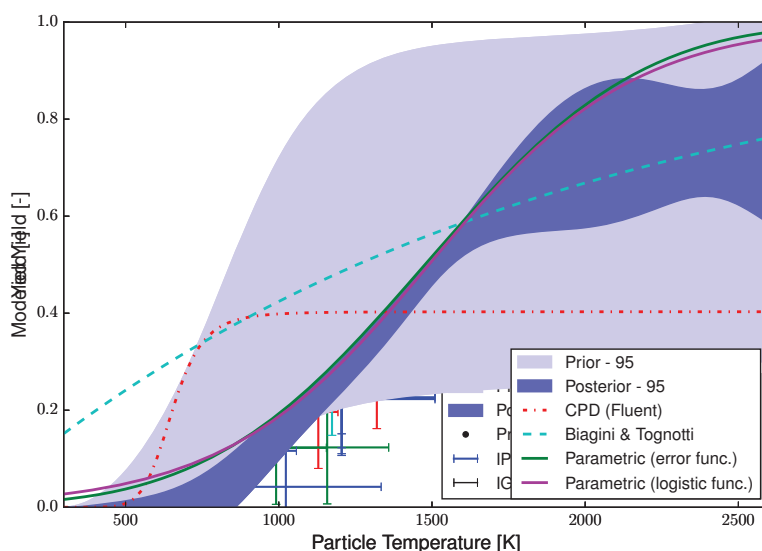


# Devolatilization – Yield

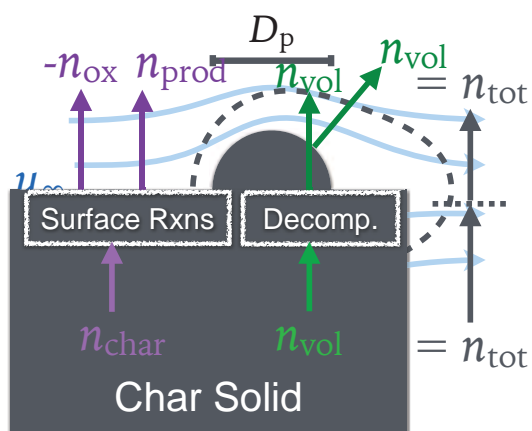




# Devolatilization – Yield



## Char oxidation – model form



- **Coupled problem:**  
mass transfer & surface reactions
- **Mass transfer:**  
convection – mass transfer coefficient,  
blowing – penetration theory,  
multi-component – (diagonal) Fickian assumption,  
reaction appears as a boundary condition.
- **Surface Reactions:**  
rate laws – Arrhenius (for oxidation & gasification),  
important to match fluxes,  
mass transfer provides gas composition.

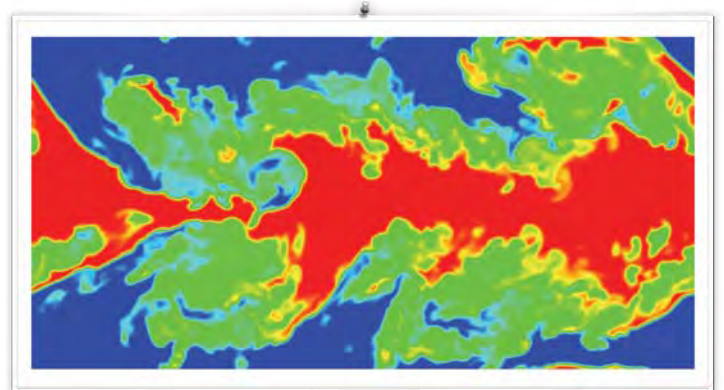
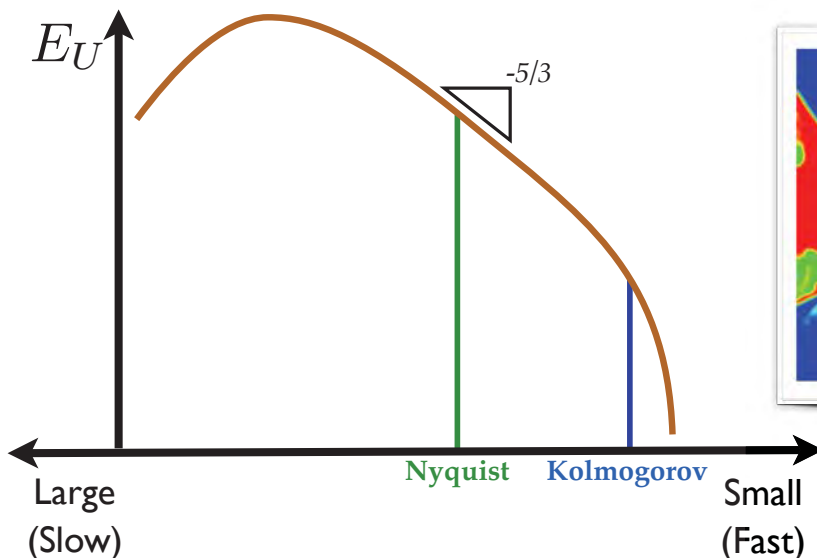


# Particle Transport

- Particle size vs. grid:  
200  $\mu\text{m}$  (largest) vs. 2 cm (smallest)
- Volume fraction:  
low ( $< 0.001$ )
- Reynolds number:  
very low ( $\sim 10$ )
- Surface blowing (due to chemical reactions):  
modeling – drag correlations w/ penetration theory
- Knudsen number:  
slip regime – for only the smallest particles  
(Young, *Aerosol Sci. & Tech.* **2011**)



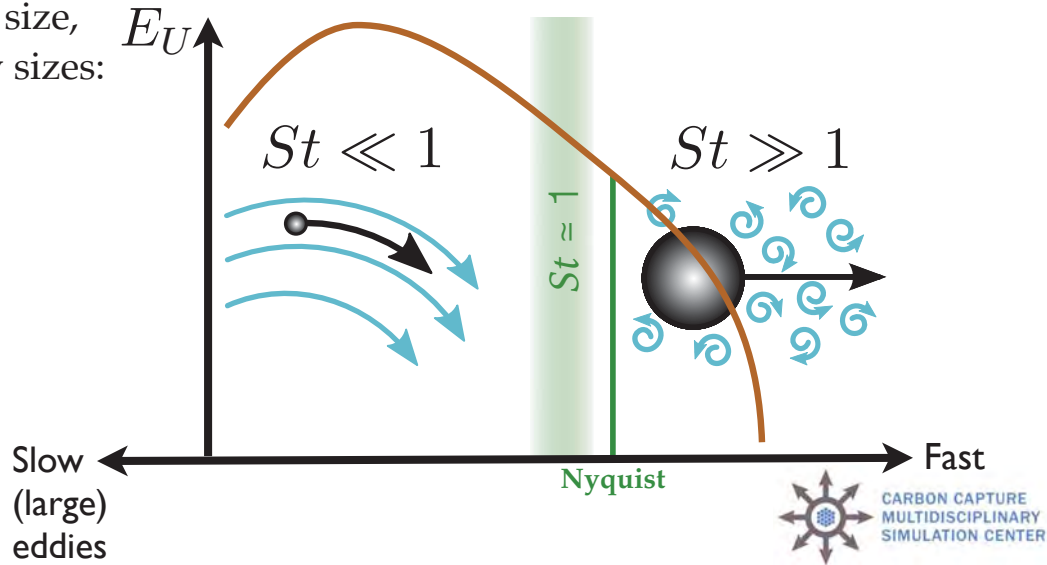
## Particles & Turbulence



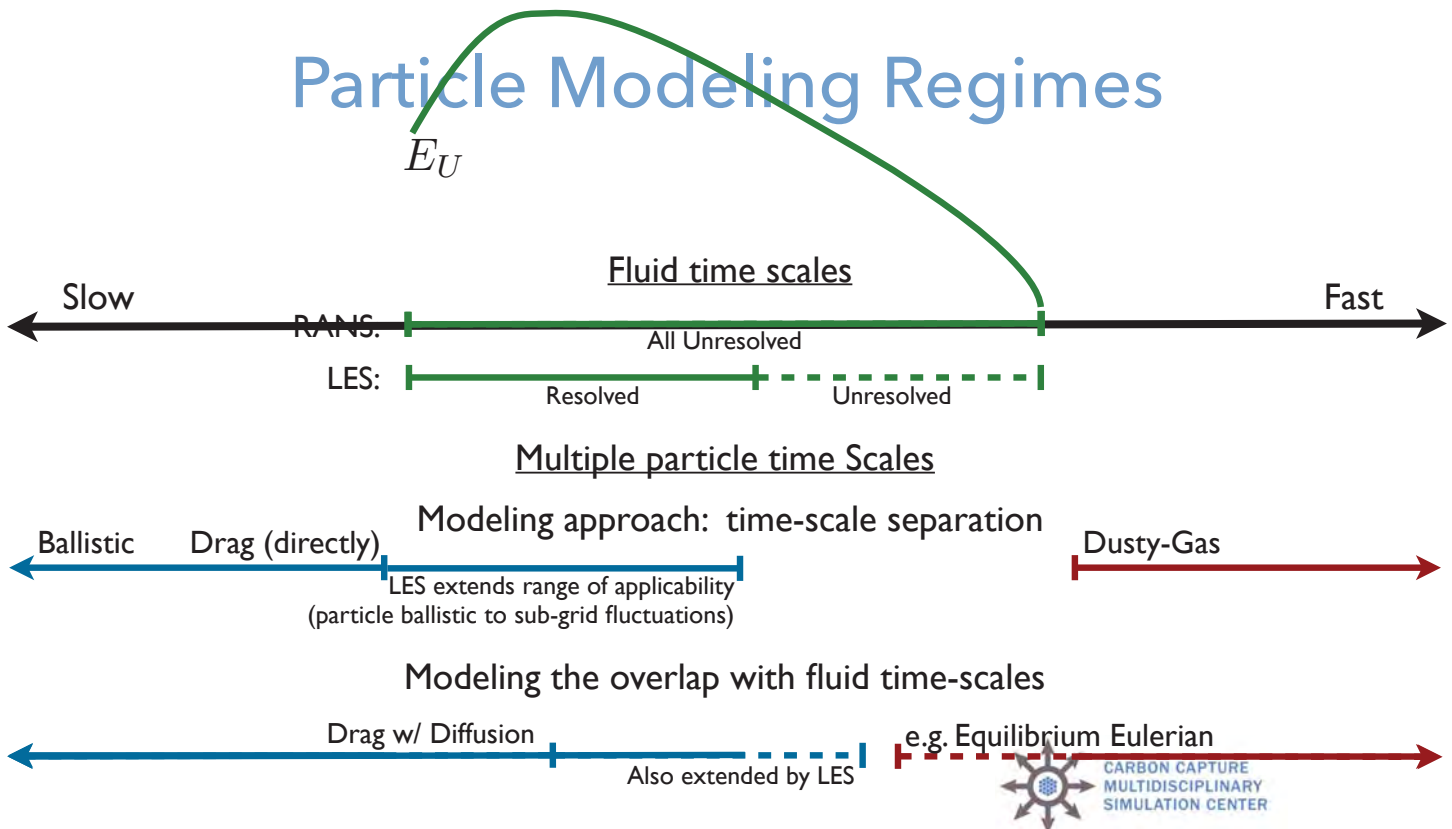
# Particle Time Scales

$$St = \frac{\text{particle relaxation time}}{\text{fluid acceleration time}}$$

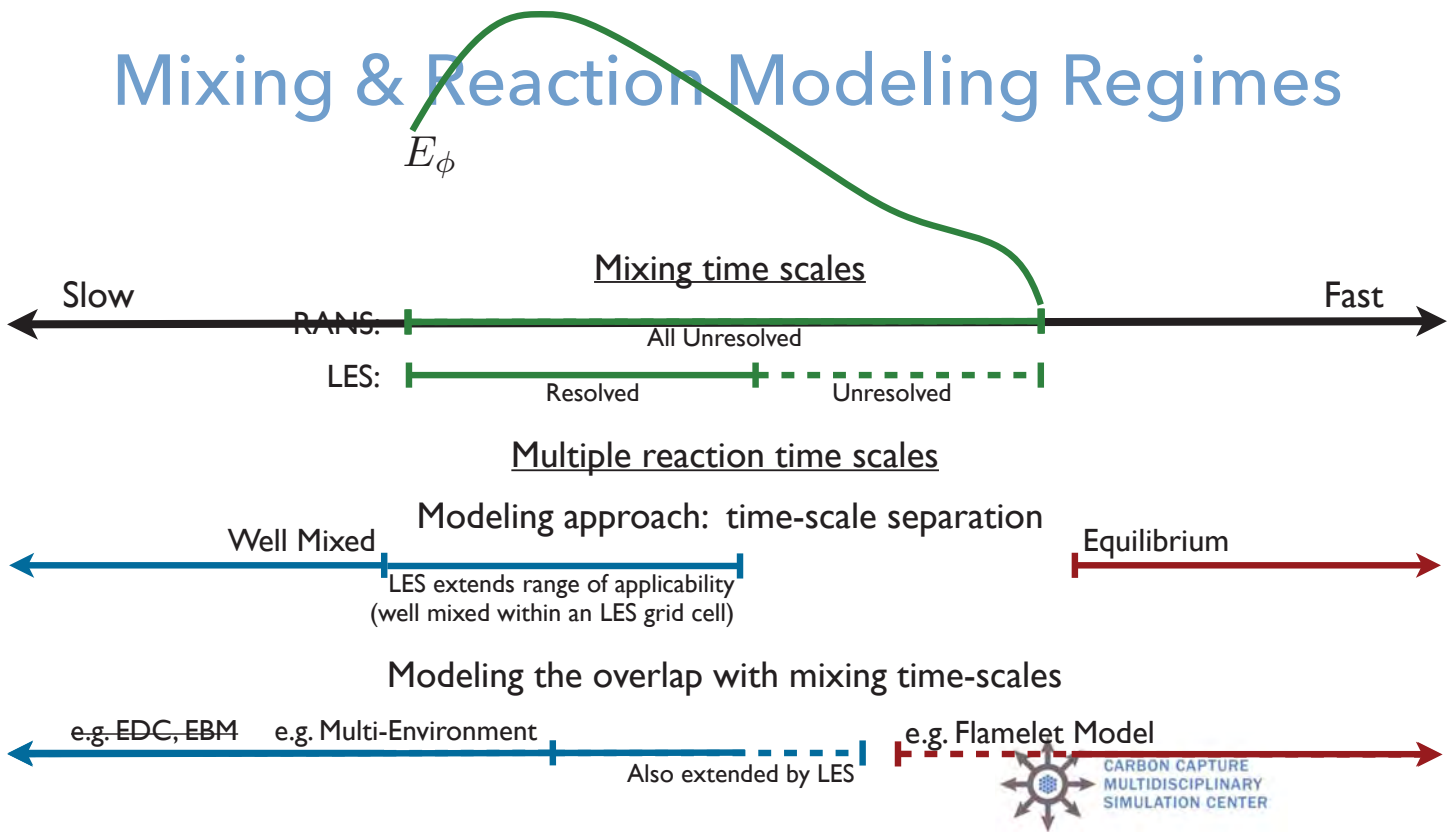
Fixed particle size,  
multiple eddy sizes:



# Particle Modeling Regimes



# Mixing & Reaction Modeling Regimes



# Microscale Modeling based on Generalized Faxén's Theorem

Subramanian Annamalai

Dept. of Mechanical & Aerospace Eng.  
University of Florida, Gainesville

Contact: **Subramanian Annamalai** | [subbu.ase@gmail.com](mailto:subbu.ase@gmail.com)



## Explosive Particle Dispersal

Photron

10000 fps  
Manual 4604  
Date: 2011/8/25

FASTCAM SA5 mode

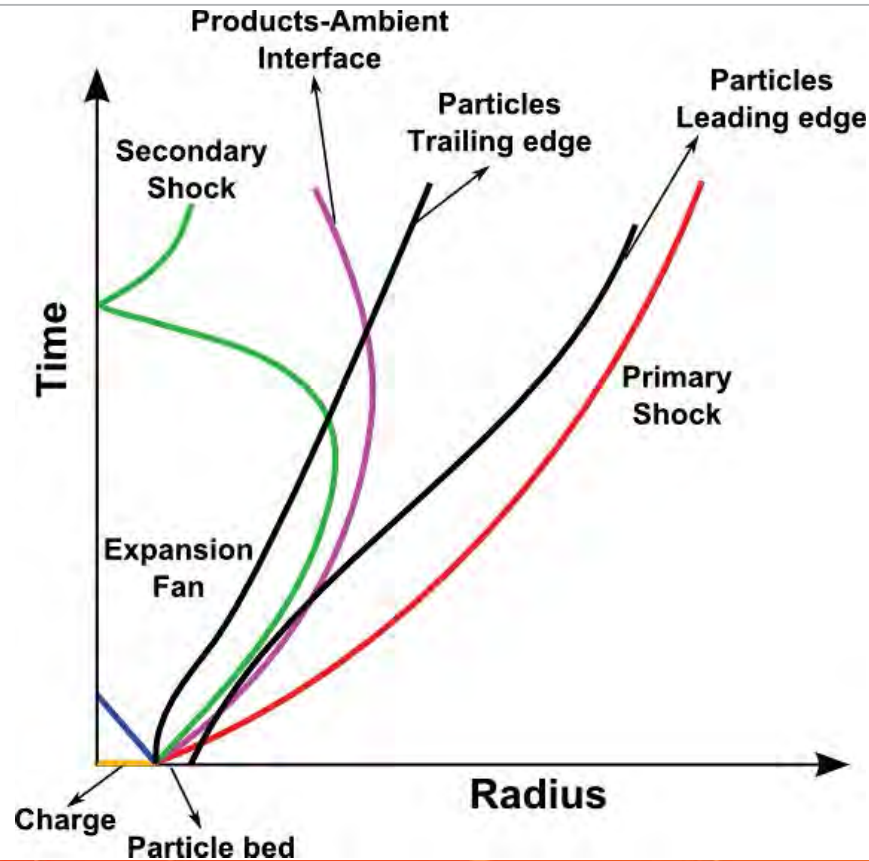
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frame: -937  
Time: 11.22

1024 x 720  
-00:00:00.0937





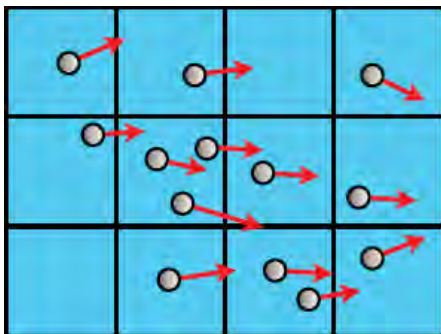
# Radius-Time Schematic



CCMT

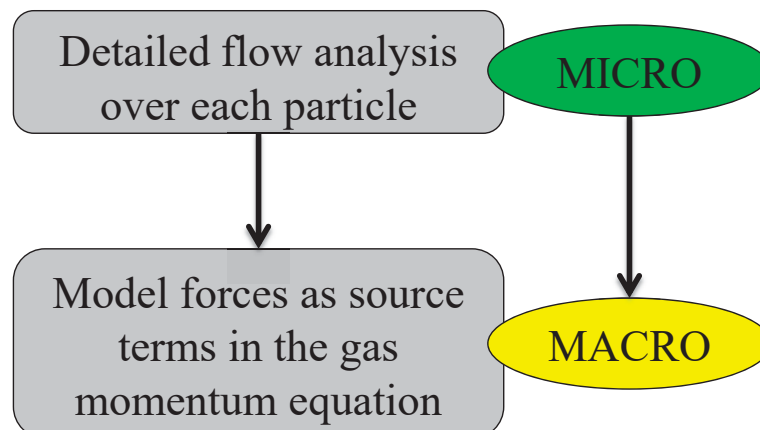
| 3

# Force Modeling: Microscale vs Macroscale



$$m_{p_i} \frac{dv_i}{dt} = \mathbf{F}_i(t),$$

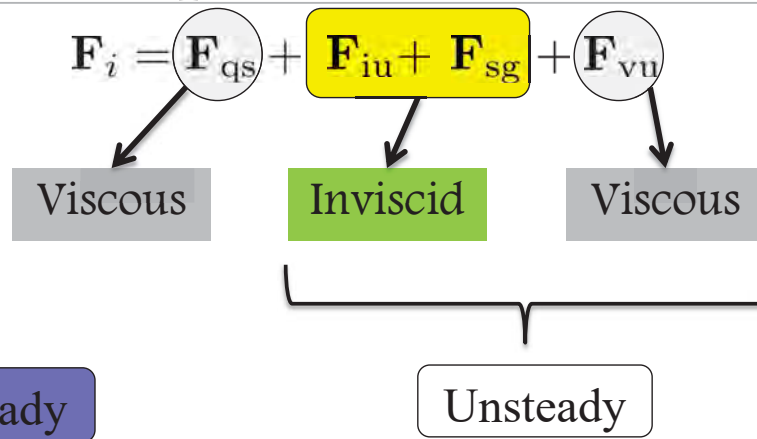
$$\frac{\partial(\rho\phi\mathbf{u})}{\partial t} + \nabla \cdot (\rho\phi\mathbf{u}\mathbf{u}) + \nabla p = -\frac{1}{V} \sum_{i=1}^{nPcls} \mathbf{F}_i(t).$$



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# Force Components



$\mathbf{F}_{qs}$  : Quasi-Steady

Considered with appropriate correction factors

$\mathbf{F}_{iu}$  : Inviscid Unsteady

$\mathbf{F}_{sg}$  : Stress Gradient

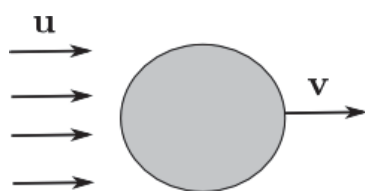
$\mathbf{F}_{vu}$  : Viscous Unsteady

Generally ignored in literature

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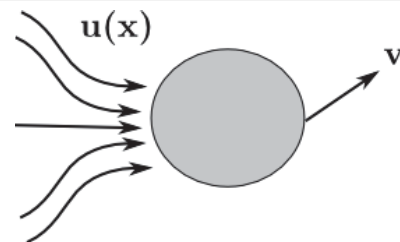
5

# Additional physics involved



Steady

Inhomogeneous

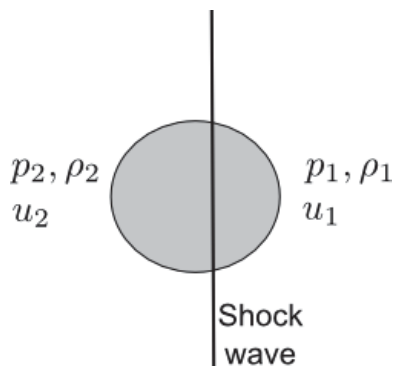


Stokes drag:  $6\pi\mu R(\mathbf{u} - \mathbf{v})$

$\mathbf{F}_{qs}$  : Quasi-Steady

Faxén's law:  $6\pi\mu R(\bar{\mathbf{u}}^S - \mathbf{v})$

$$\bar{()^S} = \frac{1}{S^p} \int_{S^p} () dS$$



Unsteadiness

Compressibility

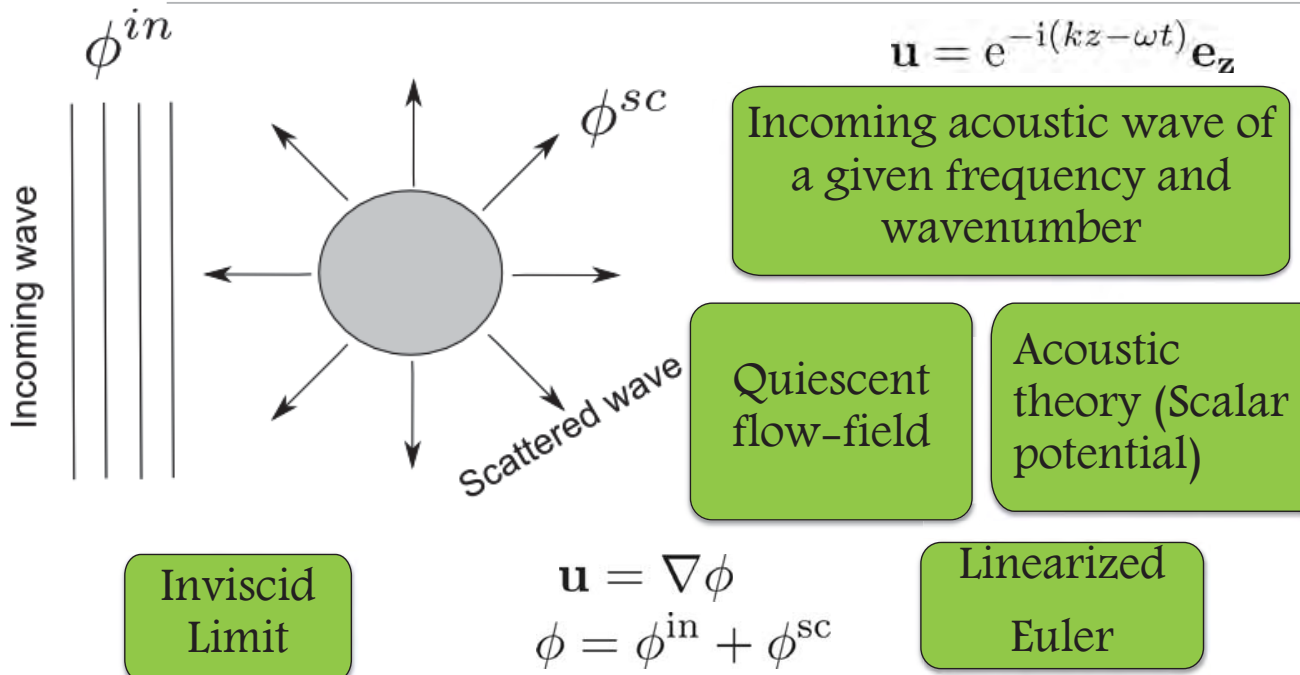
Inhomogeneity

Highly space and time dependent

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# Acoustic analogy & scalar potential



We have extended this analysis to viscous regime

## Inviscid force expression

$$\phi^{in} = \sum_{n=0}^{\infty} C_n (2n+1) (-i)^n j_n(kr) P_n(\cos \theta) e^{i\omega t}$$

$$\phi^{sc} = \sum_{n=0}^{\infty} C_n (2n+1) (-i)^n S_n h_n(kr) P_n(\cos \theta) e^{i\omega t}$$

Rigid Particle:  $\mathbf{u} \cdot \mathbf{n} = 0$  ( $u_r = 0$ )

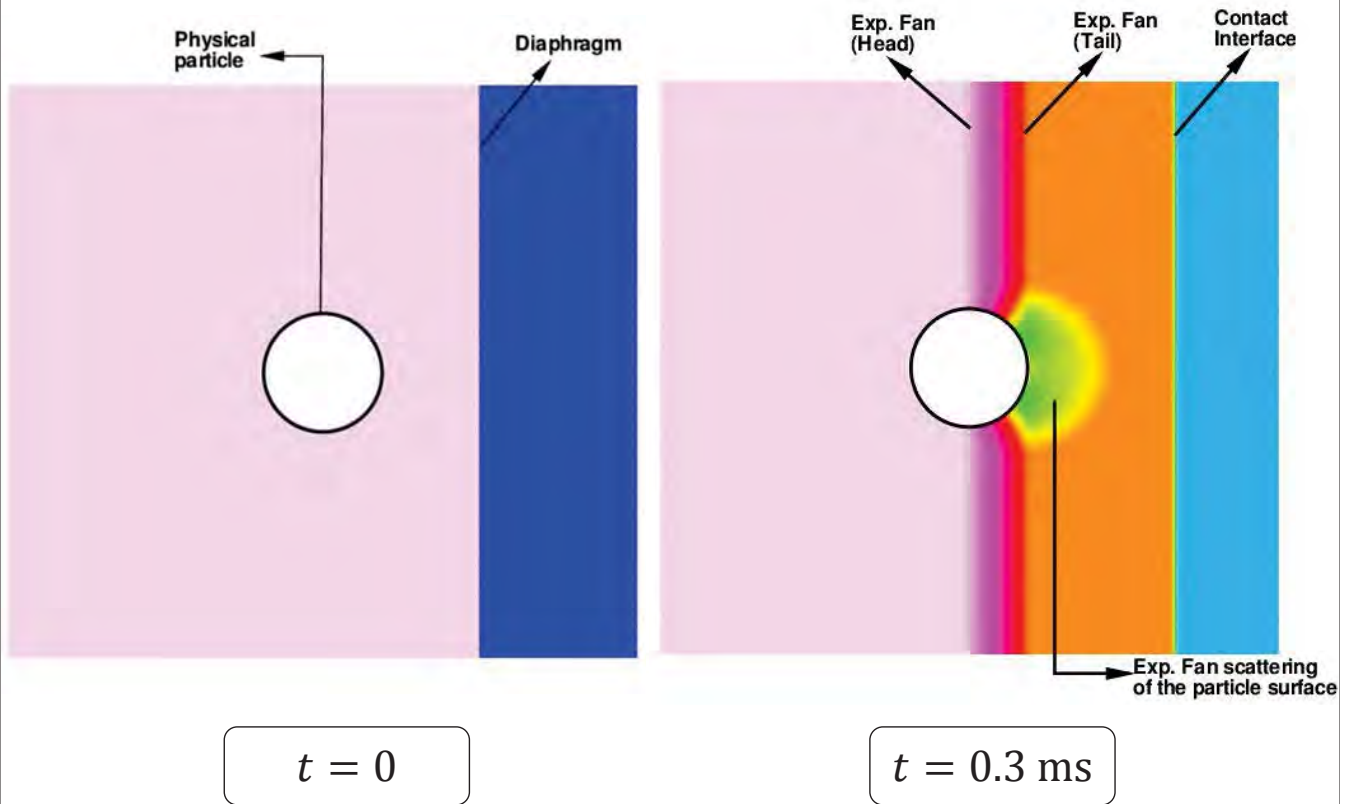
$$\mathbf{F}_{inv} = \int_{V^p} \nabla \cdot \boldsymbol{\sigma}_{inv} dV = - \int_{S^p} p \mathbf{n} dS$$

$$\mathbf{F}_{inv}(t) = \mathbf{F}_{sg}(t) + \mathbf{F}_{iu}(t)$$

$$\mathbf{F}_{sg}(t) = \frac{4}{3} \pi R^3 \rho \overline{\frac{D\mathbf{u}}{Dt}}^V$$

$$\mathbf{F}_{iu}(t) = 4\pi R^3 \int_{\tilde{\xi}=-\infty}^{\tilde{t}} K_{iu}(\tilde{t} - \tilde{\xi}) \left[ \frac{\partial}{\partial \tilde{t}} \overline{(\rho \mathbf{u}_r)}^S \right]_{\tilde{t}=\tilde{\xi}} d\tilde{\xi}.$$

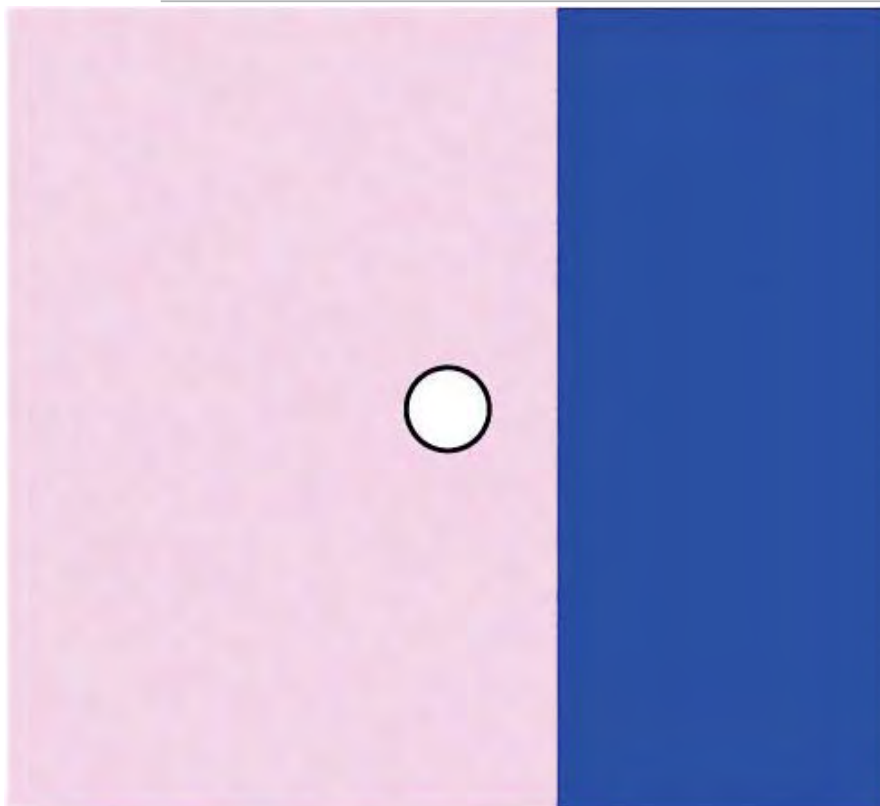
# Expansion fan over the particle



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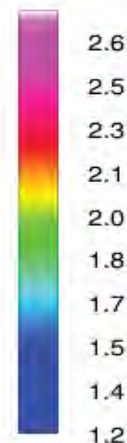
| 9

# 1d Shock Tube : Isolated Particle



Highly resolved  
axisymmetric  
flow over a  
sphere  
(5 mil. grid cells)

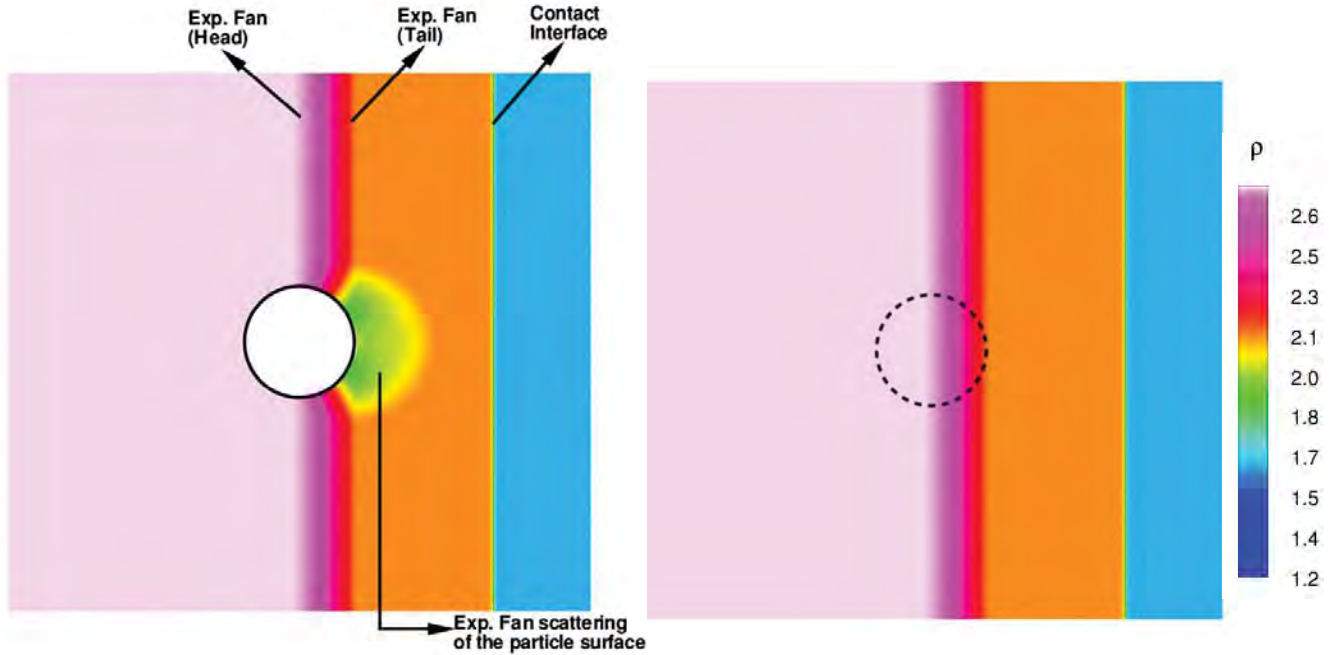
$\rho$



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# Expansion fan over the particle



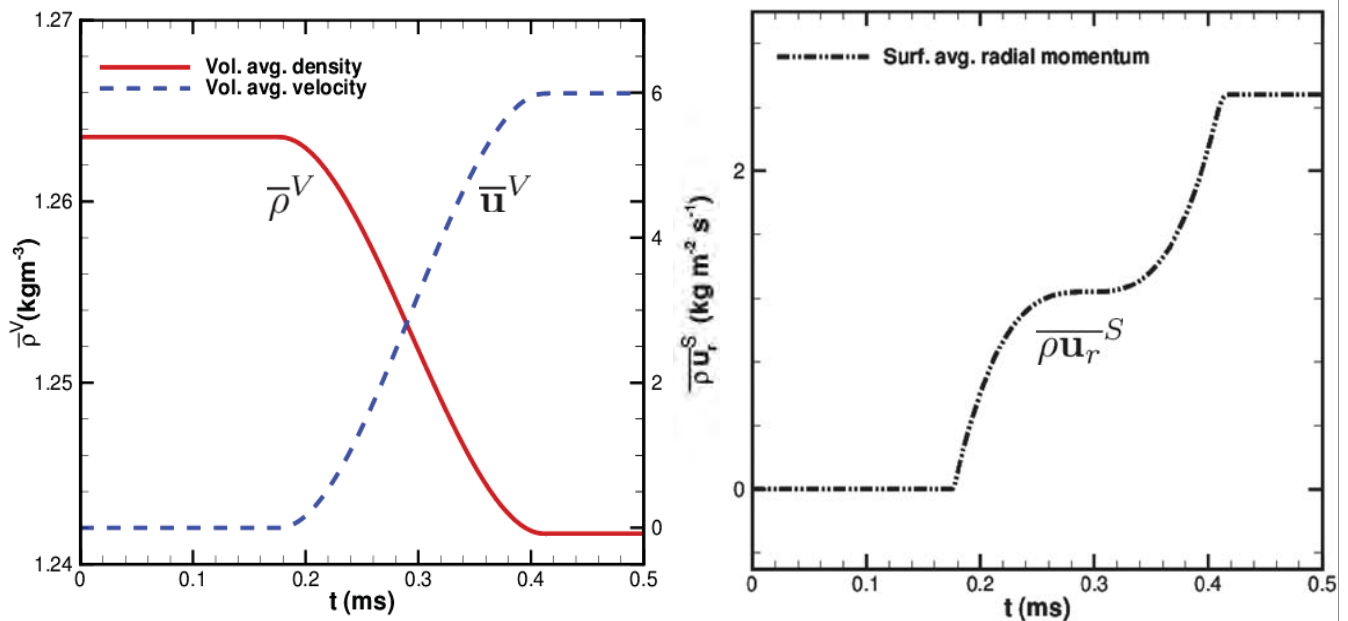
$$\mathbf{F}_{\text{inv}}(t) = \frac{4}{3}\pi R^3 \rho \frac{D\mathbf{u}}{Dt}^V + 4\pi R^3 \int_{\tilde{\xi}=-\infty}^{\tilde{t}} K_{iu}(\tilde{t} - \tilde{\xi}) \left[ \frac{\partial}{\partial t} (\rho \mathbf{u}_r)^S \right]_{\tilde{t}=\tilde{\xi}} d\tilde{\xi}.$$

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# Weak expansion fan

$$\mathbf{F}_{\text{inv}}(t) = \frac{4}{3}\pi R^3 \rho \frac{D\mathbf{u}}{Dt}^V + 4\pi R^3 \int_{\tilde{\xi}=-\infty}^{\tilde{t}} K_{iu}(\tilde{t} - \tilde{\xi}) \left[ \frac{\partial}{\partial t} (\rho \mathbf{u}_r)^S \right]_{\tilde{t}=\tilde{\xi}} d\tilde{\xi}.$$



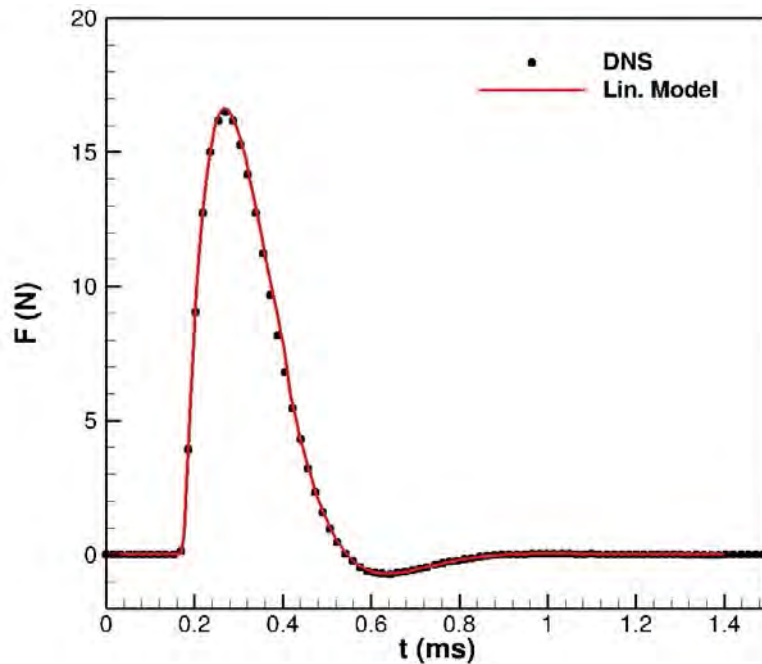
CCMT

| 12



# Weak expansion fan

$$\mathbf{F}_{\text{inv}}(t) = \frac{4}{3}\pi R^3 \rho \frac{D\mathbf{u}}{Dt}^V + 4\pi R^3 \int_{\tilde{\xi}=-\infty}^{\tilde{t}} K_{iu}(\tilde{t} - \tilde{\xi}) \left[ \frac{\partial}{\partial t} (\overline{\rho \mathbf{u}_r})^S \right]_{\tilde{t}=\tilde{\xi}} d\tilde{\xi}.$$



Averaged  
undisturbed flow  
properties

Local time  
derivative

Initial Pressure  
Ratio = 1.05

Shock Mach  
number = 1.01

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# Strong expansion fan

$$\mathbf{F}_{\text{inv}}(t) = \frac{4}{3}\pi R^3 \rho \frac{D\mathbf{u}}{Dt}^V + 4\pi R^3 \int_{\tilde{\xi}=-\infty}^{\tilde{t}} K_{iu}(\tilde{t} - \tilde{\xi}) \left[ \frac{D}{Dt} (\overline{\rho \mathbf{u}_r})^S \right]_{\tilde{t}=\tilde{\xi}} d\tilde{\xi}.$$

Exp. fan first interacts w/ particle

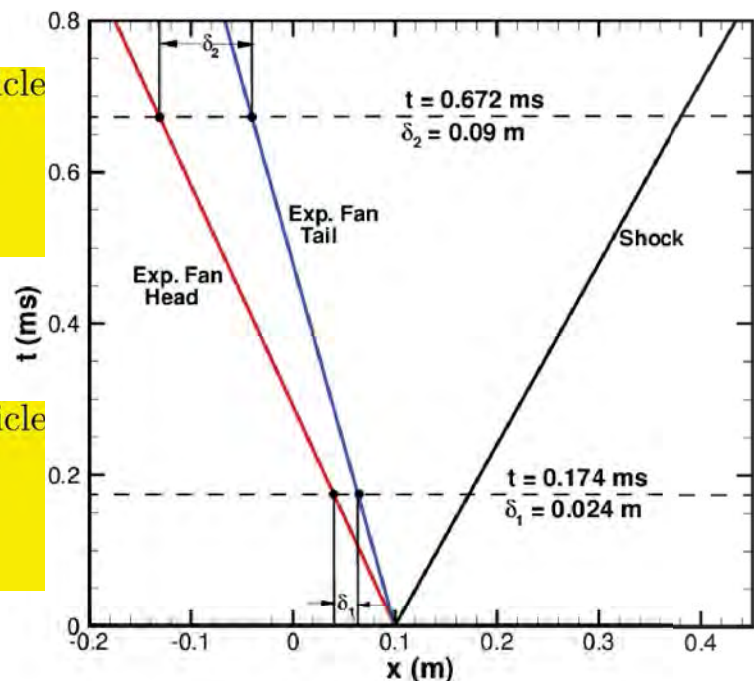
$t = 0.174 \text{ ms}$

$\frac{\delta_1}{R} = 0.6$

Exp. fan last interacts w/ particle

$t = 0.672 \text{ ms}$

$\frac{\delta_1}{R} = 2.25$

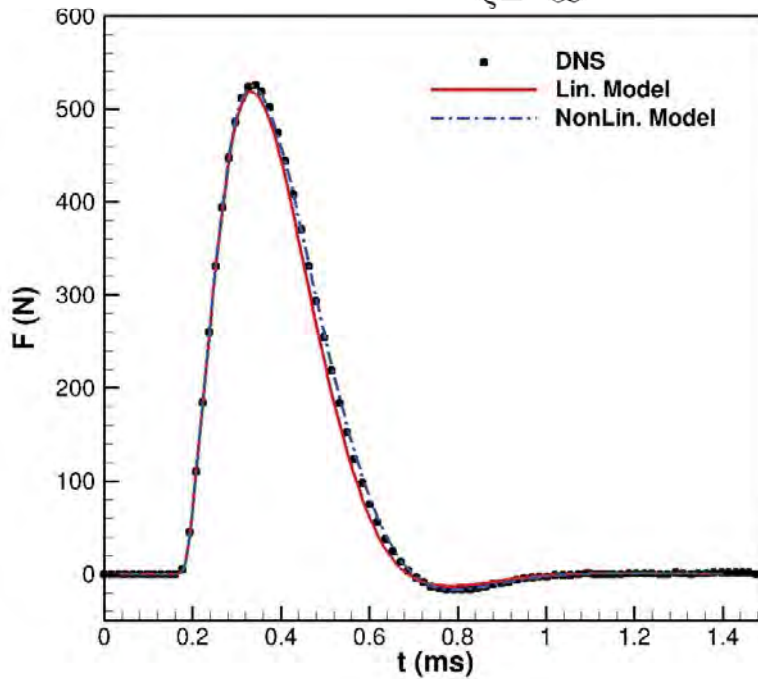


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# Strong expansion fan

$$\mathbf{F}_{\text{inv}}(t) = \frac{4}{3}\pi R^3 \rho \frac{D\mathbf{u}}{Dt}^V + 4\pi R^3 \int_{\tilde{\xi}=-\infty}^{\tilde{t}} K_{iu}(\tilde{t}-\tilde{\xi}) \left[ \frac{D}{Dt} (\rho \mathbf{u}_r)^S \right]_{\tilde{t}=\tilde{\xi}} d\tilde{\xi}.$$



$$K_{iu} = e^{-(\tilde{t}-\tilde{\xi})} \cos(\tilde{t}-\tilde{\xi})$$

Substantial time derivative

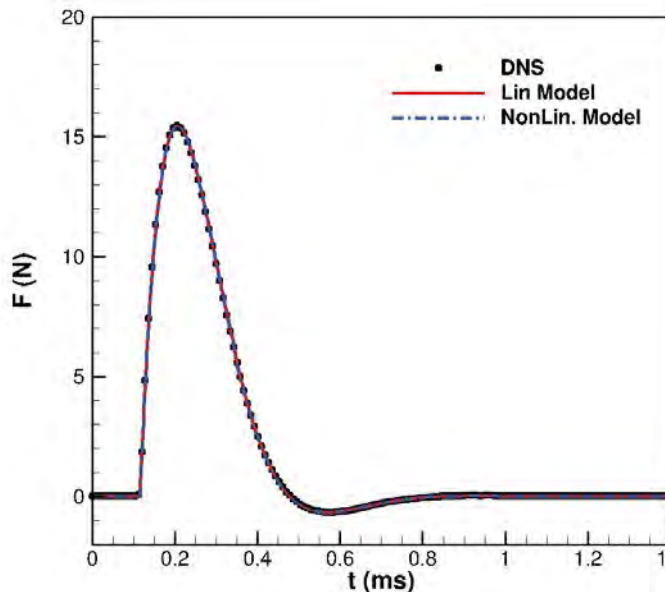
Initial pressure ratio = 2.5

Shock Mach number = 1.22

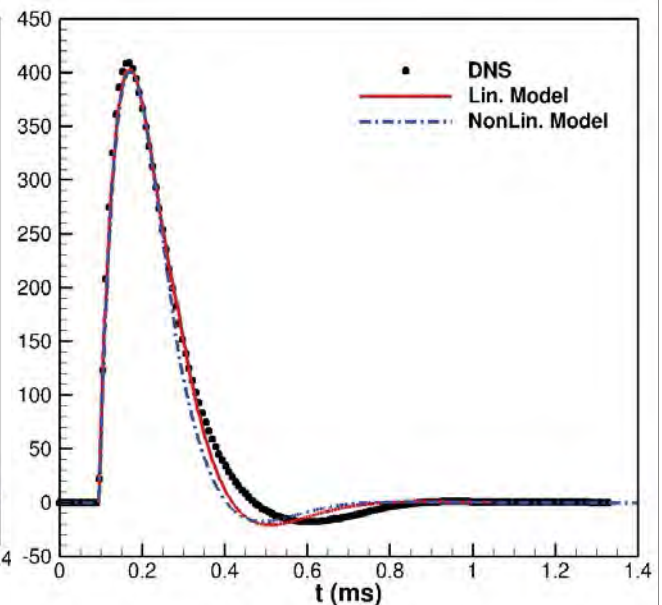
CCMT

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# Weak & Strong normal shock



Weak Shock  
( $M_s = 1.01$ )

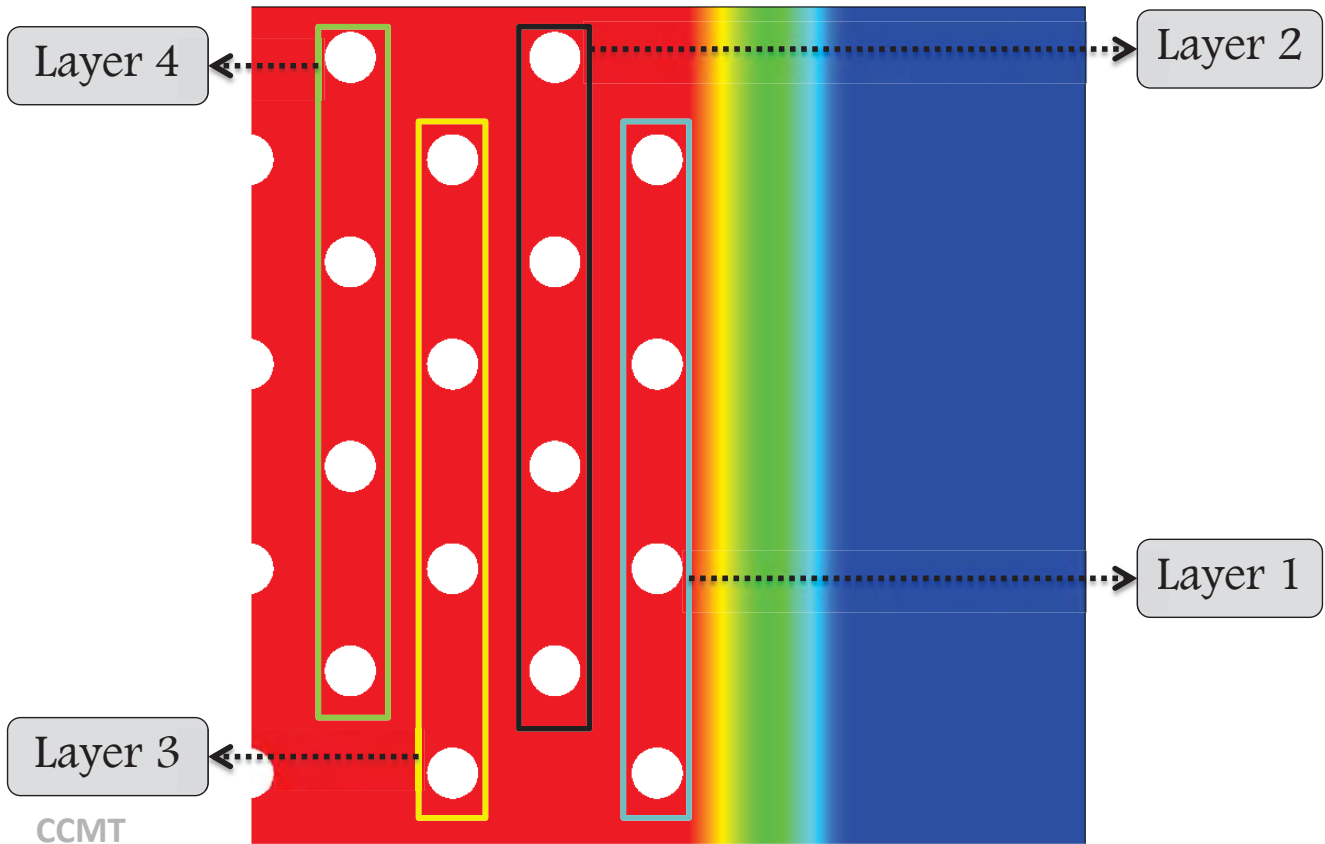


Strong Shock  
( $M_s = 1.22$ )

CCMT

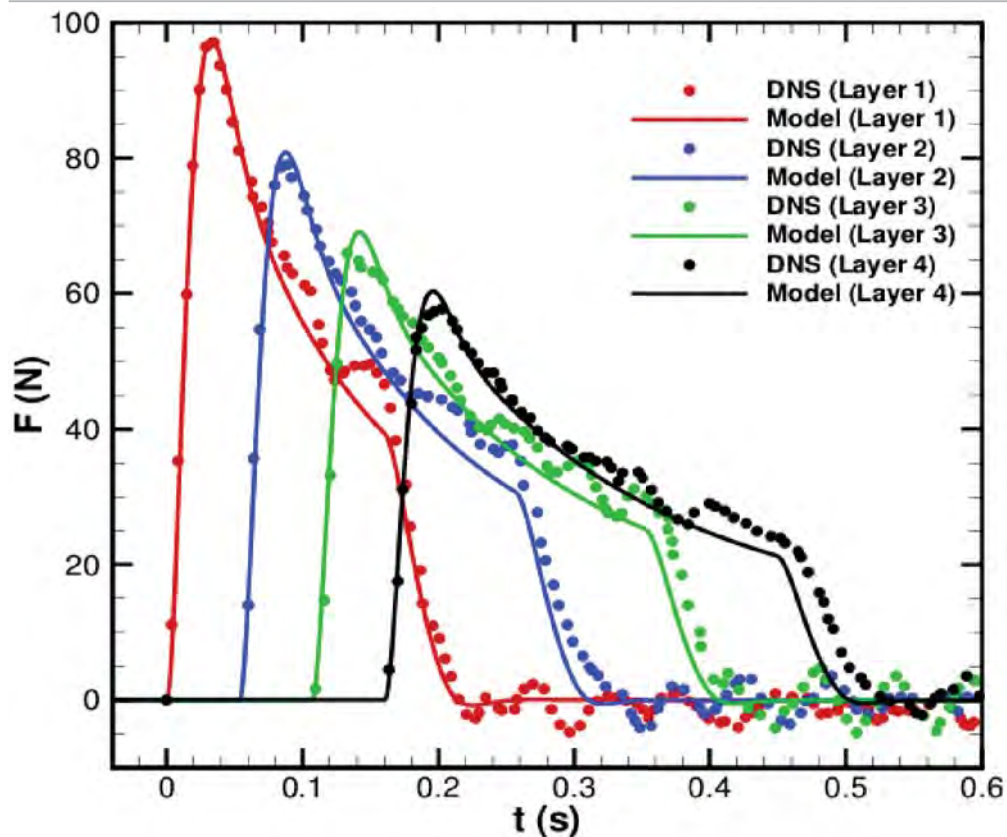
| 16

# Expansion fan – Multi-particle system



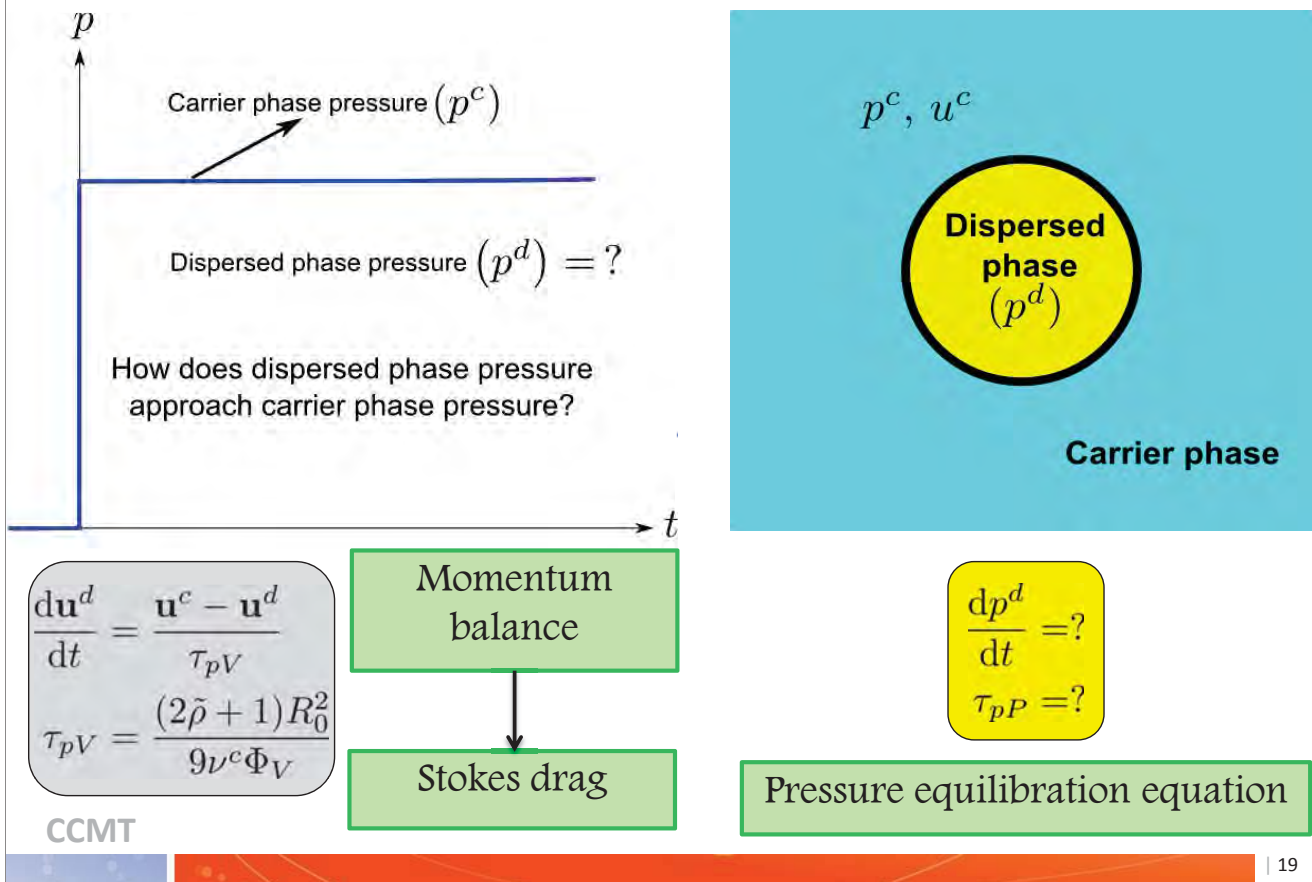
| 17

# Expansion fan – Multi-particle system



| 18

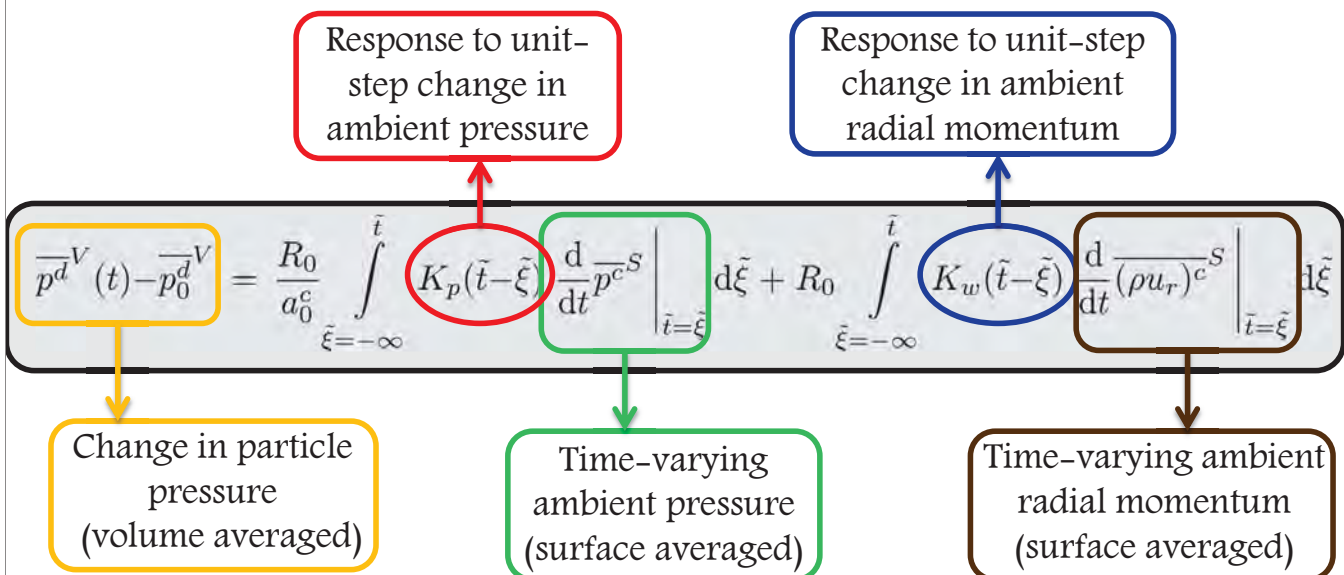
# Pressure evolution inside the particle



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## Pressure Equilibration Equation

The volume-averaged particle phase pressure expressed purely in terms of “undisturbed” time-variation of the carrier phase properties is given as below:



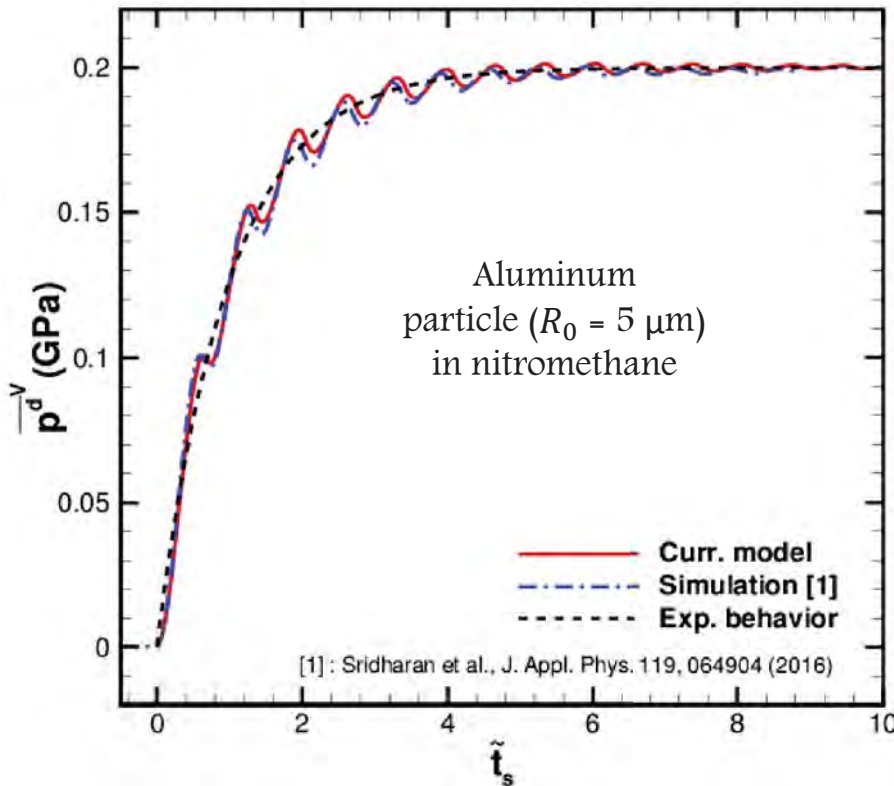
$R_0$  : Mean particle radius,  $a_0^c$  : Ambient speed of sound

$K_p, K_w = f(\tilde{\rho}, \tilde{a})$  where  $\tilde{\rho}$  : density ratio,  $\tilde{a}$  : Speed of sound ratio

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# Pressure Evolution: Shock Ma = 1.11



Exp. behavior (black-dash line) analogous to particle equation of motion:

$$\frac{d\overline{p^d}^V}{dt} = \frac{p_{\text{final}}^c - \overline{p^d}^V}{\tau_{pP}}$$

$$\tau_{pP} = R_0/u_s$$

is the particle time-scale

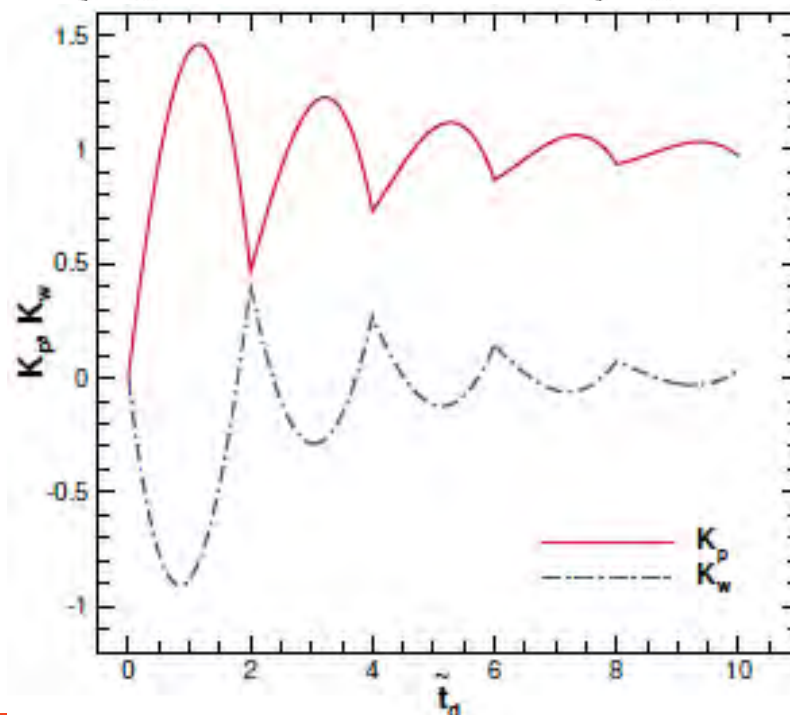
Similar agreements also observed for higher Mach numbers and particles with larger radii.

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## Pressure kernels

$$\overline{p^d}^V(t) - \overline{p_0^d}^V = \frac{R_0}{a_0^c} \int_{\tilde{\xi}=-\infty}^{\tilde{t}} K_p(\tilde{t}-\tilde{\xi}) \left. \frac{d\overline{p^c}^S}{d\tilde{t}} \right|_{\tilde{t}=\tilde{\xi}} d\tilde{\xi} + R_0 \int_{\tilde{\xi}=-\infty}^{\tilde{t}} K_w(\tilde{t}-\tilde{\xi}) \left. \frac{d(\overline{\rho u_r})^c}{d\tilde{t}} \right|_{\tilde{t}=\tilde{\xi}} d\tilde{\xi}$$

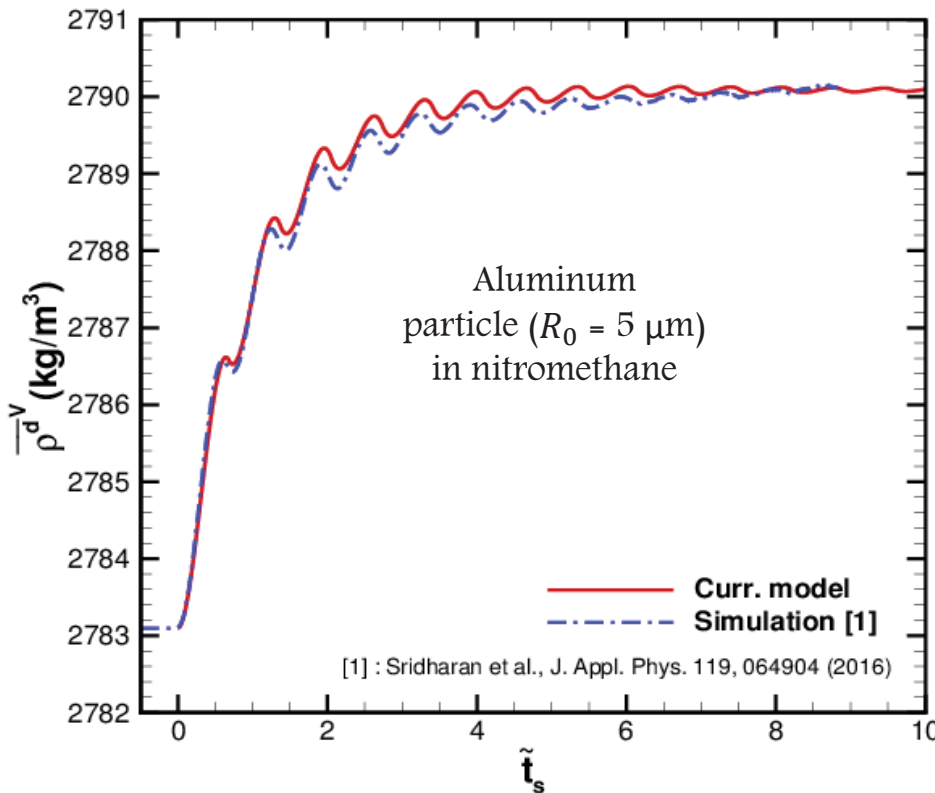


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# Density Evolution: Shock Ma = 1.11



Density evolution



Specific volume  
time history



Equation for volume  
fraction evolution

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## Future work & Challenges

- Application of the generalized Faxén theorem when there is a random cloud of particles.
- How does one handle particle compaction especially when a strong blast wave rams on a close-packed particle bed?
- A similar pressure model allowing for particle deformation.
- While we have a state of the art hydrodynamic force model (gas-particle interaction), a model for particle-particle interaction in highly compressible and unsteady environments is still in its primitive stages.

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# Any Questions





# Modeling of Ejecta Particles in the FLAG Continuum Mechanics Code

Alan K. Harrison  
Lagrangian Codes Group (XCP-1)  
Computational Physics Division

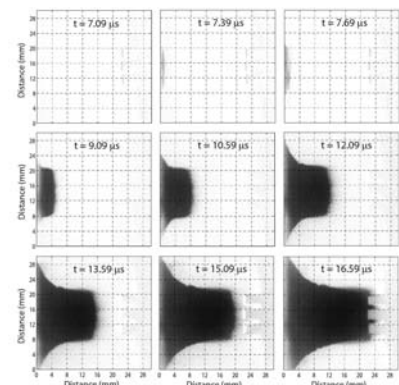
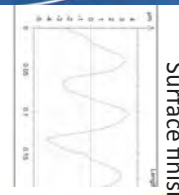
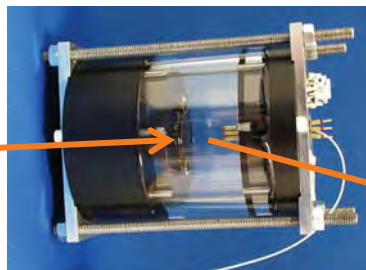
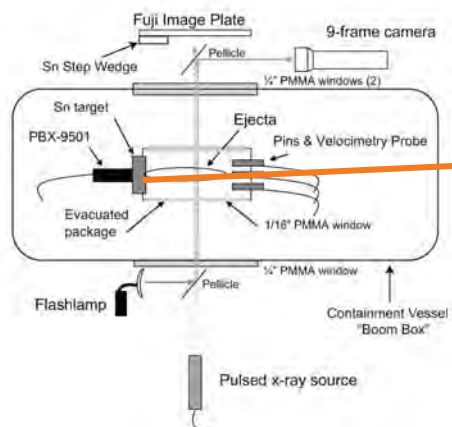
Multiphase Physics Deep-Dive  
St. Petersburg, Florida  
October 7, 2016



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## Motivation: Ejecta and modeling ejecta experiments

- Extreme shock loading may cause damage and failure at material free surfaces, producing particulate fragmentation known as ejecta.
- Theories, experiments, and modeling involve a wide range of solid and fluid mechanics at relevant spatial and temporal scales.



See many experimental papers  
by W. T. Buttler, R. T. Olson,  
M. B. Zellner and coworkers

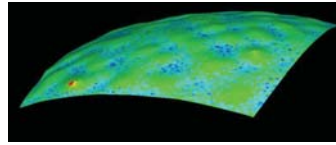
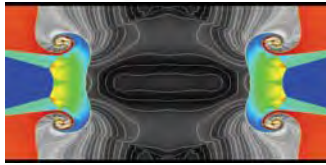
Slide 2



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# The FLAG hydrocode is our testbed to simulate the experiment.



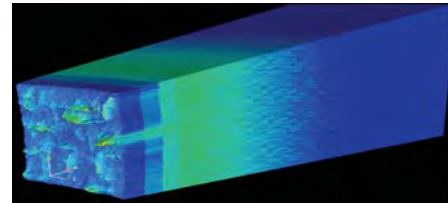
High strain rates

Strength, damage, failure

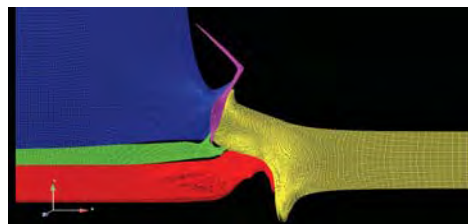


Compressibility

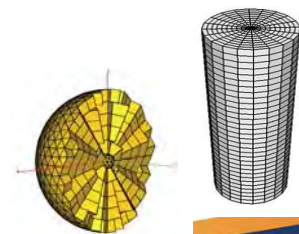
Mixtures



Impact/contact



1, 2, 3D  
Lagrange/ALE hydro  
Fully unstructured meshes  
(arbitrary polygons/polyhedra)



Slide 3



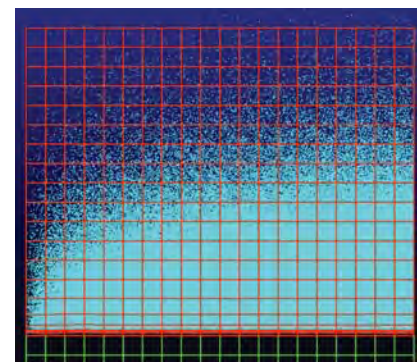
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## A particle-in-cell formulation is used to model ejecta in FLAG.

- (Super) particles represent packets of multiple physical particles.
  - Tracking individual physical particles is expensive, so allowing computational particles to represent “many” reduces cost. The tradeoff is the statistical resolution.
- While FLAG hydro advances the continuum equations of motion, a distinct solver is implemented to advance particle equations of motion.
  - Positions and velocities in space and time
  - Positions relative to the hydrodynamics mesh
- Particle-fluid coupling involves
  - Summing/averaging particle quantities over zones (to an “ejecta phase”)
  - Interpolating continuum information from mesh zones and points to particles
- How is the formulation integrated with FLAG hydro?
  1. Predictor step for continuum momentum and energy
  2. Corrector step for continuum momentum and energy
  3. Ejecta tracking and transport\*
  4. Ejecta sourcing (for launch next cycle)\*

\*Including momentum/energy exchange with continuum



Andrews, M.J. and  
O'Rourke, P.J. : MP-PIC

Slide 4



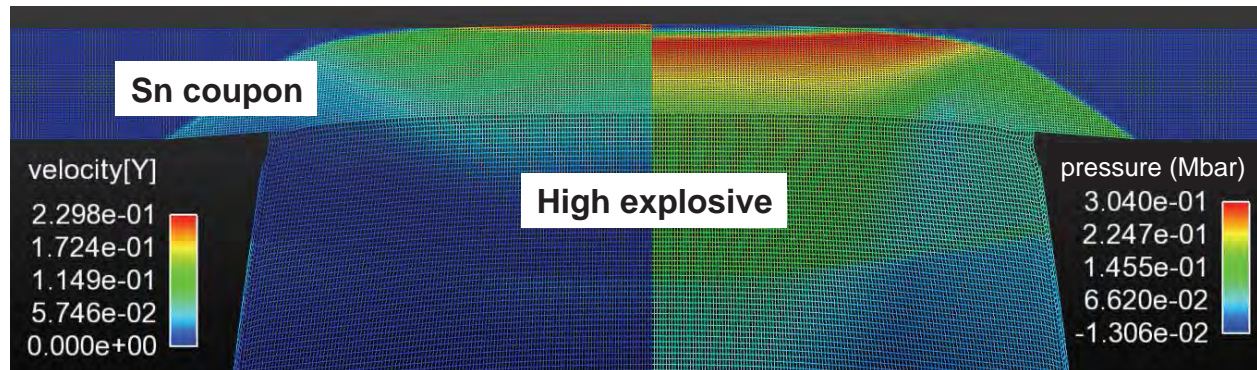
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# An ejecta calculation begins with a shock passing through the source metal

Calculation and slide  
by I. Tregillis

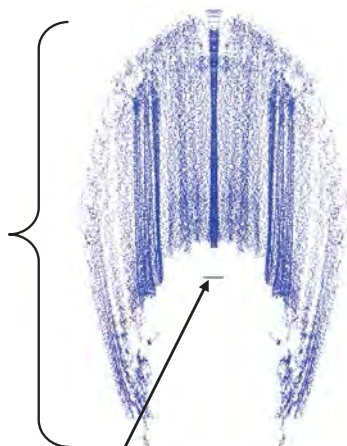


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Slide 5  
**Los Alamos**  
NATIONAL LABORATORY  
EST. 1943

## The calculation tracks ejecta within or/and beyond the problem mesh

*When the surface doesn't refreeze, ejecta production in the calculation can persist indefinitely.*



Calculation and slide  
by I. Tregillis

Size & location of central  
 $\text{LiNbO}_3$  piezo head for this shot

Melt state of Sn coupon:  
**Melted** **Solid**



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Slide 6  
**Los Alamos**  
NATIONAL LABORATORY  
EST. 1943



## The ejecta package includes pieces corresponding to different stages of ejecta development

- The **source package** determines *whether/when* to produce ejecta, the *production rate*, and the *initial conditions* (size and velocity distributions) of the particles produced
  - The production decision is based on shock detection and surface properties
  - The Richtmyer-Meshkov Instability (RMI) source model predicts
    - the production rate, and
    - initial particle sizes and velocities
- Other packages account for particle-gas forces during the **transport** phase
  - drag
  - buoyant force
  - effective force field in dense particle clouds
- Other packages account for various types of particle **transformations**
  - heat flux
  - mass flux (evaporation)
  - particle breakup
  - collisions

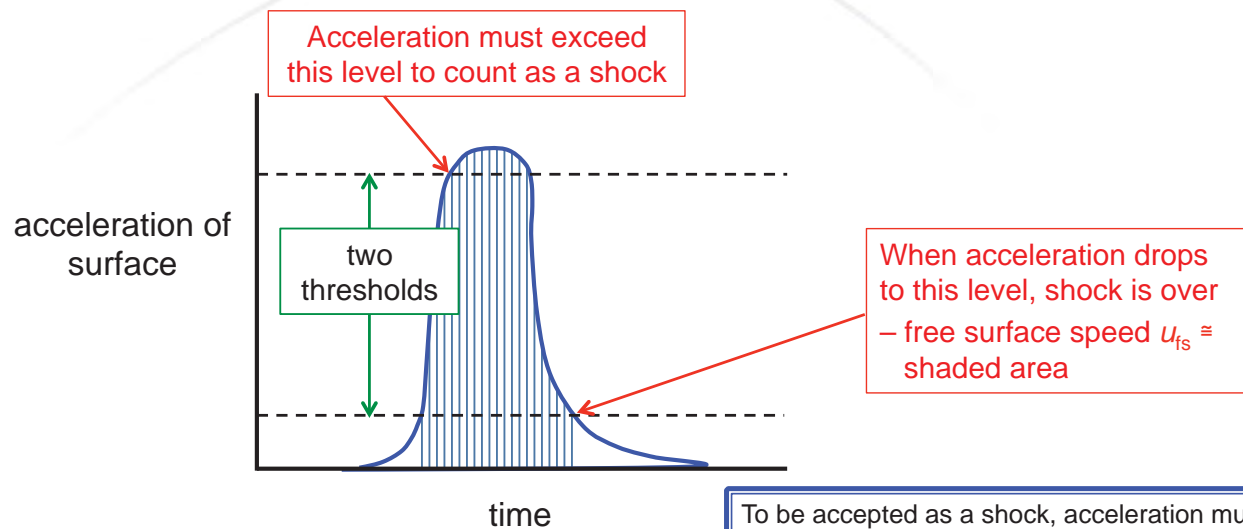
Slide 7



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## Shock detection and characterization depend on acceleration and melt state of the metal surface



To be accepted as a shock, acceleration must

- (1) start below lower threshold
- (2) exceed upper threshold
- (3) drop below lower threshold

and **surface must be melted** at time (3)

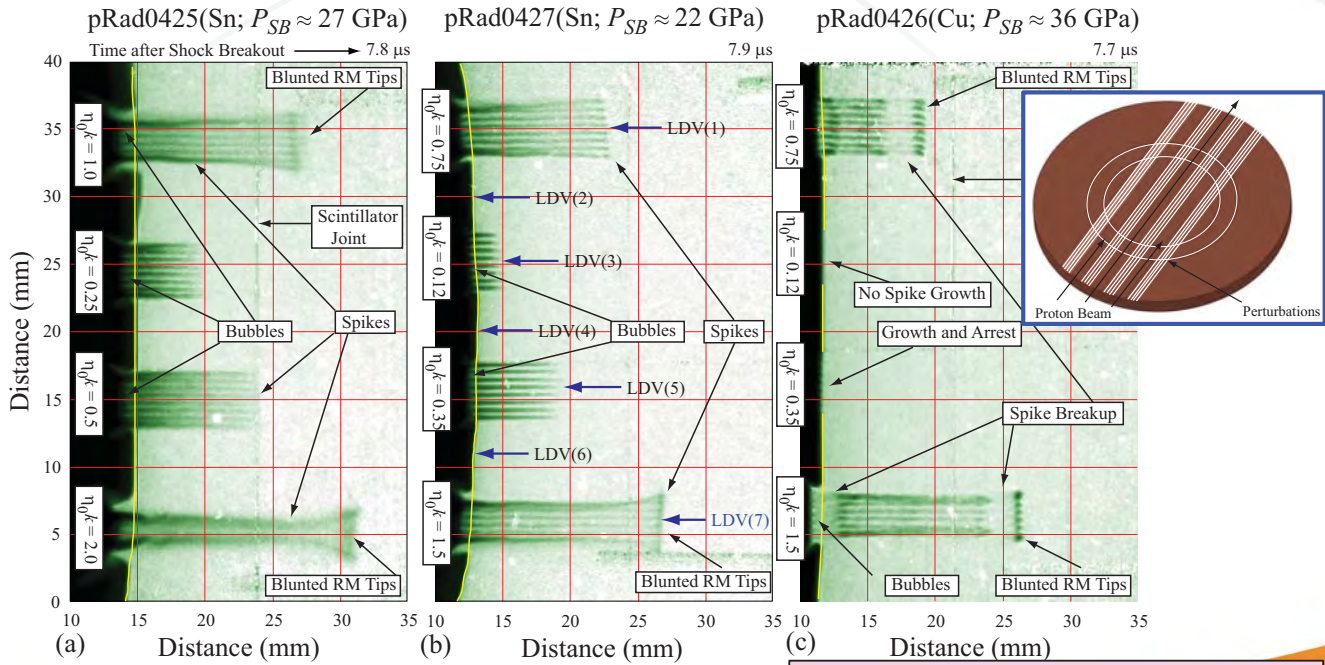
Slide 8



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# Under conditions known to produce ejecta, proton radiography shows the spikes and bubble of RM instability



W. T. Buttler, D. M. Oró, D. L. Preston, K. O. Mikaelian, F. J. Cherne, R. S. Hixson, F. G. Mariam, C. Morris, J. B. Stone, G. Terrones and D. Tupa, *Journal of Fluid Mechanics* **703**(July 2012):60-84  
DOI: 10.1017/jfm.2012.190, Published online: 13 June 2012



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## Ejecta particles are droplets broken off spike tips

Buttler *et al.* (2012) analysis of spike and bubble growth rates (based on earlier work by Mikaelian):

$$\dot{\eta}^b(t) = \frac{2\dot{\eta}_0^b}{2 + 3\dot{\eta}_0^b kt} \quad \dot{\eta}^s(t) \approx \sqrt{3} \dot{\eta}_0^s$$

with initial rates

$$\dot{\eta}_0^{b,s} = \pm F_l F_{nl}^{b,s} a u_{fs}$$

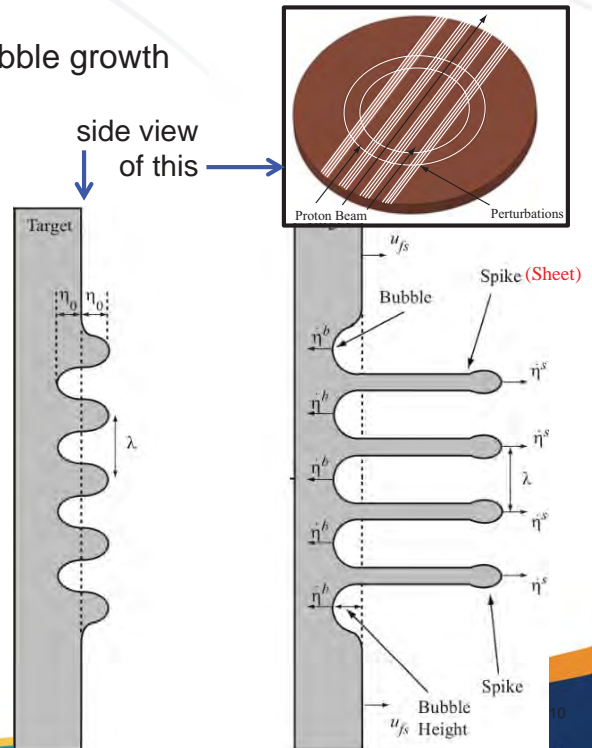
where

$$a = \eta_0 k = \frac{2\pi\eta_0}{\lambda} \quad F_l = 1 - \frac{u_{fs}}{2u_{sh}}$$

$$F_{nl}^b = \frac{1}{1 + \frac{a}{6}} \quad F_{nl}^s = \frac{1}{1 + \frac{a^2}{4}}$$

$u_{sh}$  = shock velocity

$u_{fs}$  = free surface velocity



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# We have implemented a force-field model due to particle-particle interactions

$$F_i^{pp} = -\frac{V_i^p}{\phi_i^p} \nabla \tau_{pp,i}$$

$$\tau_{pp,i} = \frac{P_s (\phi_i^p)^\beta}{\phi_c^p - \phi_i^p}$$

- This code capability does not actually predict individual collisions, but it is intended to account for their effect.
- The model is simple and designed for dense-particle flows, so it will tend to spread out particles even if they are stationary.
- Predicting individual collisions is much more expensive (future work), so a model of this type is welcome.



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Slide 11

# We have implemented a model of heat flux between particles and gas

- Convective heat transfer

$$\dot{Q}_c = \text{Nu} \pi D k_c (T_s - T_c)$$

where  $T_s$  is the particle surface temperature and  $T_c$  is the gas temperature, and we use the Ranz-Marshall correlation for Nusselt number:

$$\text{Nu} = 2 + 0.6 \text{Re}^{1/2} \text{Pr}^{1/3}$$

- This capability should be important in predicting the temperature history of ejecta particles, which is needed for proper prediction of their evaporation (next slide)
- Reference for this model and mass flux model (next slide): C. T. Crowe, J. D. Schwarzkopf, M. Sommerfeld and Y. Tsuji, *Multiphase Flows with Droplets and Particles*, 2nd ed. (CRC Press: Boca Raton, 2012).



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Slide 12

## We have implemented a model of mass flux between particles and gas (evaporation)

- Evaporation

$$\dot{m} = Sh \pi D \rho_c D_v (\omega_\infty - \omega_s)$$

where  $\omega_\infty$  is the mass fraction for the evaporated phase in the free stream and  $\omega_s$  is the surface concentration of the evaporated phase. We use the Ranz-Marshall correlation for Sherwood number:

$$Sh = 2 + 0.6 Re^{1/2} Sc^{1/3}$$

This mass flux also carries energy with it:

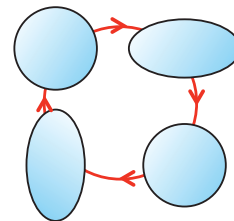
$$\dot{Q}_m = \frac{Sh}{2} \frac{12 k_c}{c_d \rho_d D^2} \frac{Pr}{Sc} \frac{h_l}{c_p} (\omega_\infty - \omega_s)$$

- As a particle loses matter from its surface, the code tracks its decrease in mass and diameter
- When its mass drops to zero it is deleted from the calculation
- This model relies on predicting the metal vapor pressure at the surface of a particle, which depends on knowing its thermodynamic state

## We model oscillations in a liquid droplet as a damped driven harmonic oscillator—an analogy

- Experimental evidence suggests that ejecta are liquid drops, not solid particles
- Accordingly, we have implemented a drop breakup model—the Taylor Analogy Breakup (TAB) model—in FLAG.

- We model an ejecta particle as a liquid drop
  - with radius  $r$ , density  $\rho_l$ , viscosity  $\mu_l$ , surface tension  $\sigma$
  - moving through a gas of density  $\rho_g$
  - with relative velocity  $u$



- The displacement  $x$  of the droplet's equator from its equilibrium position satisfies the differential equation

$$m \ddot{x} = F - kx - d\dot{x}$$

in which aerodynamics, surface tension and droplet viscosity provide the force terms:

$$\frac{F}{m} = C_F \frac{\rho_g u^2}{\rho_l r} \quad \frac{k}{m} = C_k \frac{\sigma}{\rho_l r^3} \quad \frac{d}{m} = C_d \frac{\mu_l}{\rho_l r^2}$$

- We evaluate the exact solution to the ODE and test for the breakup condition  $x = C_b r$ .



# When a particle breaks up we must model the daughter droplets

- Daughter droplets acquire a new velocity increment perpendicular to the original velocity vector

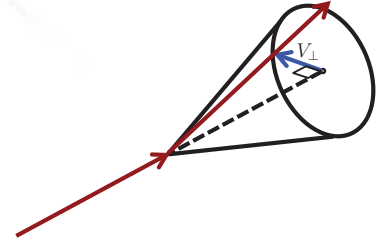
$$C_v = 1 \quad (\text{default } C_v = 1)$$

- An energy conservation argument determines the Sauter mean radius  $r_{32} \equiv \langle r^3 \rangle / \langle r^2 \rangle$  of the expected distribution of daughter droplet radii:

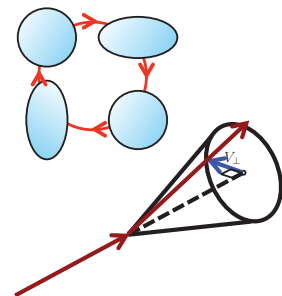
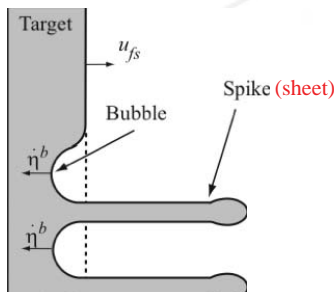
$$r_{32} = r \left[ 1 + \frac{2K}{5} + \left( \frac{K}{20} - \frac{1}{24} \right) \frac{\rho_l r \dot{x}^2}{C_b^2} \right]^{-1} \quad (\text{default } K = 10/3)$$

- It suffices to create a single daughter, with the direction of  $V_\perp$  sampled from  $[0, 2\pi]$ , and radius sampled from the indicated size distribution
- We must rescale the number of physical particles in the daughter packet

$$N^{\text{new}} = N \left( \frac{r}{r^{\text{new}}} \right)^3$$



## The TAB-SHEET and TAB-LIG models account for breakup of spikes to droplets



RMI source model:  
production of sheets

TAB-SHEET model:  
sheets tear into ligaments

TAB-LIG model:  
ligaments break up into  
droplets

TAB model: droplets  
break up into smaller  
droplets

These models are mathematically similar to the original TAB model; only the coefficients in the ODE are different.



# Acknowledgments

- The code development work reported here was done in partnership with J. Fung, N. Denissen and other code developers on the FLAG code team
- We have benefitted from collaborations with LANL research teams in experimental, model development and V&V areas
- This work was supported by the Advanced Simulation and Computing Program (ASC) of the National Nuclear Security Administration (NNSA) of the U. S. Department of Energy.

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# Summary and invitation

- We have developed a particle-in-cell capability to model mass ejection and transport (ejecta) in a Lagrange/ALE continuum dynamics code.
- We are building models of ejecta phenomena including production, transport through gases, evaporation and breakup. The models are at various levels of fidelity and maturity.
- We invite collaboration with experts in relevant phenomena—theory, experiment, simulation and modeling.
- Contact us (Alan K. Harrison or Marianne M. Francois) if you are interested.



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Slide 19

# Backup slides

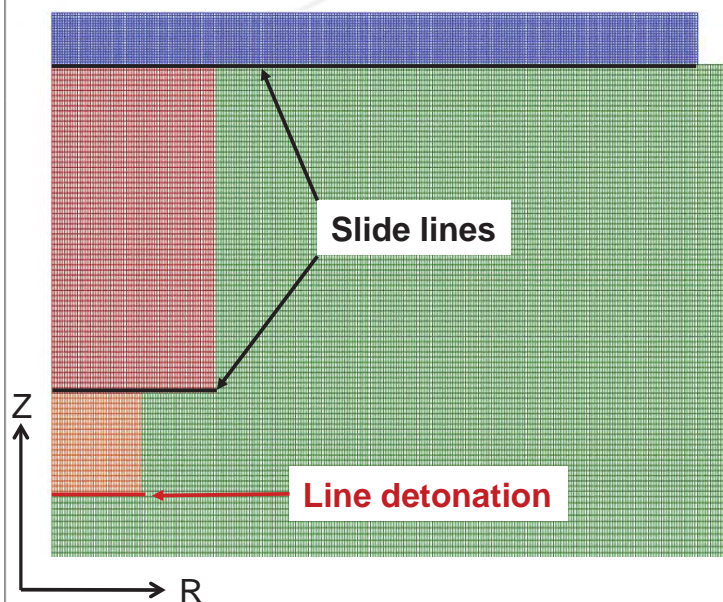


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Slide 20

# Initial mesh



- Initial zone size in the coupon:  
 $\Delta z = \Delta r = 100, 50, 25 \mu\text{m}$
- Zone sizes in the HE & sheath blocks are impedance matched in  $\Delta z$  via  $\sqrt{\rho_1/\rho_2}$ .
- All boundaries are free except the z-axis, where radial forces and velocities are set to 0.
- There is no mesh above the coupon surface. Ejecta package tracks the particles through vacuum.

- All mesh hydro is frozen after ejecta are launched to sidestep mesh tangling issues.

Calculation and slide  
by I. Tregillis

Slide 21

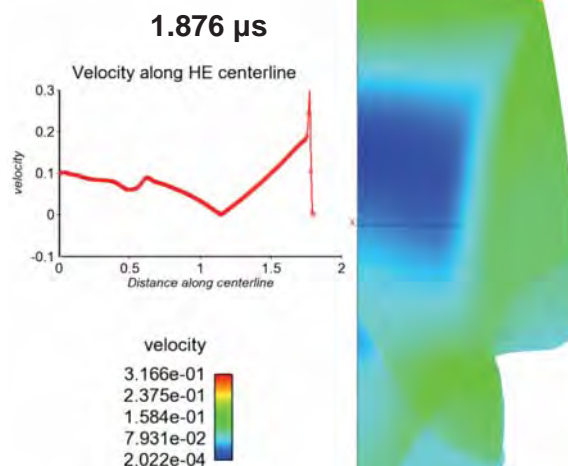
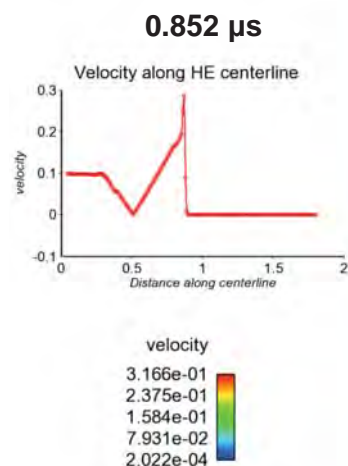


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## High explosive drive produces a “triangle wave” velocity front

Calculation and slide  
by I. Tregillis



$\Delta z = \Delta r = 50 \mu\text{m}$  in coupon



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## Ejecta production rate (volume/area/time) is inferred from equality of spike and bubble volumes

$\chi_{s,b}$  = spike, bubble area fractions

$$\chi_s + \chi_b = 1$$

Spikes and bubbles must have equal growth rates:

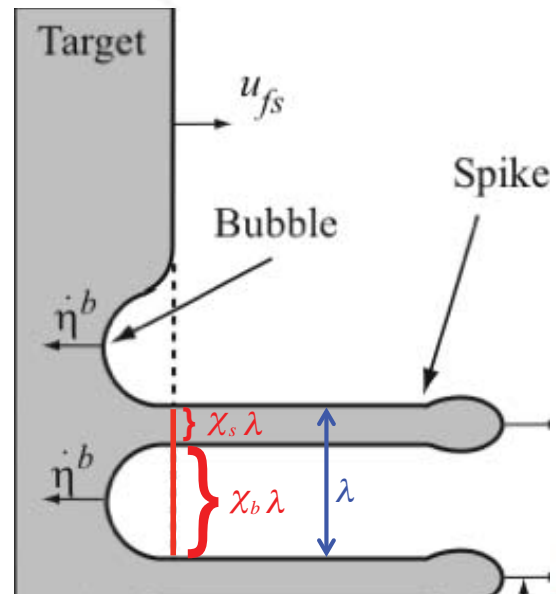
$$\dot{\Delta} = \chi_s |\dot{\eta}_s| = \chi_b \dot{\eta}_b$$

Eliminate  $\chi_{s,b}$  from equations:

$$\frac{1}{\dot{\Delta}} = \frac{1}{\dot{\eta}_b} + \frac{1}{|\dot{\eta}_s|}$$

Integrate  $\dot{\Delta}$  over one cycle from  $t_f$  to  $t_i$  (measured from shock breakout time):

$$\Delta = \frac{2}{3k} \ln \frac{t_f + t_0}{t_i + t_0} \quad t_0 = \frac{1}{\frac{F_{nl}^b}{\frac{3}{2}kaF_l u_{fs}} + \frac{\sqrt{3}F_{nl}^s}{\sqrt{3}F_{nl}^s}}$$



Slide 23



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## Solution of the damped driven harmonic oscillator

- Suppose the droplet breaks up when  $x = C_b r$ , and define  $y = \frac{x}{C_b r}$ . Then the solution of the ODE is

$$y(t) = C_0 We + e^{-t/t_d} \left\{ (y_0 - C_0 We) \cos \omega t + \frac{1}{\omega} \left[ \dot{y}_0 + \frac{1}{t_d} (y_0 - C_0 We) \right] \sin \omega t \right\}$$

in which

$$C_0 = \frac{C_F}{C_b C_k} \quad We = \frac{\rho_g u^2 r}{\sigma}$$

$$\frac{1}{t_d} = \frac{C_d}{2} \frac{\mu_l}{\rho_l r^2} \quad \omega^2 = C_k \frac{\sigma}{\rho_l r^3} - \frac{1}{t_d^2} \quad (\omega^2 < 0 \text{ only for very small drops})$$

- Default coefficient values are based on experiments and modeling hypotheses:

$$C_k = 8 \quad C_d = 5 \quad C_b = \frac{1}{2} \quad C_F = \frac{1}{3} \quad C_0 = \frac{1}{12} = \frac{1}{2 We_{crit}}$$

- At every cycle, for each particle, FLAG updates  $y$  and  $\dot{y}$  and checks for the breakup condition  $|y| = 1$ .

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# Finite-size particles in homogeneous-isotropic turbulence

## Clustering and preferential concentration

---

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Institute for Hydromechanics  
Karlsruhe Institute of Technology, Germany

October 2016

<sup>1</sup>currently on leave at CCMT, U. Florida

## Finite-size particles interacting with turbulence

(Nishino & Matsushita, ICMF 2004)



- mobility
- collective effects
- gravity

### Applications

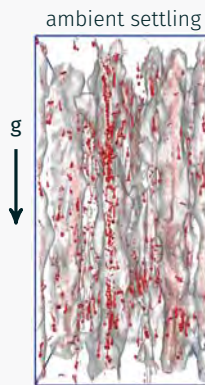
- meteorology, chemical engineering, ...

### Open questions

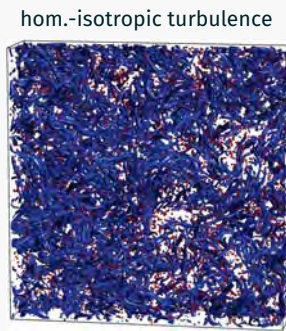
- settling velocity, cluster formation
- influence of/on turbulence



$$\rho_p / \rho_f = 1.5, \Phi_s = 0.005$$



MU & Doychev (JFM 2014)



- present work
- Homann & Bec (2010)
- Cisse et al. (2013)
- Ten Cate et al. (2004)
- Yeo et al. (2010)
- Fiabane et al. (2012)



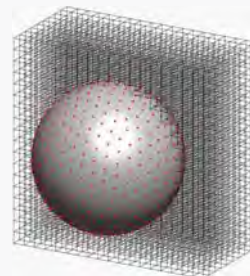
Chouippe & MU (ETC 2015)

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## Immersed boundary method

### Principle

- fixed Cartesian grid, uniform
- no-slip condition at interface imposed via body force
- smooth interpolation through regularized  $\delta$  function



MU, *J. Comput. Phys.* (2005)

### Rigid body motion

- translational & angular particle motion
- dilute  $\rightarrow$  simple repulsion model (Glowinski 1999)

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## Forcing statistically stationary turbulence

### Random forcing (Eswaran & Pope 1988)

- momentum source in low-wavenumber band
- Uhlenbeck-Ornstein processes per Fourier mode

### Present implementation

- avoids use of FFT
- validated against single-phase data
- stable procedure in presence of particles  
(Lundgren's 2003 linear forcing is not)

→ allows long-time integration

Chouippe & MU (PoF 2015)

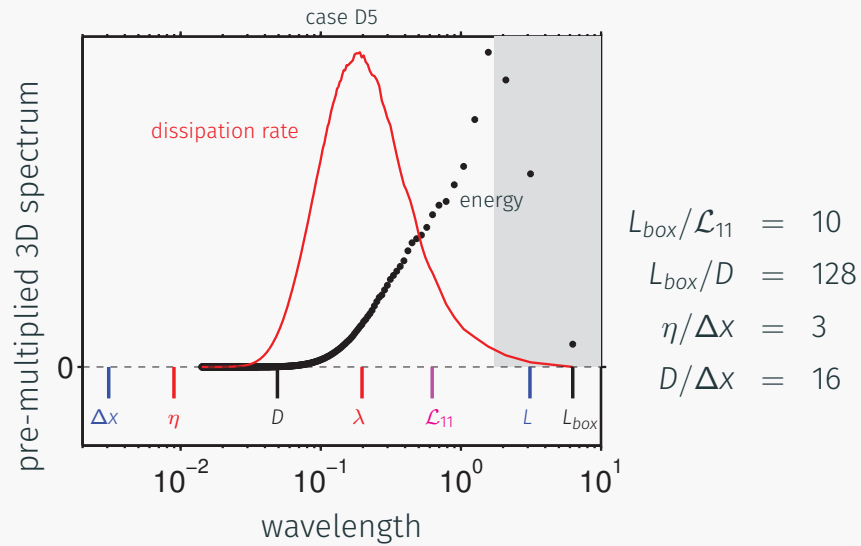
5/18

## Two-phase flow parameter values

		cases	
		D5	D11
diameter/Kolmogorov-length ratio	$D/\eta$	5.5	11.0
Reynolds number	$Re_\lambda$	125	140
density ratio	$\rho_p/\rho_f$	1.5	
solid volume fraction	$\Phi_s$	0.005	
Stokes number (Kolmogorov)	$St_\eta$	2.4	10.7
Stokes number (large-eddy)	$St_L$	0.05	0.29
number of particles	$N_p$	20026	2540

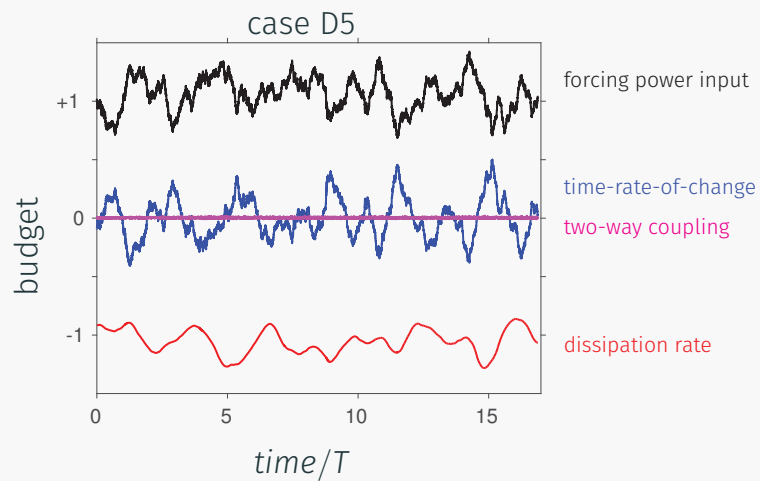
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## Energy spectrum and length scales



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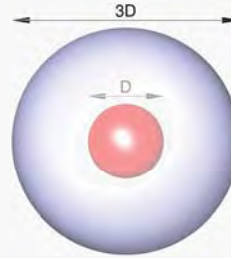
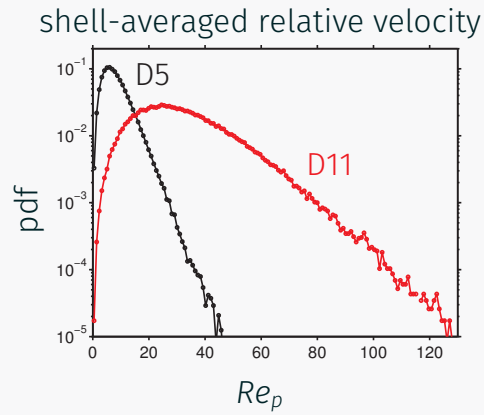
## Kinetic energy budget



$$\frac{d}{dt}\langle E_k \rangle_{\Omega} = \langle \mathbf{u} \cdot \mathbf{f}^{(t)} \rangle_{\Omega} + \langle \mathbf{u} \cdot \mathbf{f}^{(ibm)} \rangle_{\Omega} - \varepsilon_{\Omega}$$

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## Fluid-particle relative velocity



$$\mathbf{u}_f^S = \frac{1}{A_S} \int_S \mathbf{u}_f(\mathbf{x}, t) dA$$

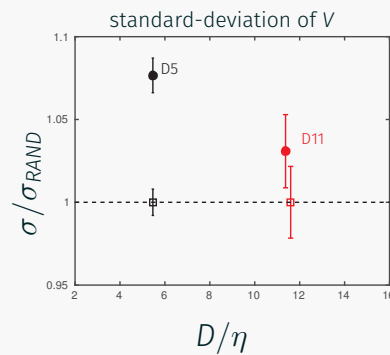
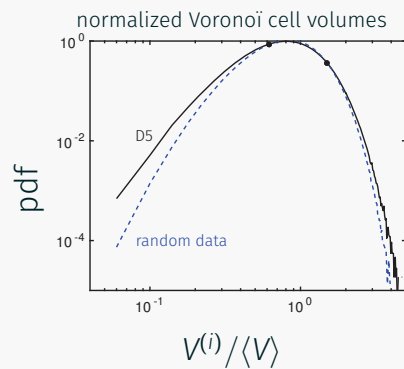
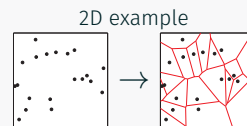
$$u_{rel} = |\mathbf{u}_p - \mathbf{u}_f^S|$$

$$Re_p = u_{rel} D / \nu$$

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## Spatial particle arrangement – clustering

### Voronoi tessellation analysis

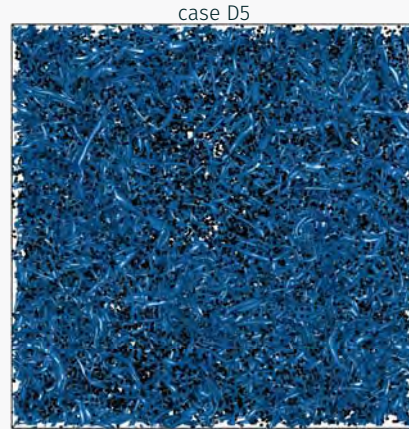


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## Relation between particles and coherent vortices

### Coherent vortex extraction

- q-criterion  
(Hunt et al. 1989)
- iso-surfaces:  
1.5·std-dev

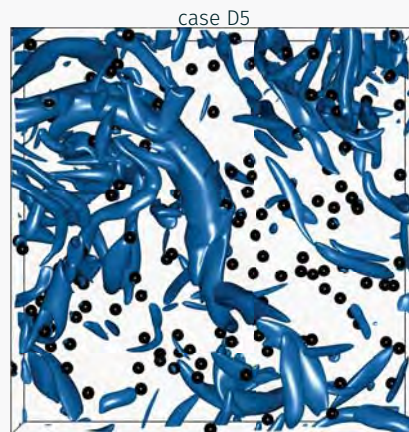


11/18

## Relation between particles and coherent vortices

### Coherent vortex extraction

- q-criterion  
(Hunt et al. 1989)
- iso-surfaces:  
1.5·std-dev



(zoom)

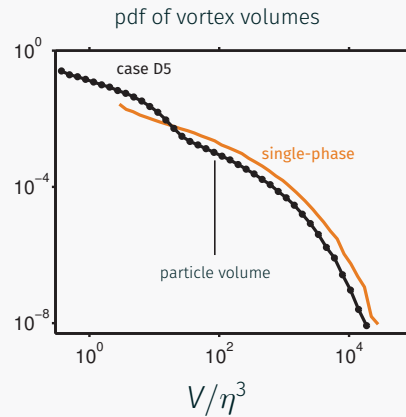
11/18



## Relation between particles and coherent vortices

### Coherent vortex extraction

- q-criterion  
(Hunt et al. 1989)
- iso-surfaces:  
1.5-std-dev
- volume statistics  
(Moisy, Jiménez 2004)

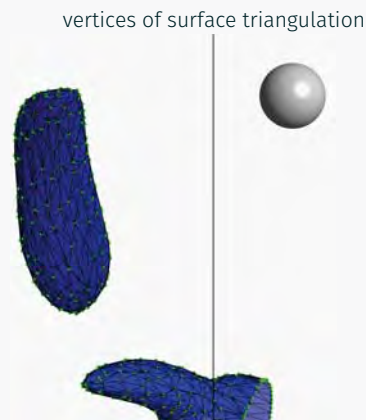


11/18

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### Coherent vortex extraction

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(Hunt et al. 1989)
- iso-surfaces:  
1.5-std-dev
- volume statistics  
(Moisy, Jiménez 2004)
- preferential locations

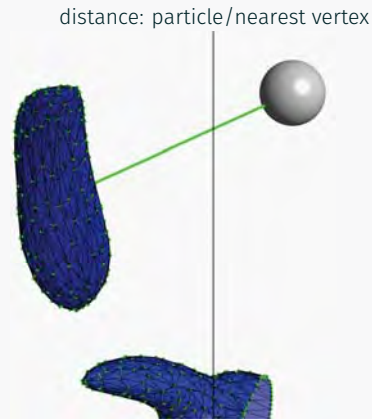


11/18

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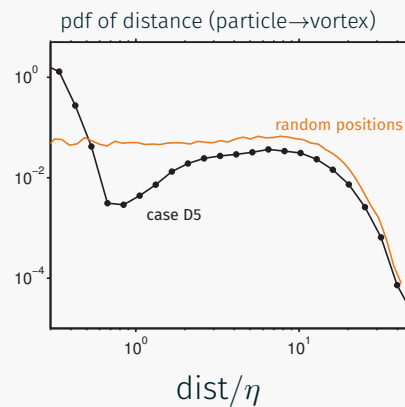


11/18

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### Coherent vortex extraction

- q-criterion  
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- preferential locations

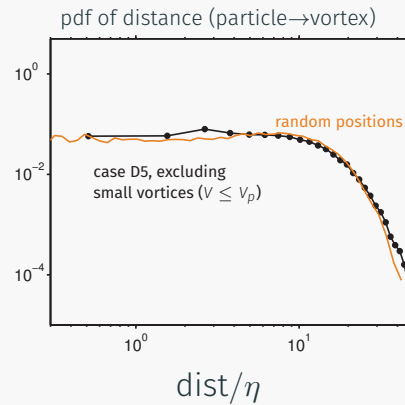


11/18

## Relation between particles and coherent vortices

### Coherent vortex extraction

- q-criterion  
(Hunt et al. 1989)
- iso-surfaces:  
1.5·std-dev
- volume statistics  
(Moisy, Jiménez 2004)
- preferential locations  
excluding small  
vortices ( $V \leq V_p$ )



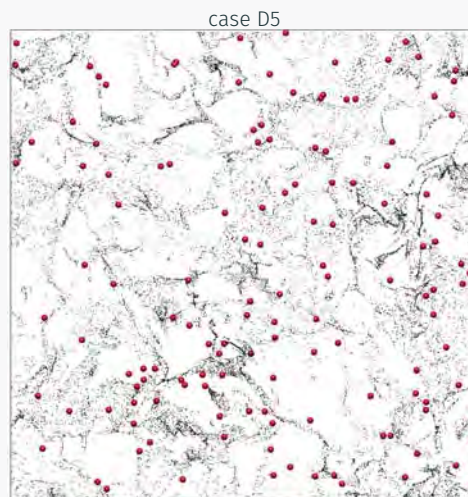
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## The role of the fluid acceleration field

### Goto, Vassilicos (2008)

- point-particles
  - 'sticky' points:  
 $\mathbf{e}_1 \cdot \mathbf{a}_f = 0$  and  
 $\lambda_1 > 0$
- particles converge

Finite-size particles?



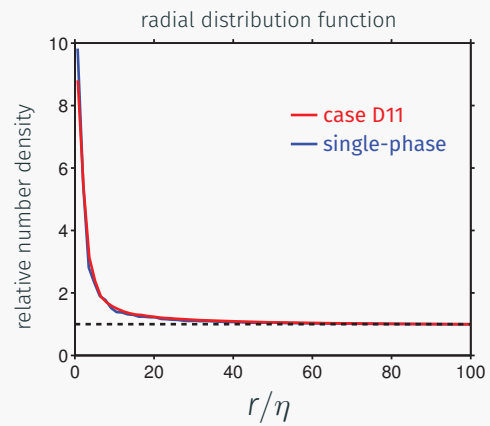
(slice:  $380\eta \times 380\eta \times 15\eta$ )

$\lambda_1$  largest eigenvalue of  $\nabla \mathbf{a}_f + (\nabla \mathbf{a}_f)^T$ ; eigenvector  $\mathbf{e}_1$

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## Spatial distribution of 'sticky points'

- 'sticky points' exhibit strong clustering

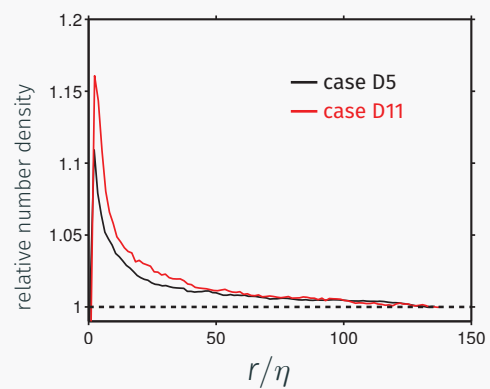


→ no apparent modification when adding particles

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## Particle-conditioned distribution of 'sticky points'

- probability of finding a 'sticky point' at distance  $r$  from particle surface

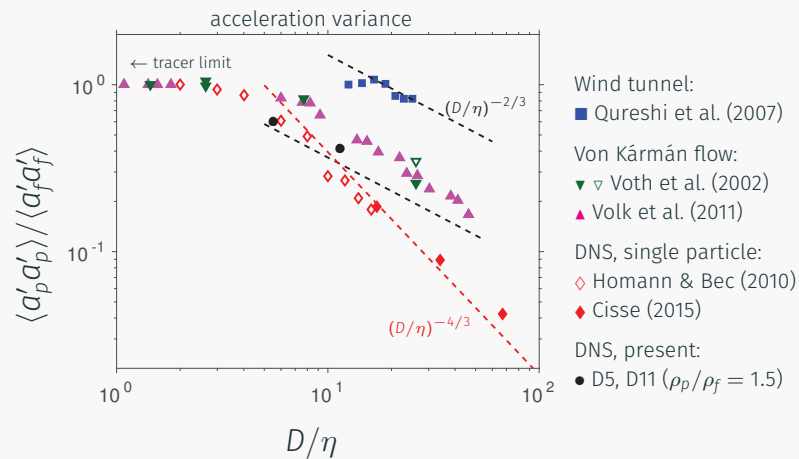


→ increased probability near particles

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## Particle acceleration statistics

- normalized pdf matches fit of Qureshi et al. (2007)



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## Conclusions so far

### Moderately dense particles in hom-iso turbulence

- acceleration variance: similar trend as density matched
- 'clustering' detected, intensity decreases with  $D$
- no significant preferential concentration w.r.t. vortex
- significant spatial correlation with 'sticky points' (Goto & Vassilicos 2008)

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## Challenges

### Relevance of numerical simulations:

- large-scale DNS can provide high-fidelity data
- the cost: €0.03/core-hour
- next generation “heroic” simulation: dilute,  $10^6$  parts.

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## Challenges

### Relevance of numerical simulations:

- large-scale DNS can provide high-fidelity data
- the cost: €0.03/core-hour
- next generation “heroic” simulation: dilute,  $10^6$  parts.

### Current bottleneck: “après-simulation”

- data transfer & long time storage
- visualization & exploring the data
- making data available to the community

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## More challenges

### Going from data → physics

- understanding: requires widening the parameter range
- scaling laws

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## More challenges

### Going from data → physics

- understanding: requires widening the parameter range
- scaling laws

### Going from physics → models

- teaching point-particle models about collective effects

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## Acknowledgements



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# Long time history of unsteady drag on shock accelerated micro-particles



Ankur Deep Bordoloi  
Adam Martinez  
Kathy Prestridge



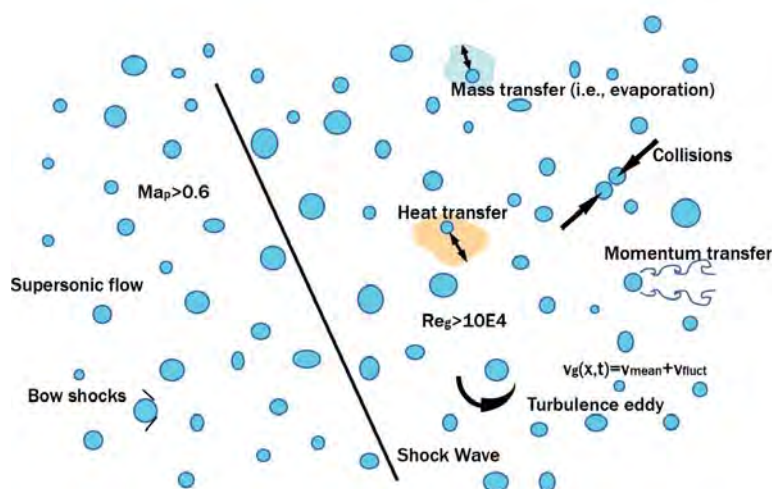
Operated by Los Alamos National Security, LLC for the U.S. Department of Energy's NNSA

extremefluids.lanl.gov

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## Multiphase flows in high-speed are extremely complex



- ☐ Mass exchange
- ☐ Momentum transfer
- ☐ Evaporation
- ☐ Collision
- ☐ Deformation & breakup
- ☐ Turbulence modulation

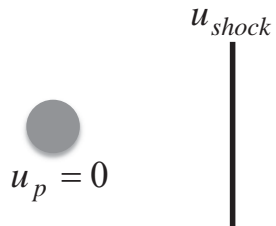
Experimental envelop is designed to simplify the problem

- Drag measurement
- Solid spherical particles (no deformation/breakup)
- Low volume fraction

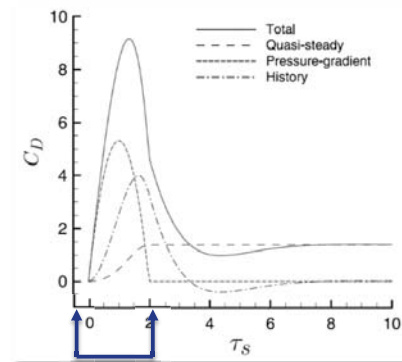


## Previous models describe unsteady forces on a stationary particle only over a short time scale

$$F(t) = \underbrace{F_{qs}(t)}_{\text{quasi-steady}} + \underbrace{F_{iu}(t)}_{\text{inviscid unsteady (added mass) (pressure gradient)}} + \underbrace{F_{vu}(t)}_{\text{viscous unsteady (Basset history)}} + \underbrace{F_l(t)}_{\text{lift}} + \underbrace{F_{bg}(t)}_{\text{buoyancy/gravity}}$$



$$O(d_p/u_{shock})$$



(Parmar & Balachandar, 2008)

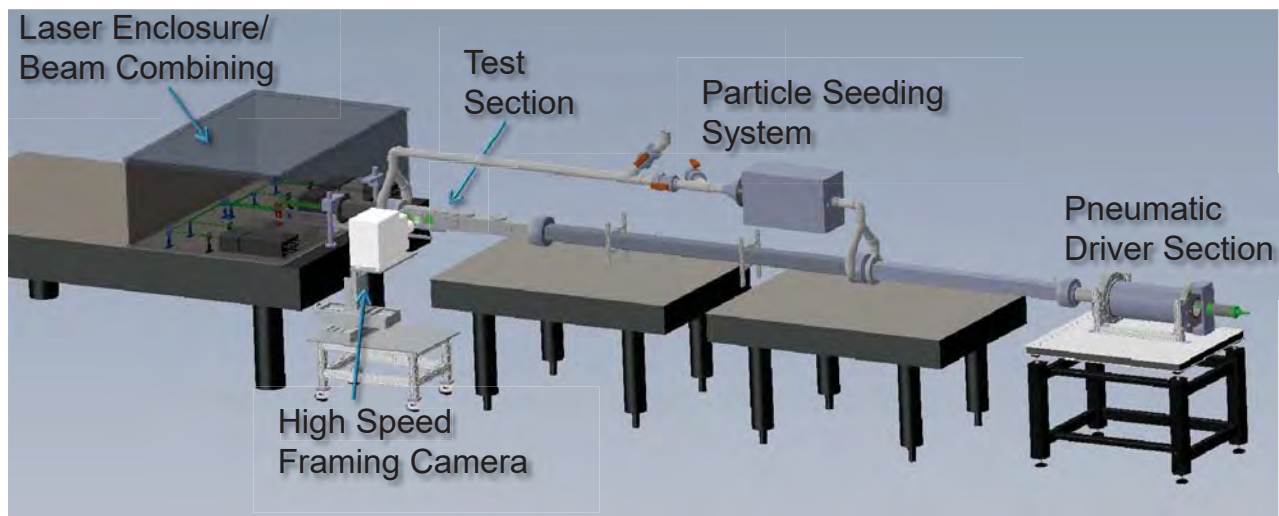
### Objectives

- Estimate unsteady drag on shock accelerated particle
- Drag history over longer time period,  $1000 \times O(d_p/U_{shock})$



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## The horizontal shock-tube facility is capable of repeatable multi-frame particle & shock tracking



### Camera:

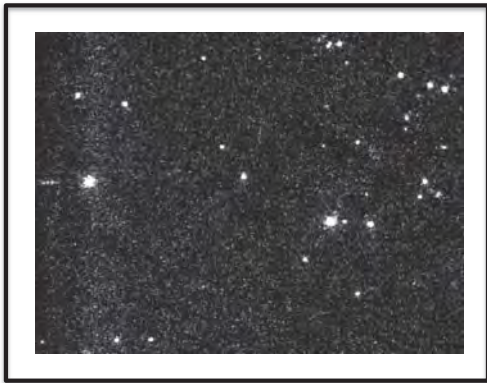
5 ns min exposure duration  
7 million fps over 8 frames

8 independent frames and 532 nm laser pulses

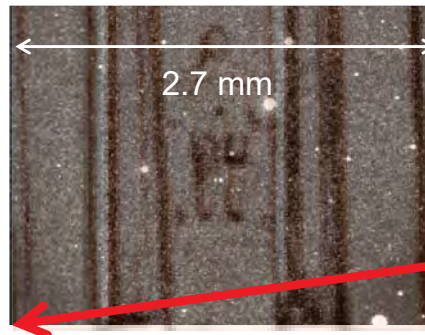


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# High resolution imaging allows precise tracking of particle

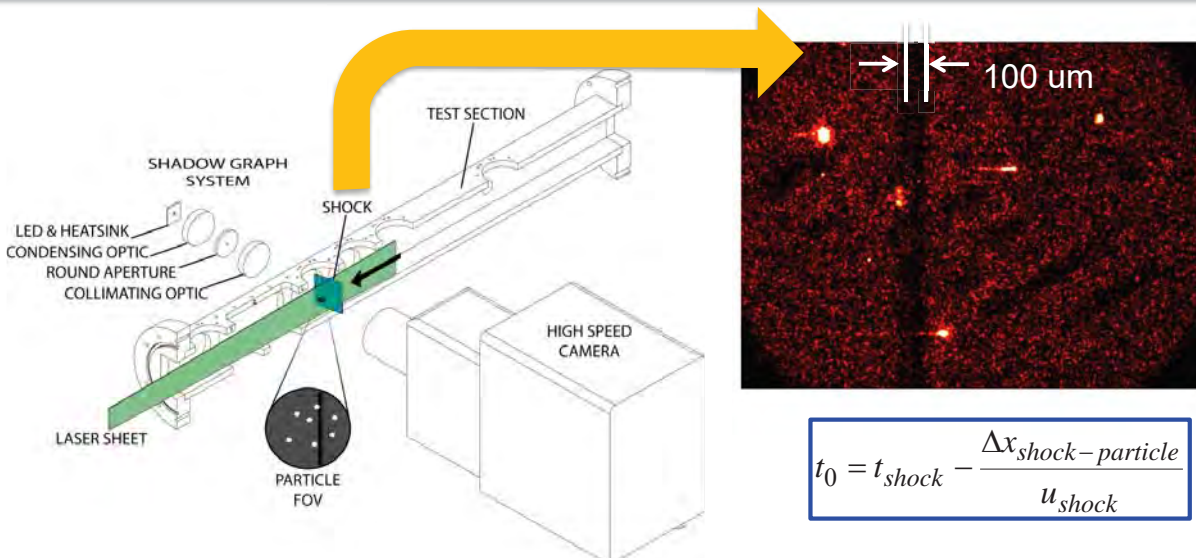


Spatial resolution: 2.14  $\mu\text{m}/\text{pixel}$



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## A shadowgraph system lets us measure the shock location more precisely than with pressure transducers

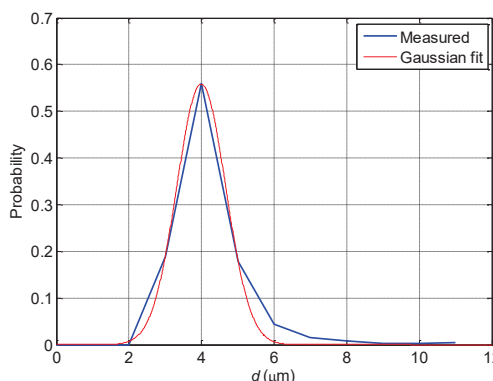


- Simultaneous imaging of shock and particle
- Shock detection on multiple frames
- Better estimates of  $t = 0$



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## Heavy, microscopic particles show polydispersed size distribution



**Manufacturer:**

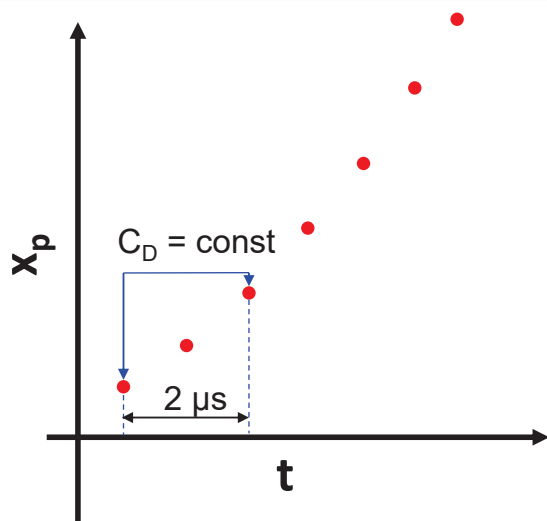
mean: 4  $\mu\text{m}$   
std dev: 1.5  $\mu\text{m}$

**Measured:**

mean: 4  $\mu\text{m}$   
std dev: 1  $\mu\text{m}$

Incident Mach number:	$M = 1.2, 1.3, 1.4, 1.5$
Particle Mach number:	$M_{p,0} = 0.1, 0.3, 0.5, 0.7$
Particle Reynolds number:	$Re_{p,0} = 25, 35, 55, 72$
Gas phase velocity:	$u_{f2} = 109\text{-}246 \text{ m/s}$
Particle Volume Fraction:	$C \approx 1\text{e-}7$

**Drag is estimated based on piecewise curve fitting of experimental data on force equation**



### 5<sup>th</sup> order Taylor Expansion

$$x_p = u_2(t - t_0) - \frac{\log(Au_2(t - t_0) + 1)}{A} + x_0 + TSE$$

$$\frac{du_p}{dt} = \frac{3\rho_2(u_{f2} - u_p)^2}{4\rho_p d_p} C_D$$

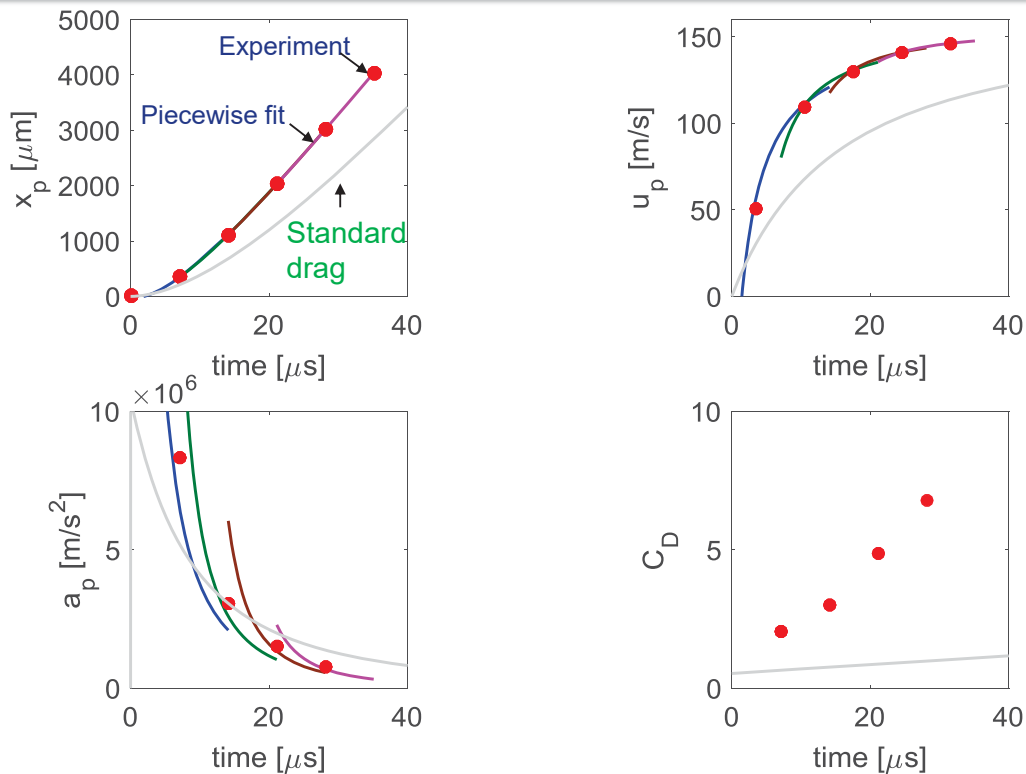
$$u_p = \frac{u_f^2 At}{1 + u_f At}; \quad A = \frac{3\rho_2 C_D}{4\rho_p d_p}$$

$$x_p = u_2 t - \frac{\log(Au_2 t + 1)}{A} + x_0$$

3 fitting parameters:  $x_0$ ,  $t_0$ ,  $A$

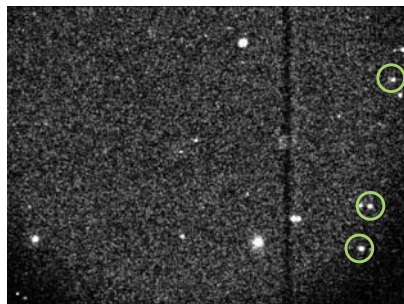
$$C_D = \frac{4\rho_p d_p A}{3\rho_g}$$

# Example showing piecewise fit on particle location, and estimated velocity, acceleration and unsteady drag



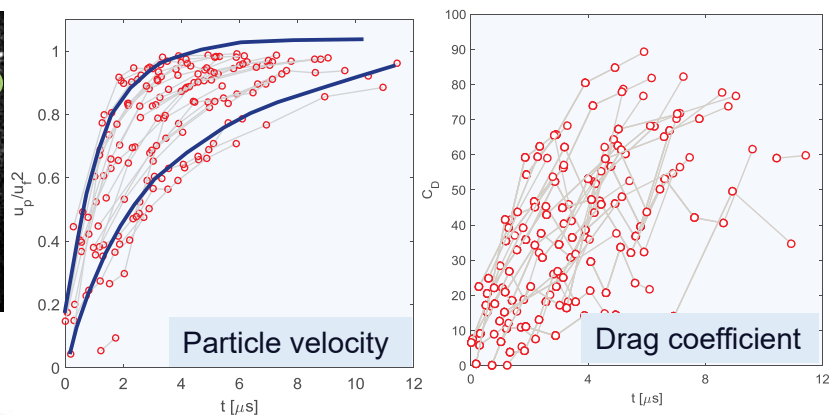
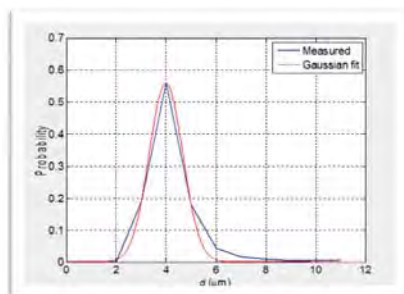
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## Due to diameter distribution particles accelerate at different rates



$\times 0(50)$

Diameter distribution



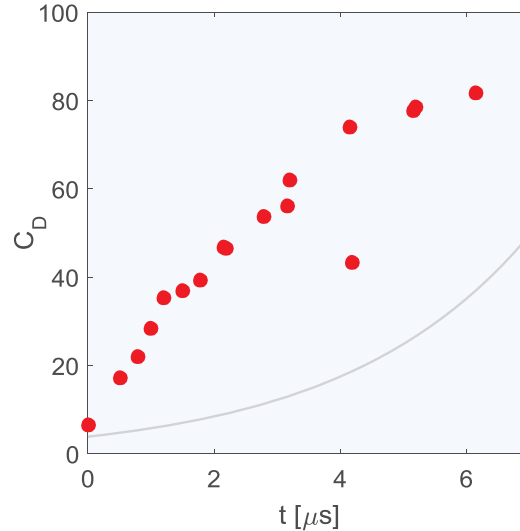
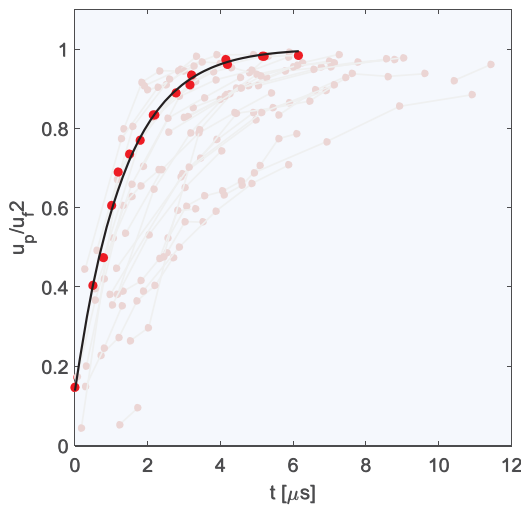
- Drag increases dramatically at later post-shock times
- The increase rate of  $C_D$  is sensitive to particle size



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# Particle velocity ( $u_p$ ) asymptotically catches up with the post-shock fluid velocity ( $u_{f2}$ )



- The maximum drag is order of magnitude higher than earlier predictions



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## Why is measured drag higher than earlier predictions?

### Quasisteady drag force

(Parmar et al. PRL, 2011)

$$\text{Re} = \frac{Vd_p}{\nu} = 53 \quad \text{Kn}' = \frac{M}{\text{Re}} = 0.02$$

$$C_{D,qs} = \frac{24}{\text{Re}}(1 + 0.15\text{Re}^{0.687}) + 0.42\left(1 + \frac{42500}{\text{Re}^{1.16}}\right)^{-1}$$

Continuum is valid; no compressibility effect on quasisteady force

### Unsteady drag force

$$\text{Acoustic time scale, } \tau = \frac{c_0 t}{d_p}$$

$$\text{Convective time scale, } \tau_c = \frac{Vt}{d_p}$$

- ❑  $F_{iu}(t)$  acts on a very short acoustic time scale (validated for static particle, Parmar 2009)
- ❑ Basset history kernel in  $F_{vu}(t)$  is valid only for  $\tau_c \ll 1/\text{Re}$  (Mei and Adrian, 2009)

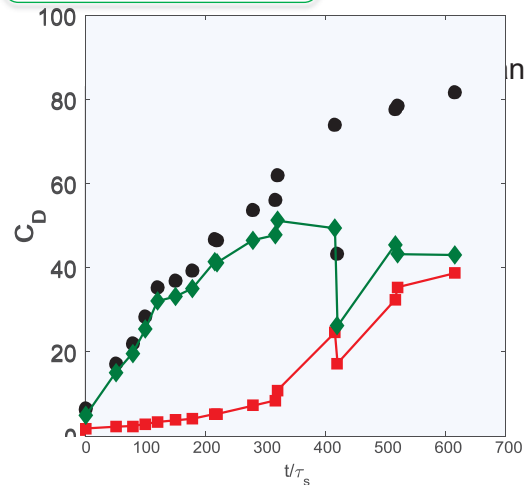
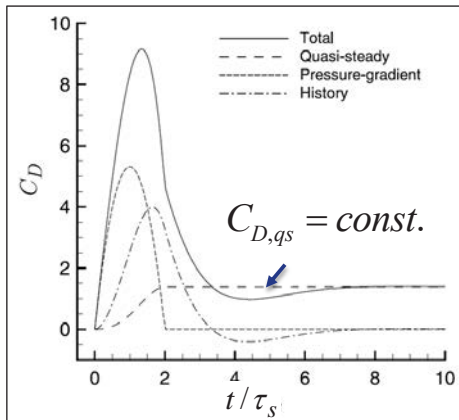


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# Earlier model captures drag at earlier times for stationary particle

$$F(t) = F_{qs}(t) + F_{iu}(t) + F_{vu}(t)$$



andar, 2008)

$$\tau_s = d_p / u_{shock}$$

$$\tau_s = 0.01 \mu s$$

$$C_{D,qs} = \frac{24}{Re} (1 + 0.15 Re^{0.687}) + 0.42 \left( 1 + \frac{42500}{Re^{1.16}} \right)^{-1}$$



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# Earlier model captures drag at earlier times for stationary particle

$$F(t) = F_{qs}(t) + F_{iu}(t) + F_{vu}(t)$$

- ❑ Consistent with Parmar model at acoustic time scale,  $\tau_s \ll 10$  (2008)
- ❑ We expect late time CD to be contributed by  $F_{qs}(t)$
- ❑ Large contribution from unsteady forces in intermediate time
- ❑  $F_{vu}(t)$  is not modeled by Basset history forces



$$\tau_s = d_p / u_{shock}$$

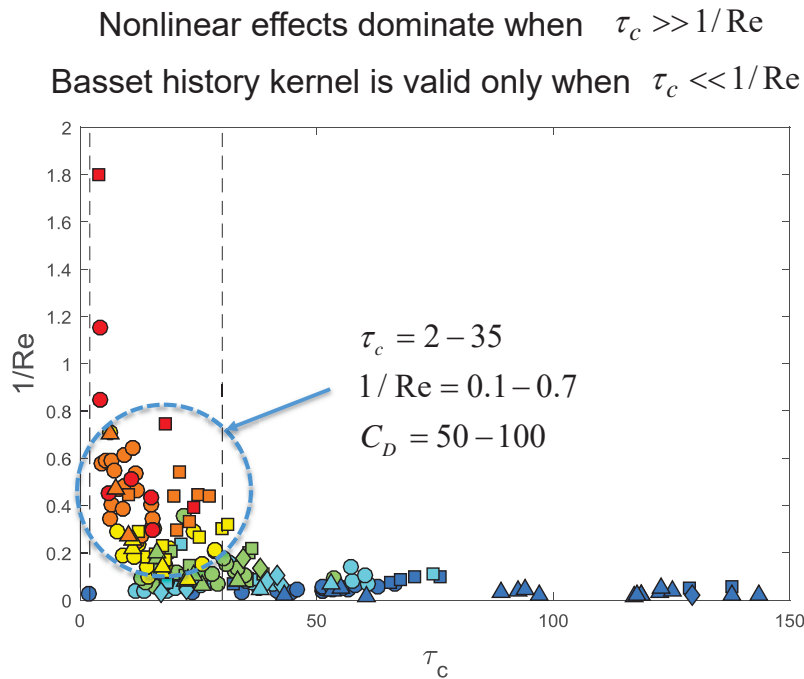
$$\tau_s = 0.01 \mu s$$

$$C_{D,qs} = \frac{24}{Re} (1 + 0.15 Re^{0.687}) + 0.42 \left( 1 + \frac{42500}{Re^{1.16}} \right)^{-1}$$



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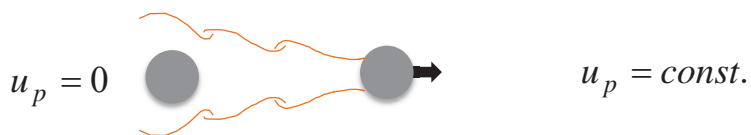
# Non-linear effects should dominate in the high drag regime



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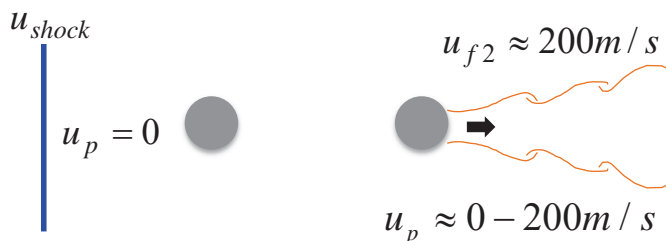
## Drag acts differently on shock accelerated particles

### Standard particle drag



$C_D$  decelerates particle

### Shock accelerated particle drag

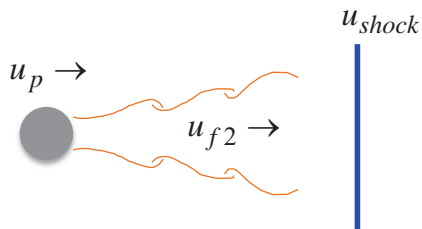


$C_D$  accelerates particle



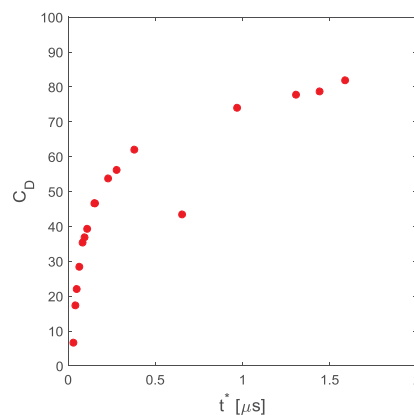
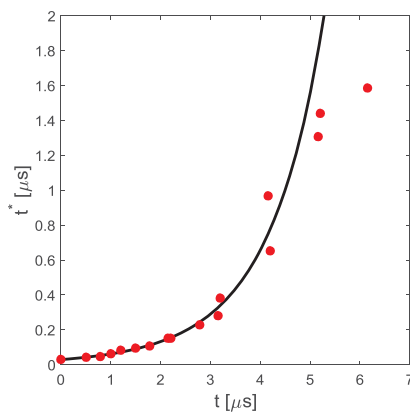
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# A time scale $t^*$ is chosen based on particle slip velocity



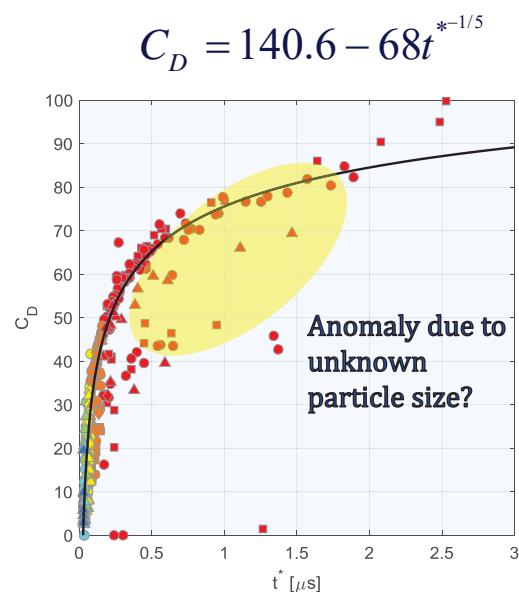
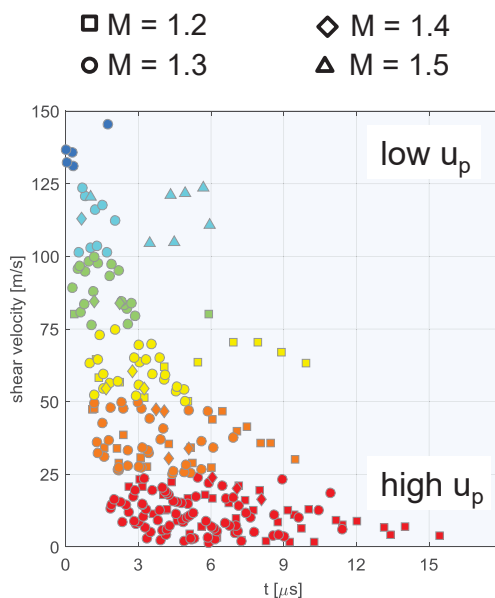
$$t^* = \frac{d_{p,mean}}{|u_{f2} - u_p|}$$

$$t^* \rightarrow \infty \Rightarrow u_p \rightarrow u_{f2}$$



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## $C_D$ increases with particle velocity following a power law relation with $t^*$



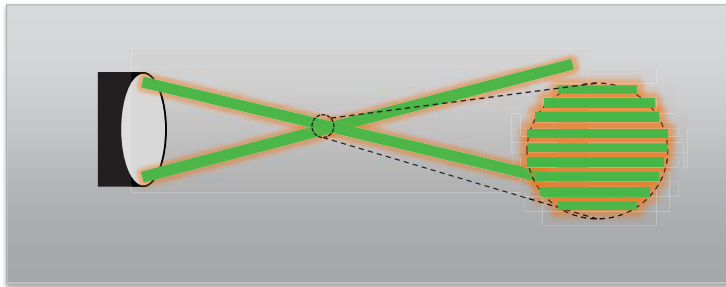
- Shows a power-law fit to  $C_D$  when  $t^*$  is used
- $C_D$  vs  $t^*$  is less sensitive to incident Mach number



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# Diagnostic improvement

## Dynamic particle sizing using PDPA technique



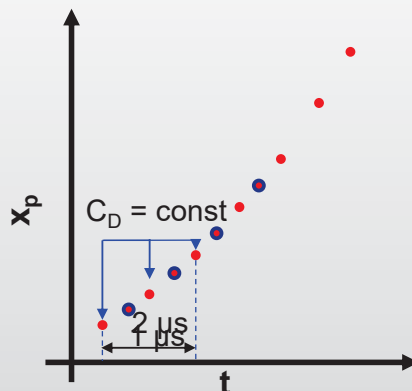
PDPA is capable of simultaneously measuring particle size and velocity



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## Work in progress..

### Improve $C_D$ measurement



Smaller inter-frame  $\Delta t$  to obtain a better piecewise fit

- Predictor/corrector: Predict a relationship between  $C_D$  and  $t^*$  and correct the piecewise fit
- Noise reduction: Apply Kalman filter



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## Open for discussion

- Drag is order of magnitude larger in the long time range
- Previous models and simulations are limited to short time
- The nonlinearity of flow contributes to large drag
- Drag follows a power law relationship with slip velocity time scale
- Drag is highly sensitive to particle size
- Mach number effects are small on  $C_D$





PSAAP II  
Particle-laden Turbulence in a Radiation Environment

# Including Real Experimental Effects in Validation of Numerical Models for Confined Particle-Laden Flows

Laura Villafañe, Andrew Banko, Mahdi Esmaily,  
Chris Elkins, Ali Mani, John Eaton

Multiphase Physics Deep-Dive  
October 6-7, 2016



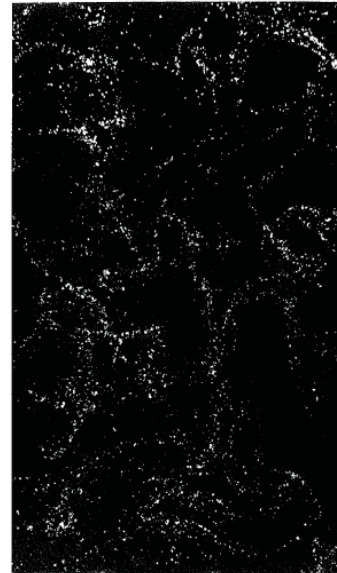
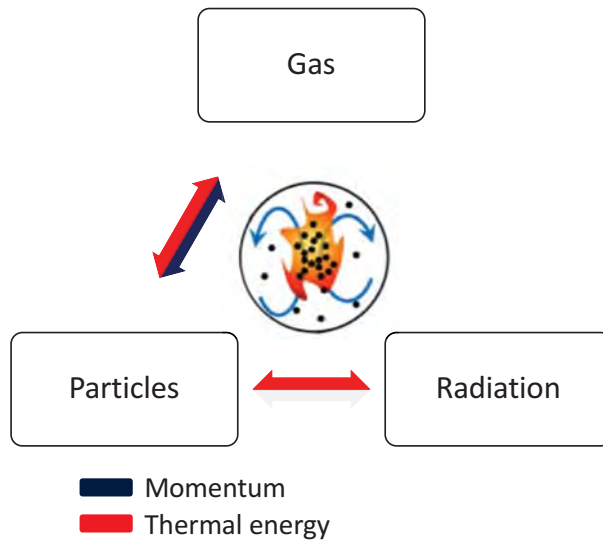
## Outline

- Concept
- Experimental/simulation overview
- Ideal vs. real
- Influence of real experimental effects for data comparison
  - ✓ Particle concentration distributions
  - ✓ 2D preferential concentration statistics
  - ✓ Particle velocities

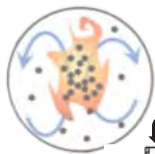




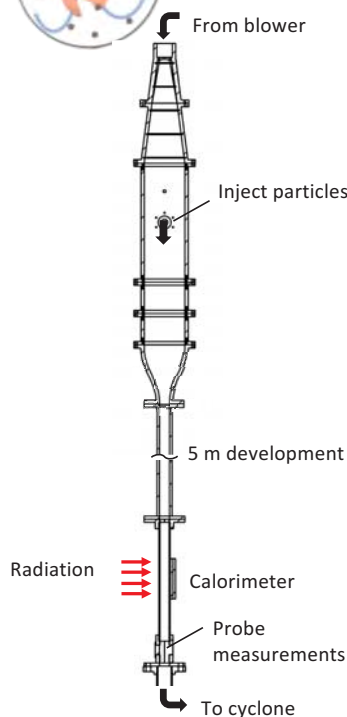
# Turbulence-Particle-Radiation Interaction



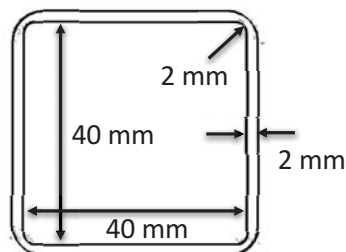
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## Channel Flow Experiment



- Fully developed, vertical, turbulent channel flow of air
- Laden with small Nickel particles
- Optical and probe-based measurements



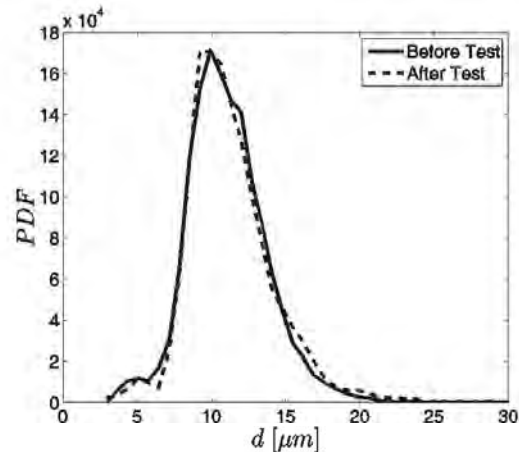
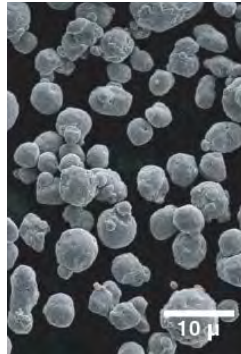
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## Parameters

- Nickel particles:



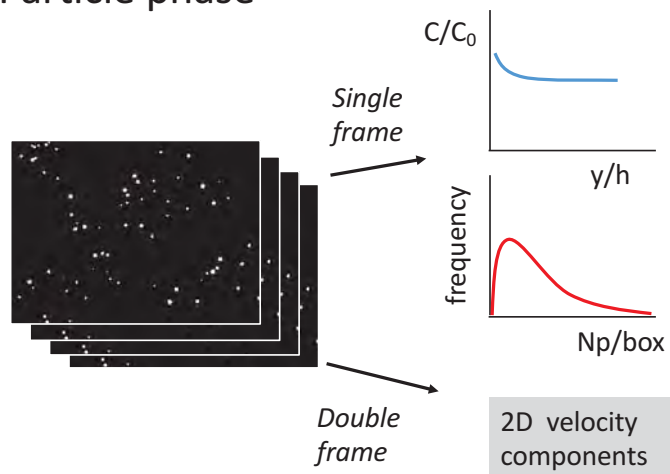
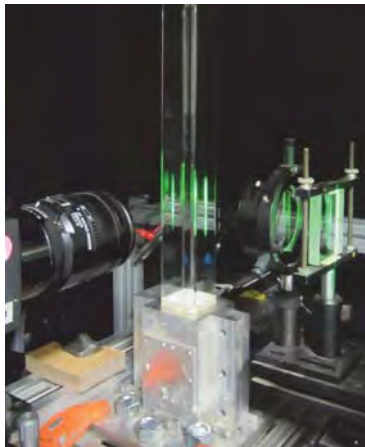
- Flow conditions:

$Re_H$	$St_\eta$	$d_p/\eta$	$MLR$	$\phi_V$
10,000	5	0.11	0.01-0.1	$10^{-6}$ - $10^{-5}$
20,000	12	0.17	0.01-0.1	$10^{-6}$ - $10^{-5}$
40,000	31	0.27	0.01-0.1	$10^{-6}$ - $10^{-5}$



## Experimental Techniques

### Particle phase



Laser Thickness (mm)	0.5-1.6
FOV (mm)	$\leq 35$



# Overview of the DNS

Governing equations:

$$\frac{\partial}{\partial t} \rho + \nabla \cdot (\rho \mathbf{u}) = 0$$

$$\frac{\partial}{\partial t} (\rho \mathbf{u}) + \nabla \cdot (\rho \mathbf{u} \otimes \mathbf{u}) = \nabla \cdot \mathbf{T} + \rho \mathbf{g}$$

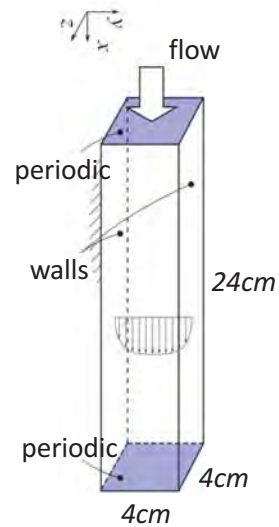
$$\mathbf{T} = -p\mathbf{I} + \mu \left( \nabla \mathbf{u} + (\nabla \mathbf{u})^T - \frac{2}{3} \nabla \cdot \mathbf{u} \mathbf{I} \right)$$

$$\dot{\mathbf{x}}_p = \mathbf{u}_p$$

$$m_p \dot{\mathbf{u}}_p = \mathbf{g} - \mathbf{f}_p$$

$$\mathbf{f}_p = m_p \tau_p^{-1} (\mathbf{u}_p - \mathbf{u})$$

- Point particle
- Hard collision model, k=1 (p-p, p-wall)



$Re = 10000, 20000$   
 $U_{bulk} = 3.93 \text{ m/s}, 7.86 \text{ m/s}$   
 $D_p \in [4, 16] \mu\text{m}$

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## Ideal vs. Real

### Ideal

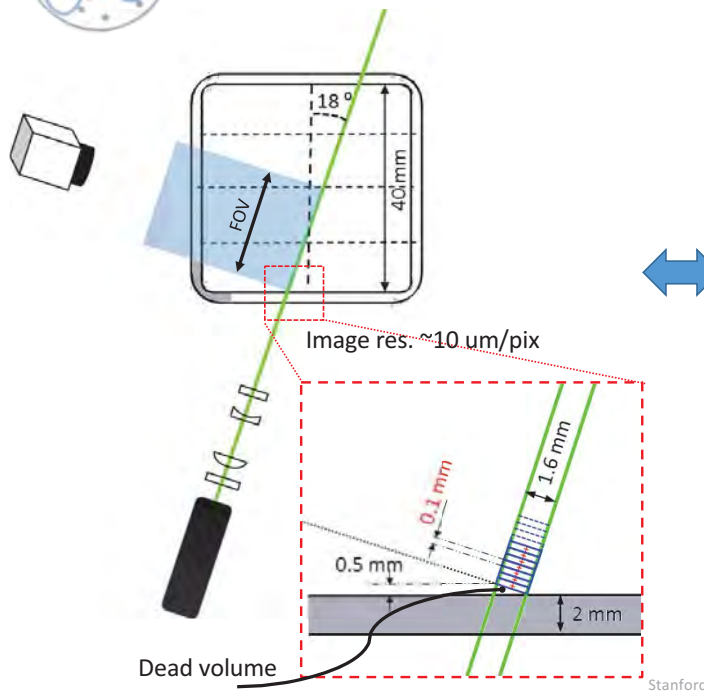
Planar laser illumination, point measurements	Finite thickness Gaussian profile laser, volume average, particle overlap
Collimated scattered light from 2D plane	Diffraction and aberrations, no particle size information
Square channel, near wall optical access	Duct with real corners, reflections
Infinite number of sampling particles	Finite number of particles

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# Particle Concentration Distribution

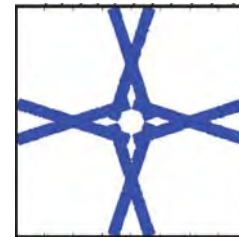


Numerical data processing

-Traditional:



-Experimental like:

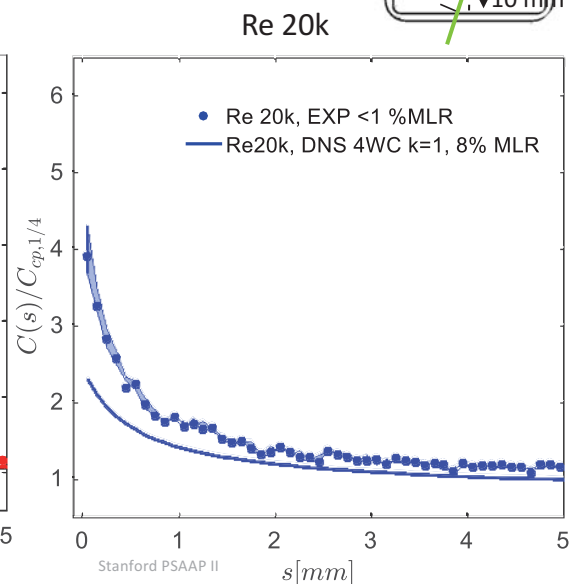
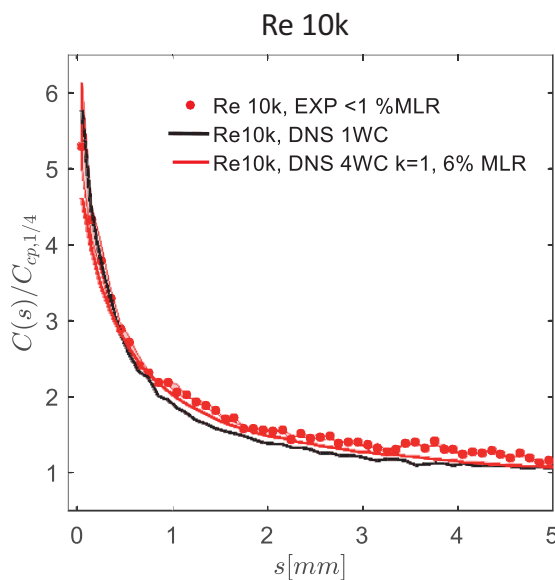
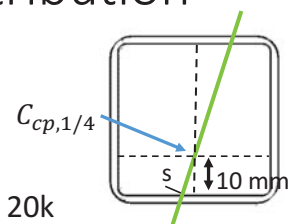


Re	10k	Re 20k
$y^+$ resol.~	8-10	16-20



# Particle Concentration Distribution

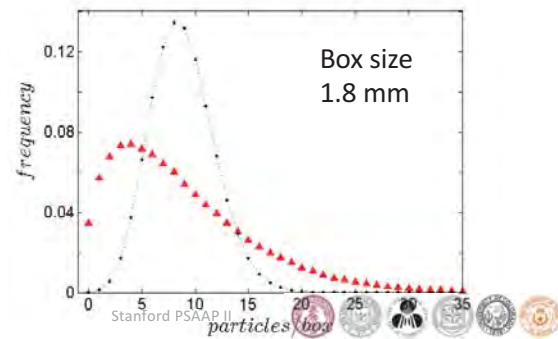
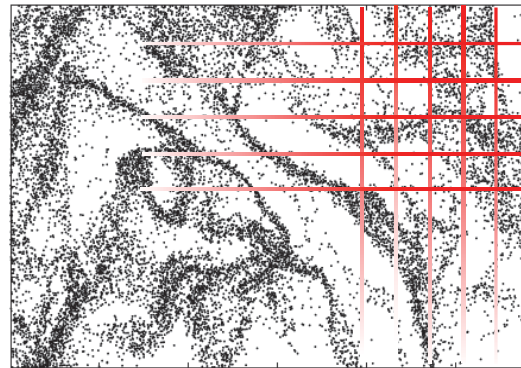
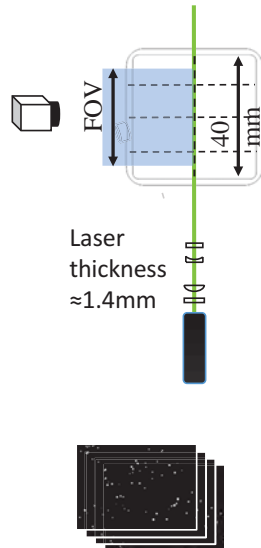
Experiments vs DNS



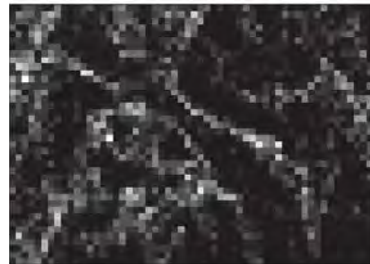
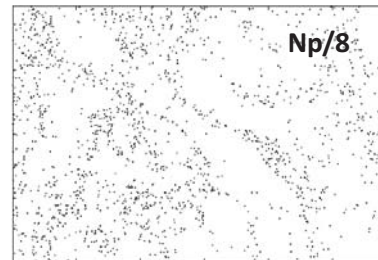
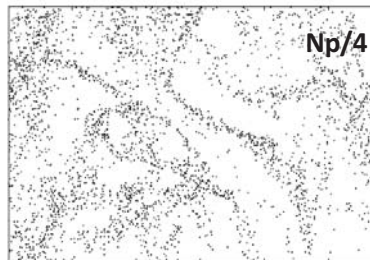
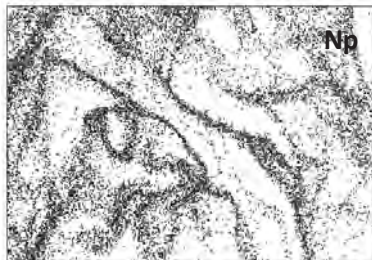




# Preferential Concentration: Box Counting



$$\left(\frac{\sigma}{\lambda}\right)_c = \sqrt{\frac{\sigma_s^2 - \lambda_s}{\lambda_s^2}} \xrightarrow{\lambda_s \rightarrow \infty} \frac{\sigma_s}{\lambda_s}$$



	Np	Np/2	Np/4	Np/8
$\sigma_s/\lambda_s$	0.9	1.0	1.2	1.4
$(\sigma/\lambda)_c$	0.8	0.8	0.8	0.8



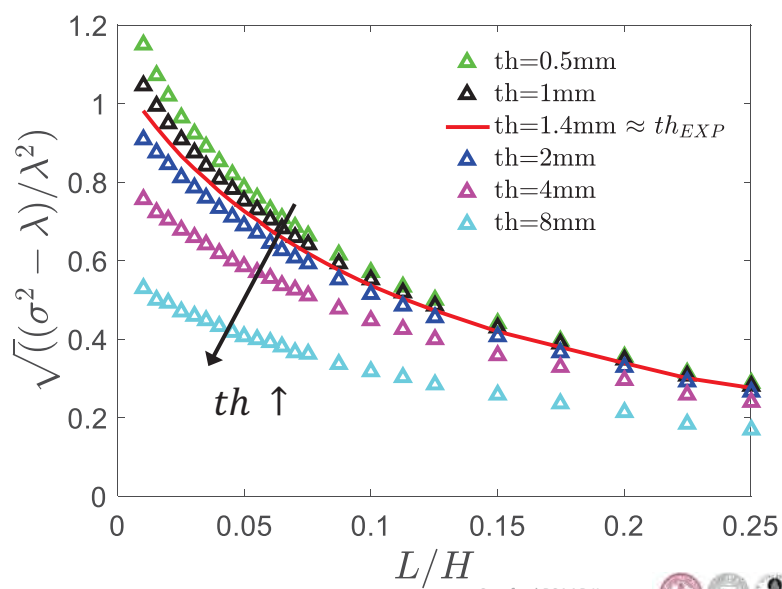
## Preferential Concentration:

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## Preferential Concentration: *Laser thickness*

DNS , Re= 10k, different thickness



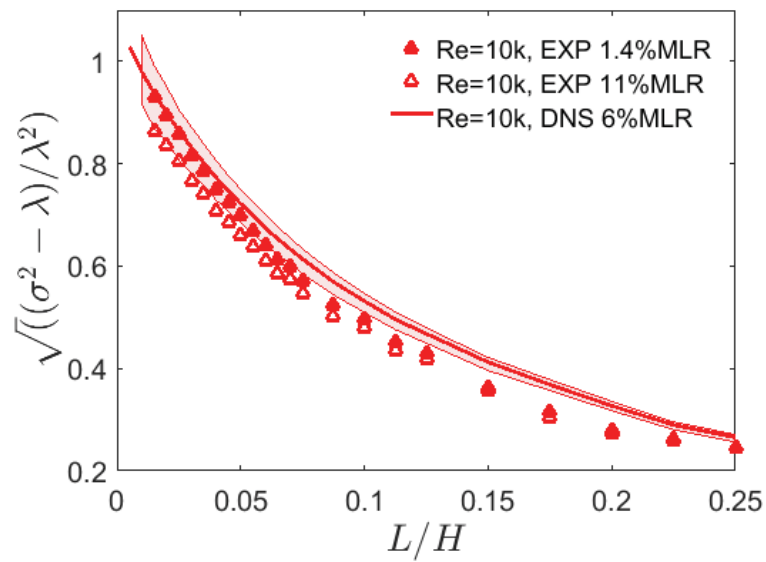
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# Preferential Concentration

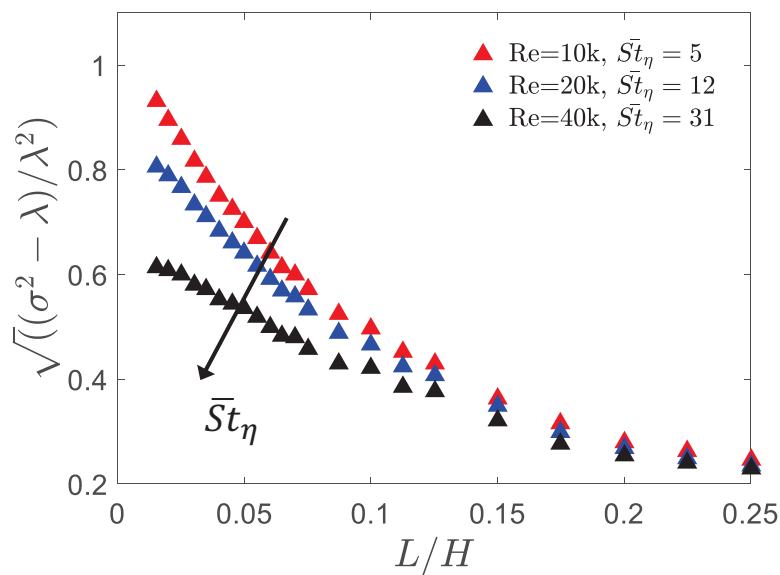
## EXP-DNS Comparison



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## Preferential Concentration: *Stokes number*

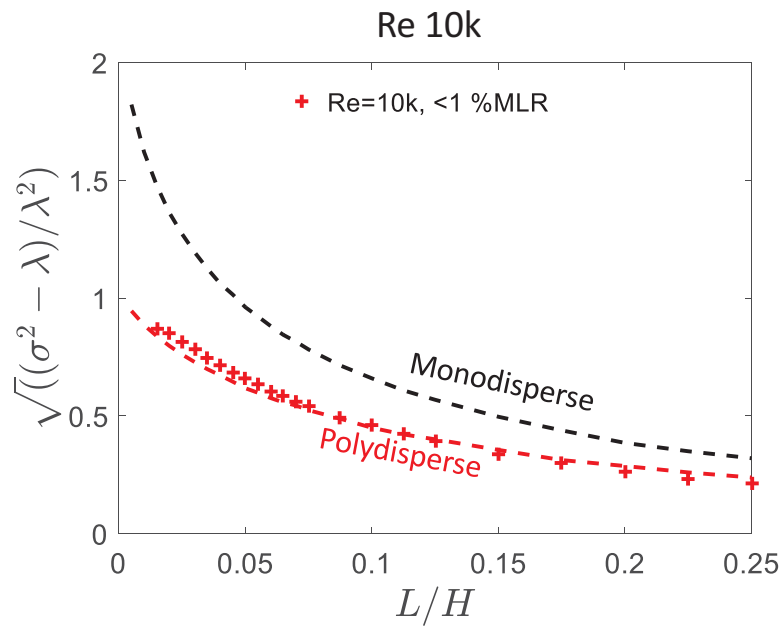


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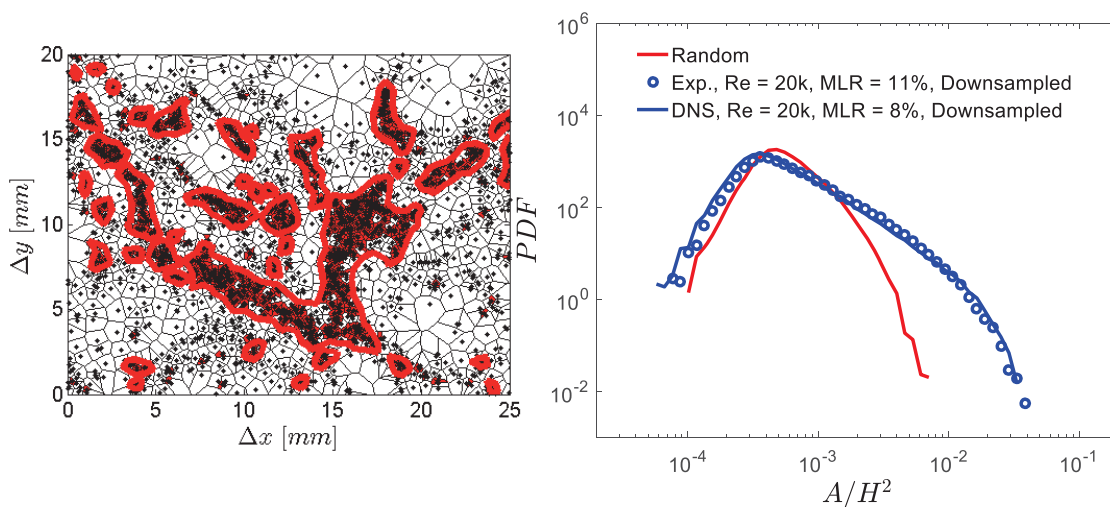
## Effect of polydispersity



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## Preferential Concentration Cluster/Void Identification



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# Particle Velocities

## Experiments

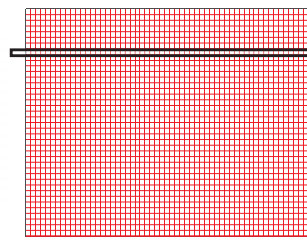
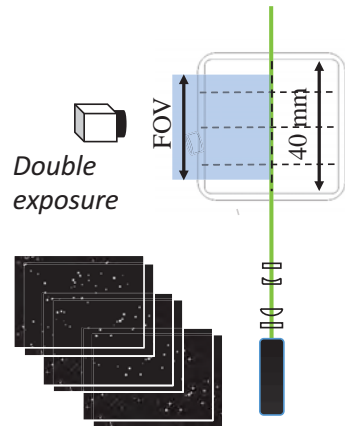
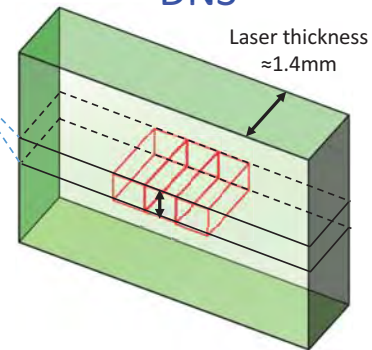


Image Plane (2D)

PIV-Interrogation windows:  
 $0.5 \times 0.5 \text{ mm}^2$

## DNS



DNS-Interrogation volumes:  
 $0.5 \times 0.5 \times 1.4 \text{ mm}^3$



Projected 2-D Velocity field

Mean and fluctuating particle velocity

Eulerian & Lagrangian mean and  
fluctuating particle velocity

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# Conclusions & Challenges

- Particle phase statistics are volumetric measurements. Comparison between cases requires integration in equivalent volumetric units.
- Statistics of preferential concentration are affected by finite number of particles. Limit of increasing  $N_p$  challenging both in exp. & simulations
- Agreement was achieved between DNS and experiments in terms of particle clustering at the center of the duct when including real effects.
- Measuring near wall concentration data remains a challenge.
- Careful comparison of particle velocity statistics. PTV processing techniques are being implemented. Processing of DNS particle velocities by class.
- Experimental particle velocities at high MLR?

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# Meso-Scale Simulations of Shock-Particle Interactions

F. Najjar

Multiphase Deepdive Workshop

Tampa, FL

October 6-7, 2016



LLNL-PRES-702745

This work was performed under the auspices of the U.S. Department of Energy by Lawrence Livermore National Laboratory under contract DE-AC52-07NA27344. Lawrence Livermore National Security, LLC



## This talk discusses hydrodynamic simulations to investigate Shock-Particle Interactions

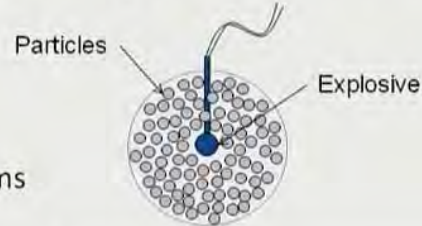
- Introduction & Goals
- Multiphase Simulations:
  - Shock-Particle interactions EM
  - Meso-scale Simulations of particle clusters with shock waves
- Summary & Future Work

## Our goal is to understand using computational tools the formation of coherent clusters of particles or jet-like particle structures during MBX detonation

### Particle jetting

*Is a common feature of explosive dispersal of solid particles or liquids:*

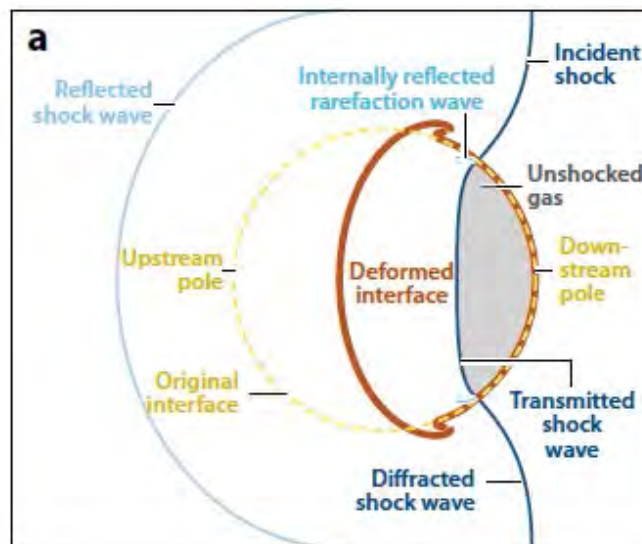
- Jets influence:
  - particle-gas mixing/burning
  - blast wave propagation
- Many possible instability mechanisms



*Spherical dispersal of sand particles*

Frost et al.

## Complex mechanisms of Shock-Particle Interaction (SPI) are summarized



Acoustic Impedance:  $\mathcal{R} = \rho c$

Positive Impedance:  $\delta \mathcal{R} > 0$

(Rajan et al., Ann. Rev Fluid Mech, 2011)

# LLNL Hydrodynamics Code is used and is based on Arbitrary Lagrange-Eulerian approach

- Perform high-resolution multiphysics simulations for SPI problems
- Why Hydrodynamics Code?
  - Multiphysics coupled code using Operator-Split Method
  - Extensive suite of EOS models
  - Massively parallel on HPC platforms
- Hydro code is based on Lagrange+Remap steps

$$\frac{D\phi}{Dt} + \vec{u} \cdot \vec{\nabla} \phi = F$$

$\frac{D\phi}{Dt} = F$  **Lagrange**

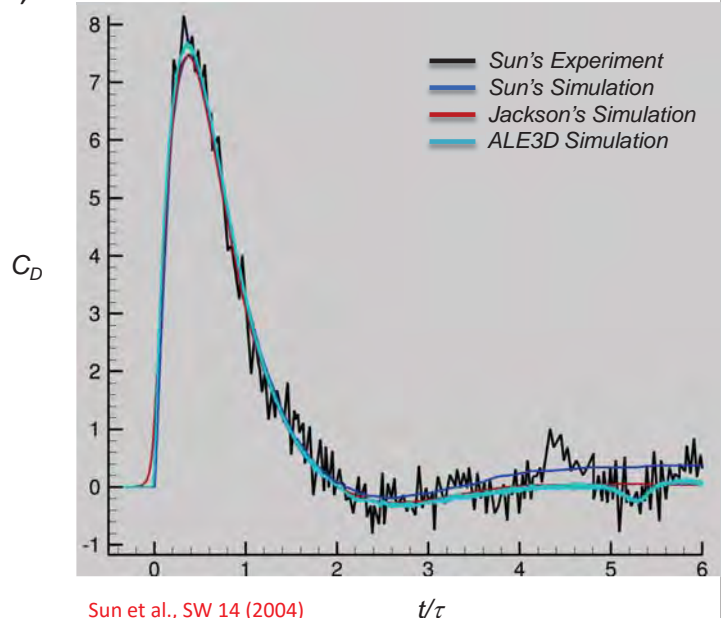
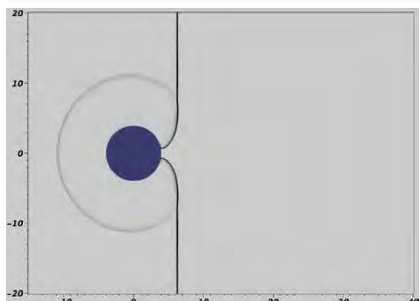
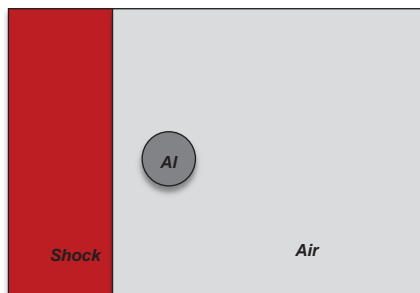
$\vec{u} \cdot \vec{\nabla} \phi = 0$  **Advection**



## ALE3D results compare quite well with Sun's experiments and numerical results by Sun et al & Jackson et al.

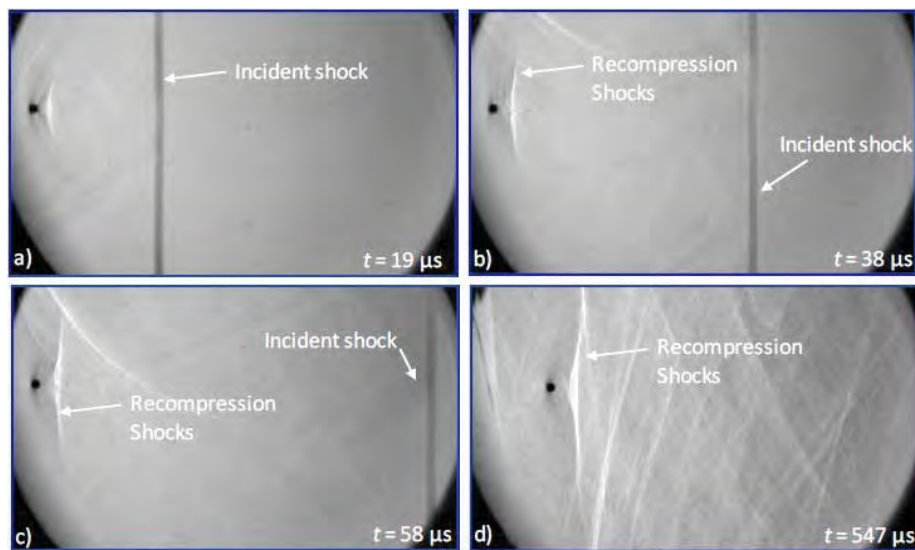
### 2-D Axisymmetric Simulations

- Particle size (d) 80mm; P=1.53MPa; Ma=1.22



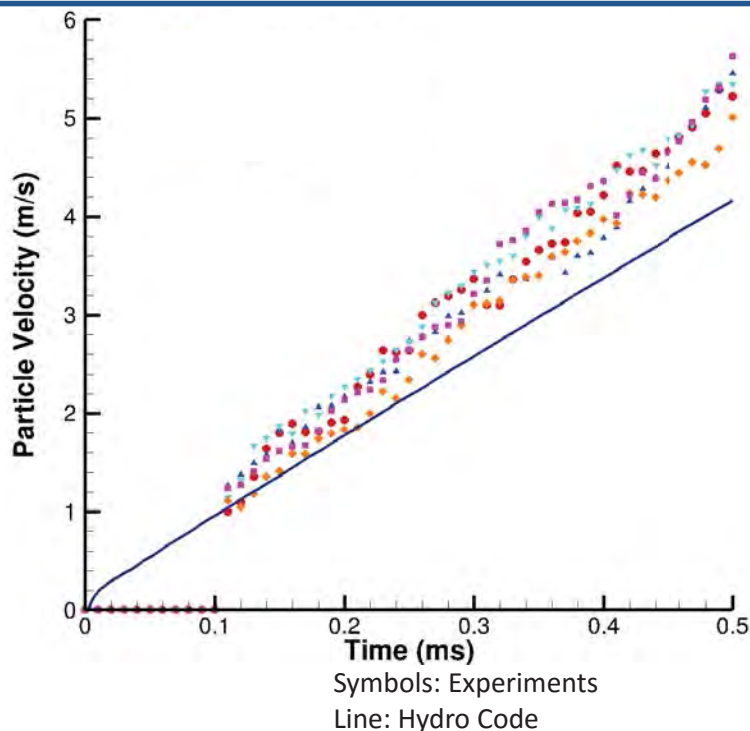
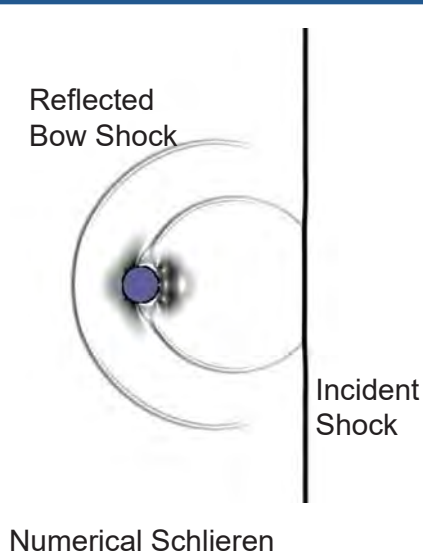
Sun et al., SW 14 (2004)  
Jackson et al., JAP 117, (2015)

## Wagner's shock tube experiments provide data for Steel particles accelerated in shock-induced flows



Wagner et al. SW (2012)

## The 1-mm steel particle is predicted in the current simulations to speed up at a lower rate than Wagner's experiments



We are collaborating with Dr. Wagner at SNL to identify key differences



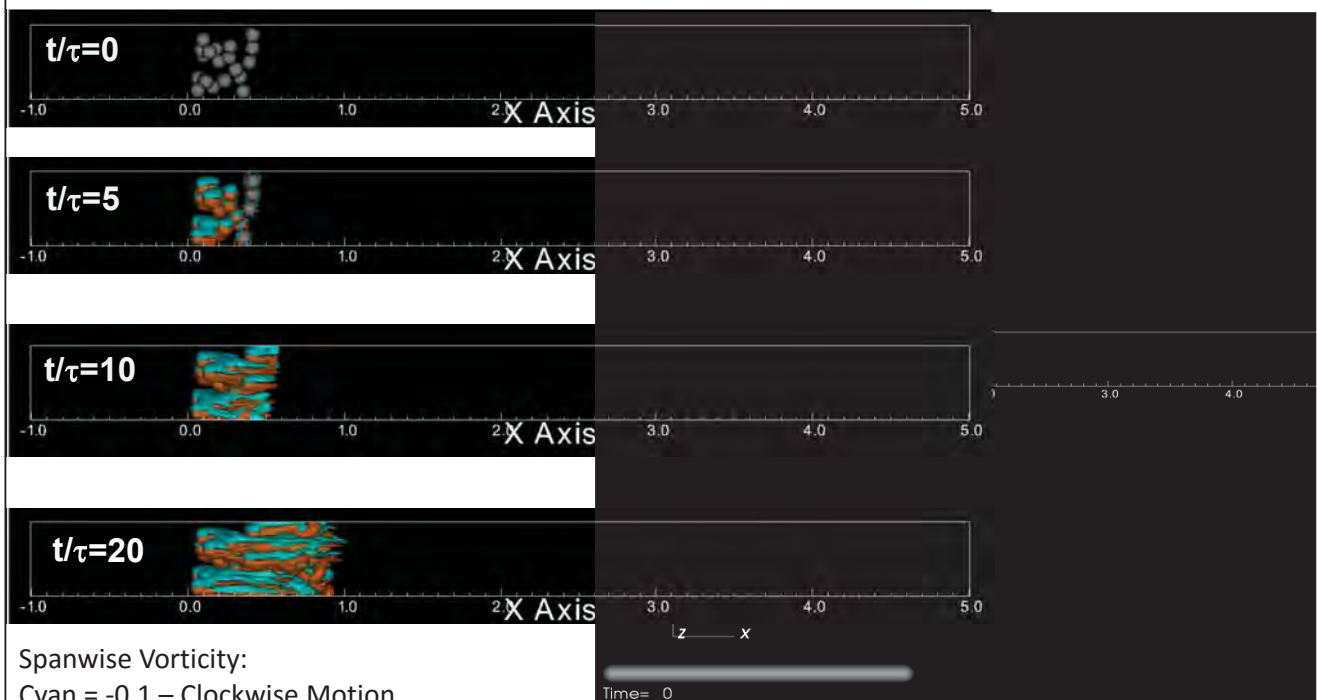
## Meso-scale simulations of multi-particle pack are studied to investigate 3D effects and particle-particle interactions

- N-particle pack of Aluminum particles in 3D volume
  - $d_p=1\text{mm}$
  - $\phi=0.1$  ( $N=20$ ),  $0.2$  ( $N=40$ ), &  $0.3$  ( $N=60$ )
  - Shock:  $M=1.5$
  - Mesh Resolution = 4.5M zones
- Individual particle positions, velocities and accelerations are tracked in time using an Embedded Grid Method.



## Compaction phase is observed followed by an expansion phase

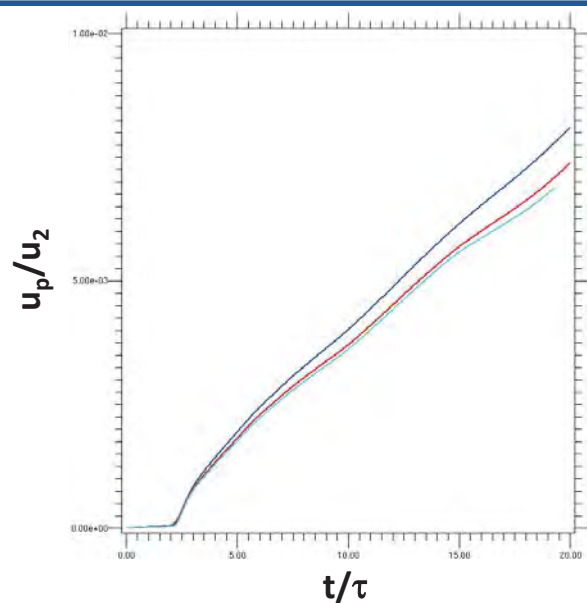
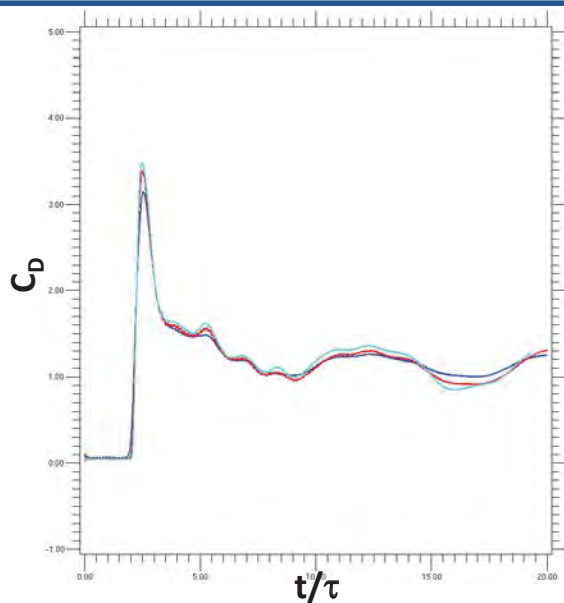
$M=1.5$ ,  $\phi=0.1$



Spanwise Vorticity:  
Cyan = -0.1 – Clockwise Motion  
Orange = +0.1 – Counterclockwise Motion



## Mesh resolution study shows that 16 zones across each particle are sufficient to capture the SPI dynamics



Particle 19 is located at front

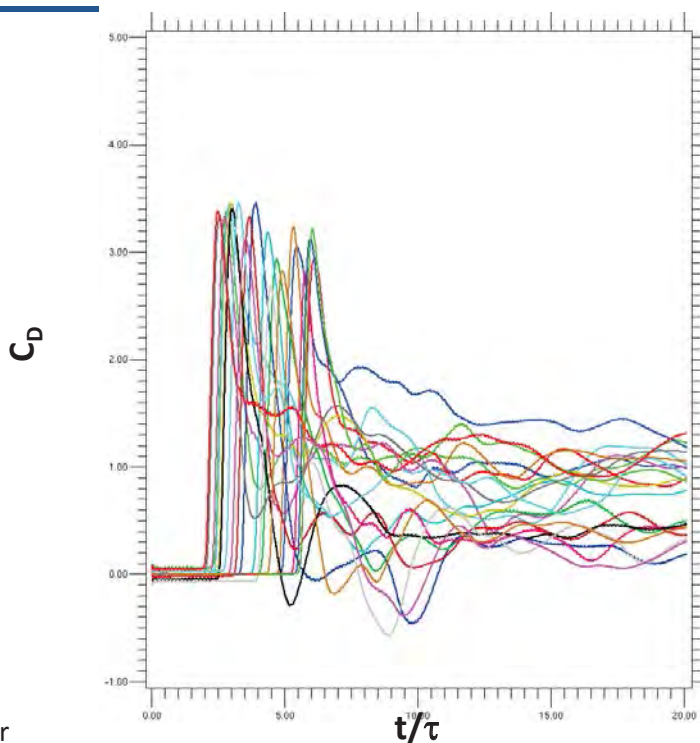
Blue: Mesh=1.3M zones

**Red: Mesh = 4.5M zones**

Cyan: Mesh=10.8M zones

$M=1.5,$   
 $\phi=0.1$

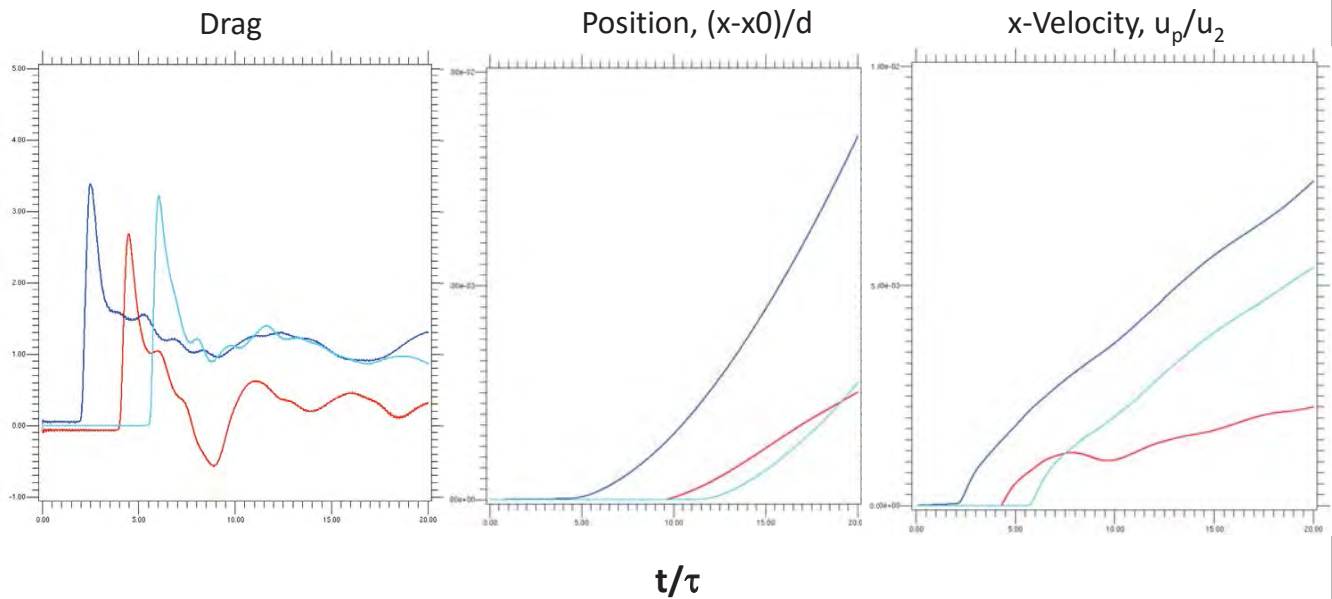
## The maximum in drag coefficient reaches $\sim 3.5$ and steady state values range from $\sim 0$ to $\sim 1.4$ for the 20-particle cluster



All particles in cluster

$M=1.5,$   
 $\phi=0.1$

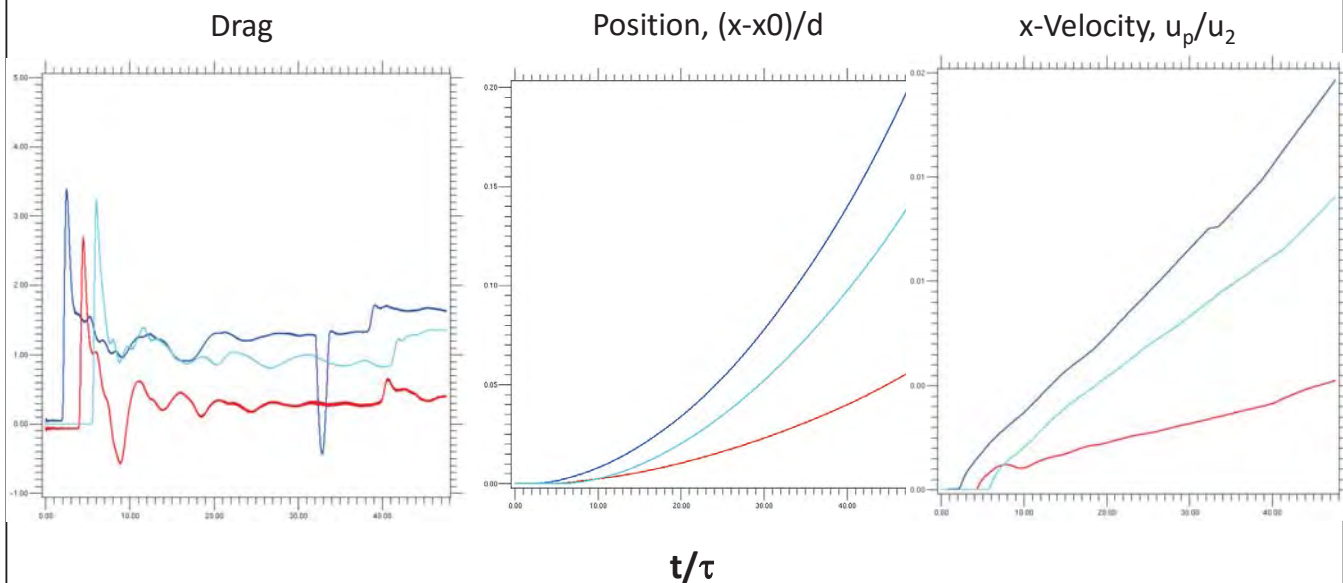
## Particles at 3 key locations are identified and their time history behavior is tracked, showing slow velocity and minimal motion



Particle 19: Front-Blue  
Particle 14: Middle-Red  
Particle 17: Back-Cyan

$M=1.5$ ,  
 $\phi=0.1$

## Drag, Position & Velocity at $M=1.5$ : long time span- $t/\tau=45$

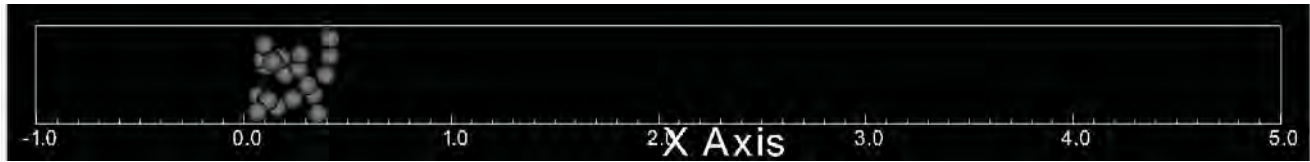


Particles 19, 14, & 17 are located at front (blue), middle (red) and back (cyan) of pack.

$\phi=0.1$

## 3D computations performed with varying volume fractions of 0.1, 0.2 & 0.3 to evaluate its effects on cluster dynamics

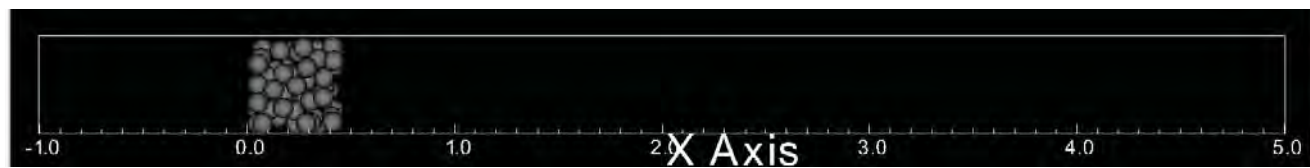
$\phi=0.1$ :  $N_p=20$



$\phi=0.2$ :  $N_p=40$

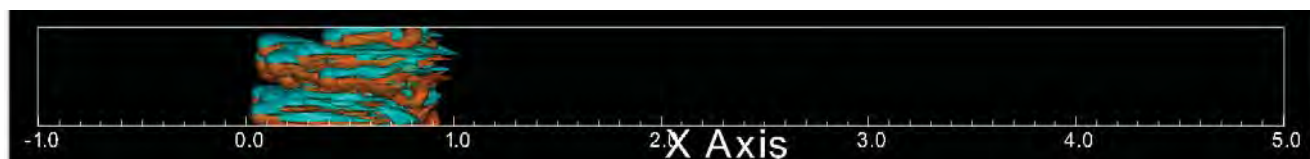


$\phi=0.3$ :  $N_p=60$



## With increasing volume fraction, flow field in the cluster breaks down into smaller scales

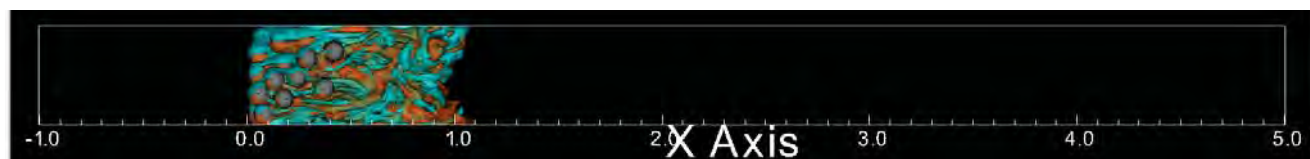
$\phi=0.1$



$\phi=0.2$

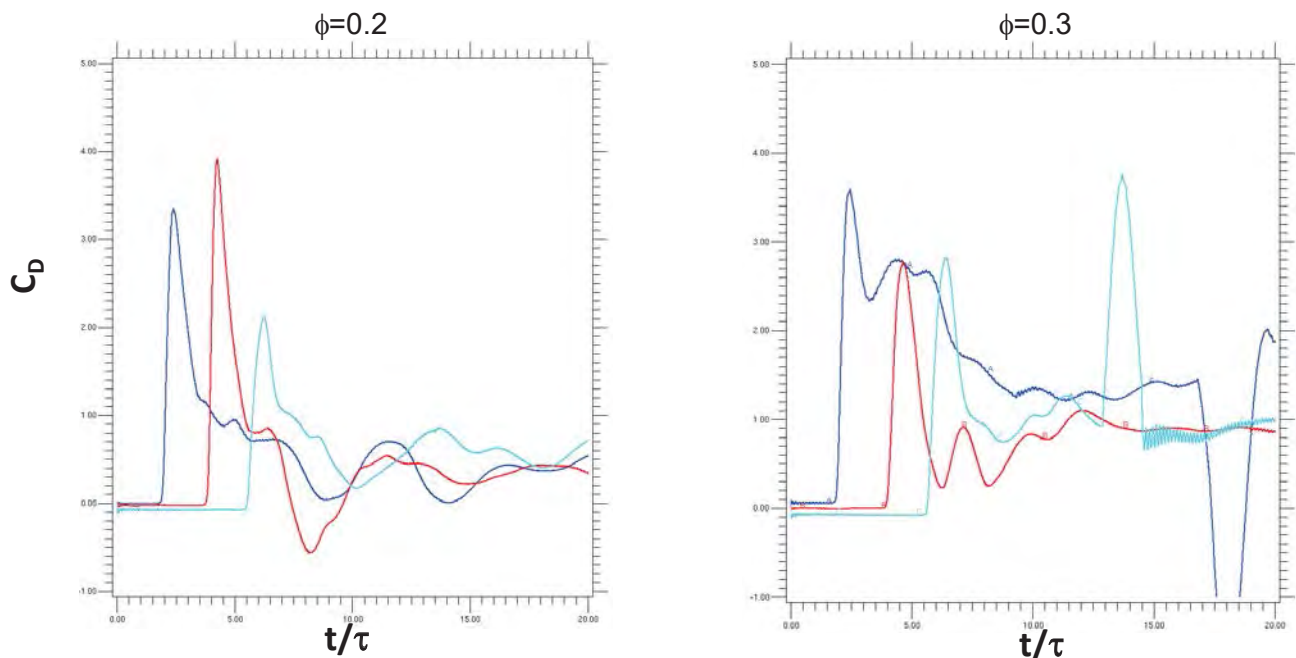


$\phi=0.3$



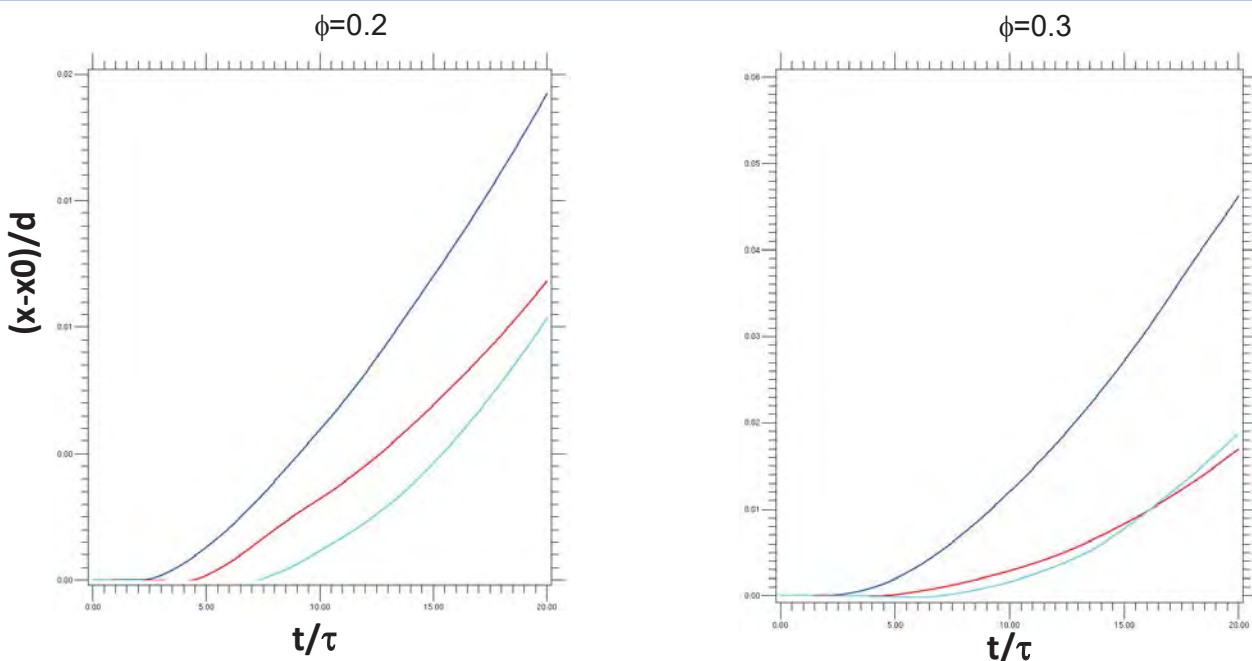
$t/\tau \sim 20$

**With increasing volume fraction, collision events between particles in the pack occur more frequently and earlier in time**



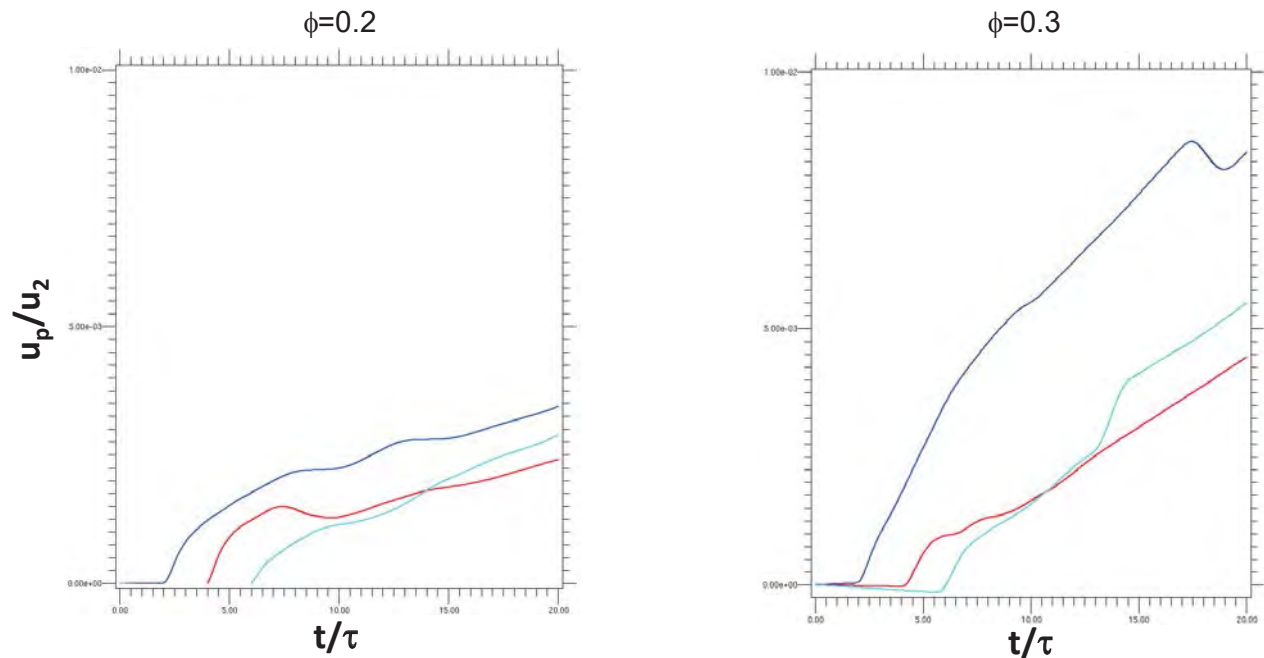
Particles are located at: front (blue), middle (red) and back (cyan) of pack.

**As volume fraction increases, particles in the cluster are able to move farther compared to low volume fraction case. Hence an assumption of “frozen” pack no longer holds for higher volume fraction**



Particle locations are at front (blue), middle (red) and back (cyan) of pack.

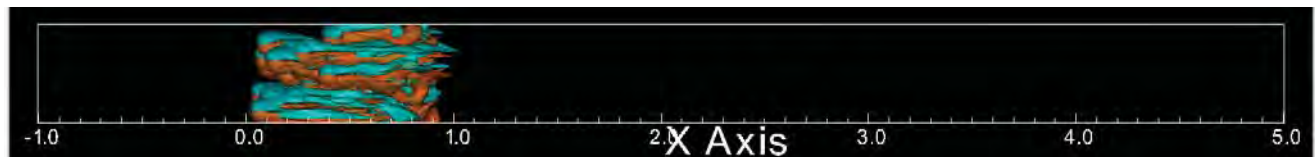
With higher volume fraction, particle velocities in the cluster increase due to the complex wake dynamics



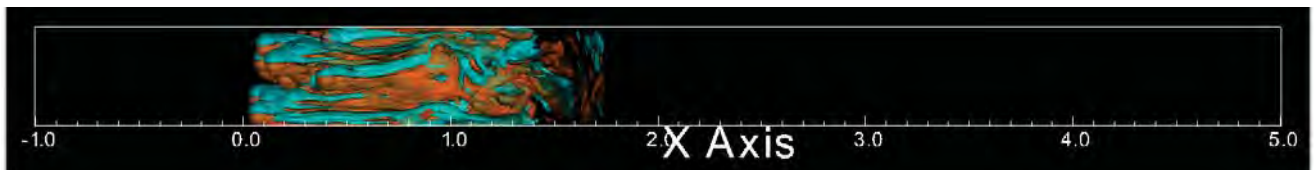
Particle locations are at front (blue), middle (red) and back (cyan) of pack.

Effects of shock strength are also studied to understand pack dynamics- $t/\tau \sim 20$

$M=1.5$



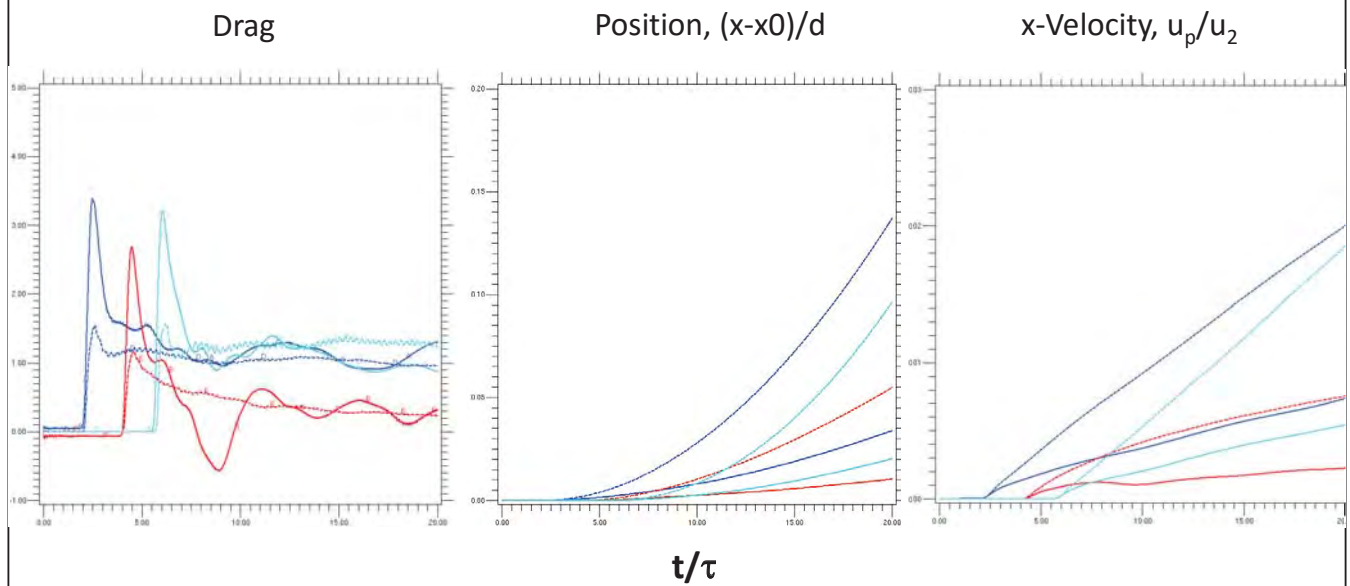
$M=3.0$



$\phi=0.1$



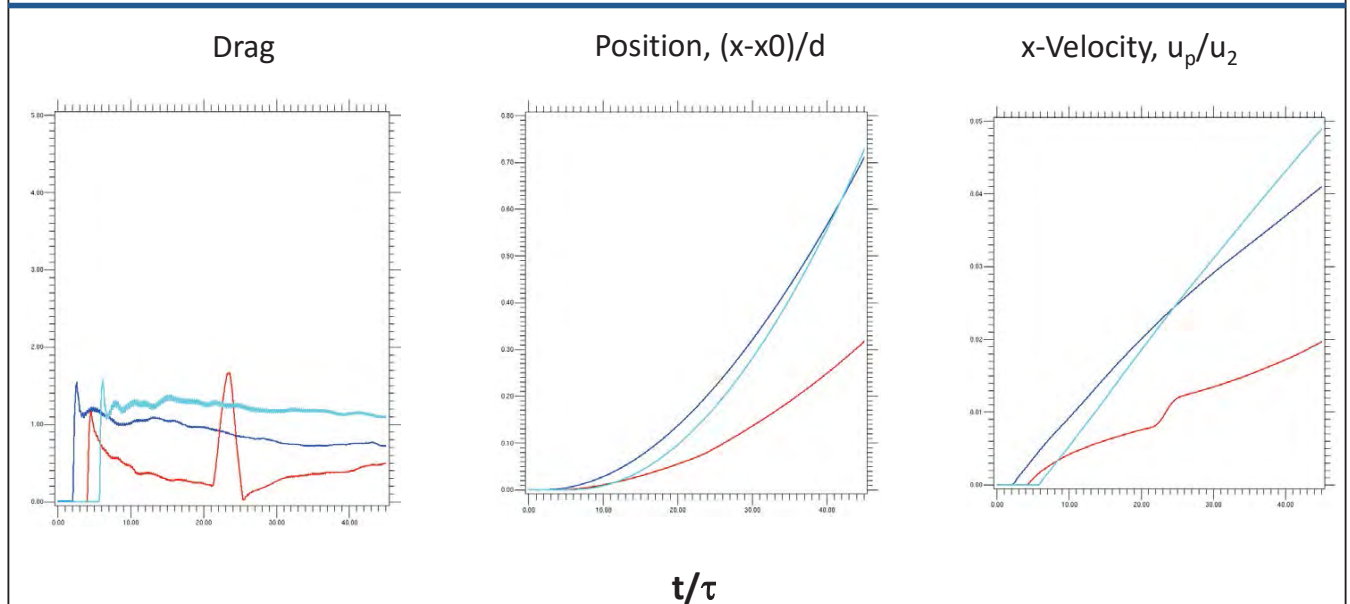
**With increasing Mach number, the particle moves and accelerates faster. Hence, an assumption of “frozen” pack no longer holds for higher Mach number**



Particles 19, 14, & 17 are located at front (blue), middle (red) and back (cyan) of pack.  
Solid:  $M=1.5$ ; Dashed:  $M=3.0$

$\phi=0.1$

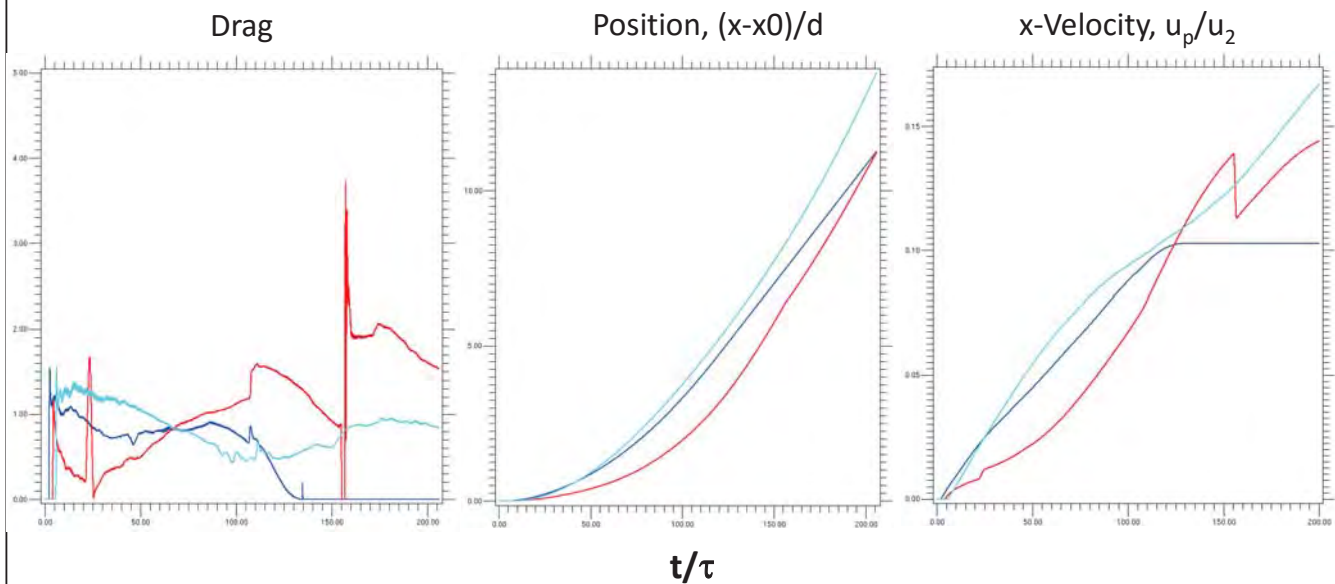
## Drag, Position & Velocity at $M=3$ : long time span- $t/\tau=45$



Particles 19, 14, & 17 are located at front (blue), middle (red) and back (cyan) of pack.

$\phi=0.1$

## Drag, Position & Velocity at $M=3$ : long time span- $t/\tau=200$



$\phi=0.1$

Particles 19, 14, & 17 are located at front (blue), middle (red) and back (cyan) of pack.

## Summary & Future Work

- Multiphase computations provide a powerful tool to study complex effects that are challenging to measure experimentally
- SPI mechanisms are validated using available experimental data
- Meso-scale simulations of shock-particle clusters are investigated for various volume fractions and shock strength & highlight the complex interactions
- Future work:
  - Perform 3D simulations of shock-particle cluster with larger particle count  $\sim 10^3$ - $10^4$
  - Create advanced drag models based on meso-scale simulations
  - Understand clustering mechanisms in MBX systems

Thank you for listening  
Questions?

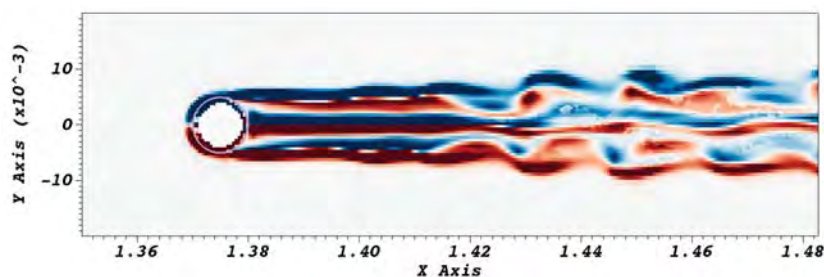
Fady Najjar, Ph.D.  
Physics Design Division  
LLNL

Email: najjar2@llnl.gov

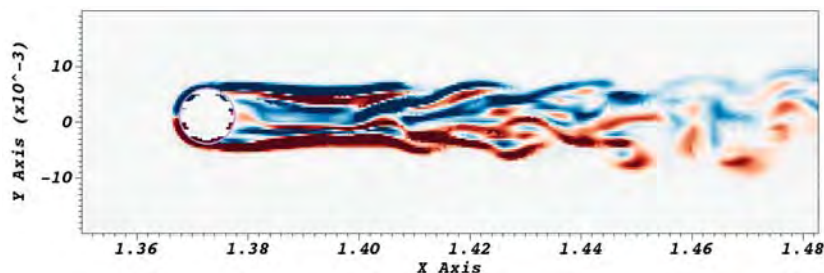


**The shear layers break up faster when the viscous effects are active as the**

Inviscid



Viscous

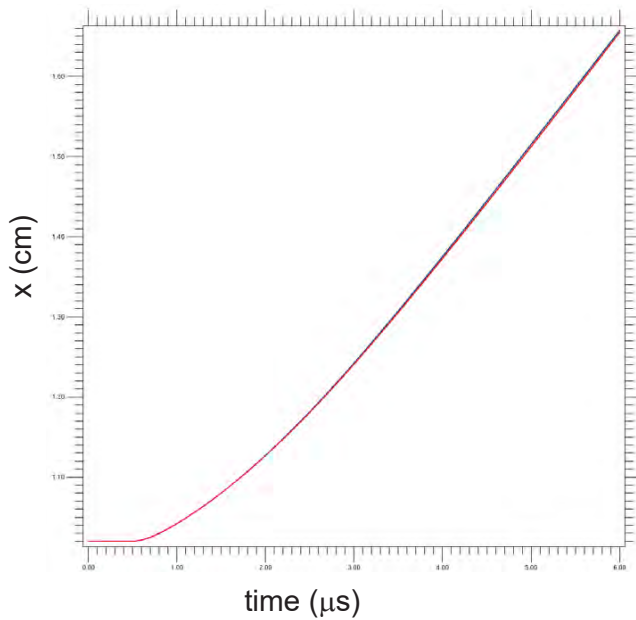


Z-vorticity at  $t=4\mu\text{s}$

W-Particle has moved to  $x \sim 1.37\text{cm}$

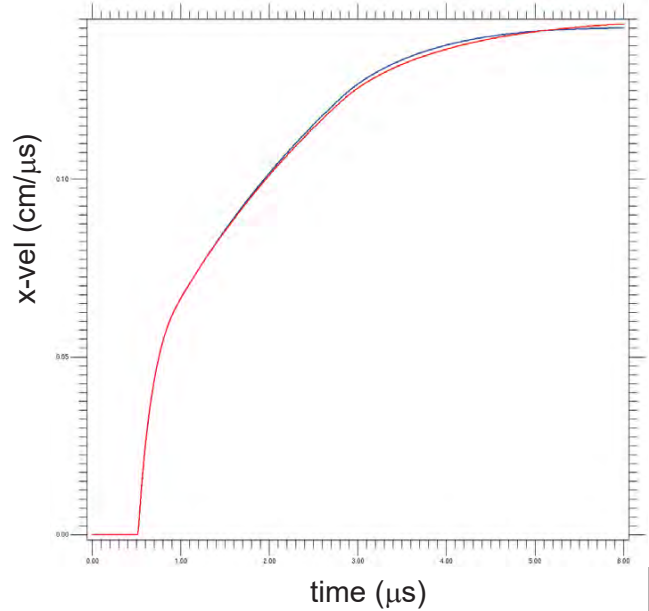
## Minimal differences in particle position and its speed are observed with and without viscous effects

Position



Blue Line: Inviscid  
Red Line: Viscous

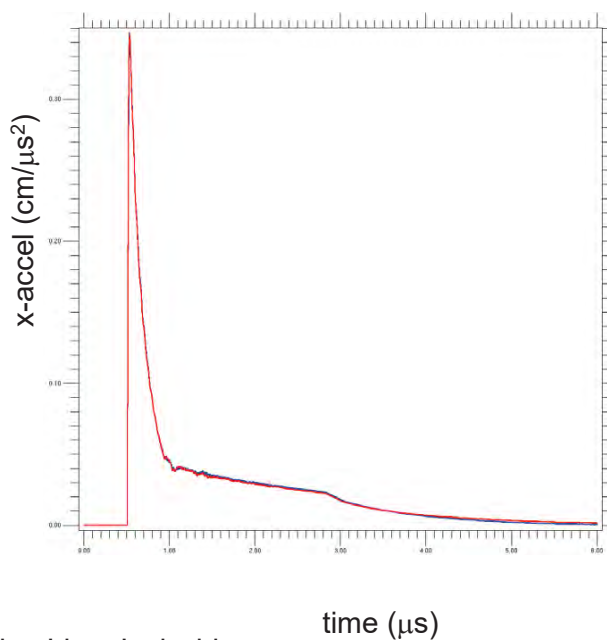
Speed



Particle "Coasting" speed is  $= 0.14 \text{ cm}/\mu\text{s}$

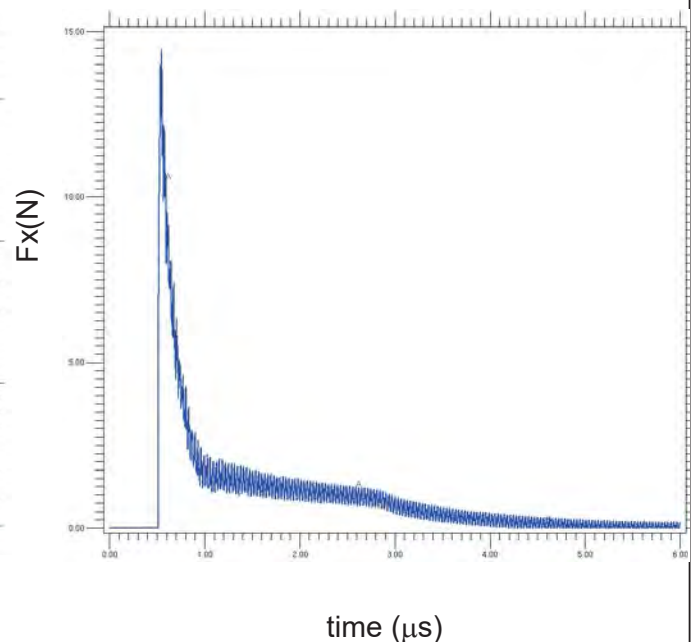
## Minimal difference in its acceleration is observed with and without viscous effects

Acceleration



Blue Line: Inviscid  
Red Line: Viscous

X-Force on Particle





# A systematic study of turbophoresis

four-way-coupled simulation of Stokesian particles in channel flow

**Mahdi Esmaily**  
**Ali Mani**

Stanford University

Deep Dive  
Oct. 6-7, 2016

## Outline

- Challenges
- Objectives
- Short notes on
  - Boundary conditions for particles
  - Computing near-wall statistics
- Systematic study of particles in the viscous layer
  - Effect of flow Reynolds number, Stokes number, density ratio, volume fraction, and restitution coefficient
  - Concentration and velocity statistics
- Open questions



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# Challenges

- Particle-flow interaction
  - Finite Reynolds number effects: Particle resolved vs. unresolved Lagrangian/Eulerian
  - Point-particle assumption: Stokes drag, history terms, added mass, pressure gradient, ...
  - Two-way-coupling effects: disturbed vs. undisturbed velocity
  - Boundary conditions for particles
- Particle-wall interaction
  - Restitution coefficient
  - Wall roughness
  - Wall vibration
  - Electrostatic, Saffman lift, and thermophoretic forces
- Particle-particle interaction
  - Restitution coefficient
  - Polydispersity
  - Particle shape effects

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# Objectives

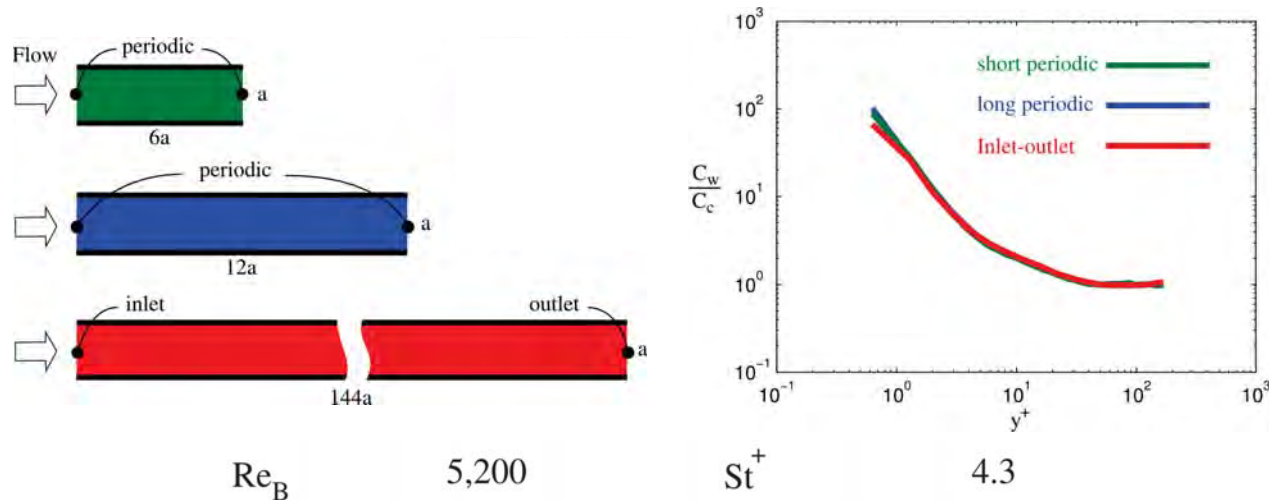
- Targeting regimes with:
  - Low mass loading ratio (no two-way-coupling)
  - Small spherical particles (no finite Reynolds number effects)
  - High particle to fluid density ratio (Only linear Stokes drag)
  - Absence of body force
  - Mono-dispersed distribution
  - Smooth walls with no vibration
  - Absence of electrostatic, thermophoretic, and Saffman lift forces
- Systematically investigating the effect of:
  - Flow Reynolds number :  $Re$
  - Particle Stokes number (particle-to-fluid density ratio) :  $St^+$
  - Particle size (relative to the viscous length scale) :  $Dp^+$
  - Restitution coefficient (particle-particle  $\equiv$  particle-wall) :  $k$
  - Volume fraction (one-way-coupled and one-way-coupled+collision) :  $\Phi$

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# Boundary conditions for particles

- Periodic boundary condition can be considered as a substitute for a long (case-dependent) inflow-outflow boundary condition
- Flow-based streamwise computational period is also sufficient for particles



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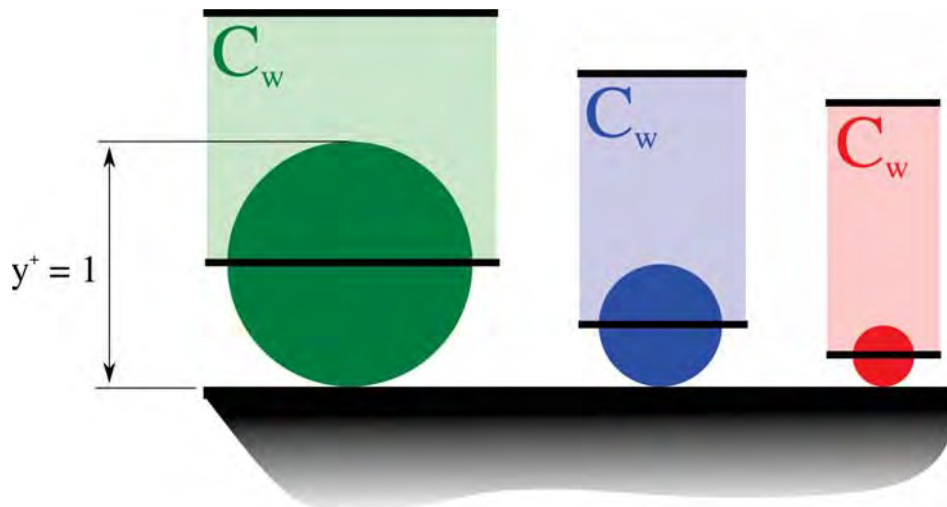
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## Computing number density

- It is important to adjust the position of the first control-volume for Lagrangian-to-Eulerian mapping based on the particle diameter



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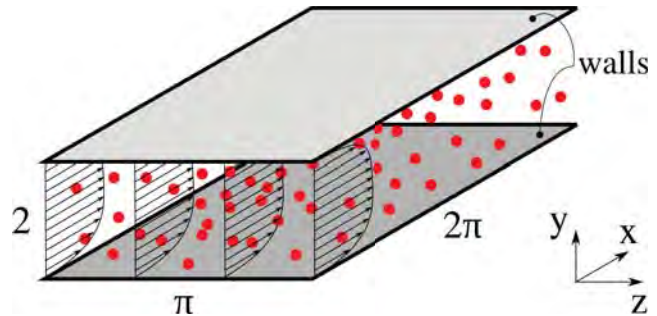
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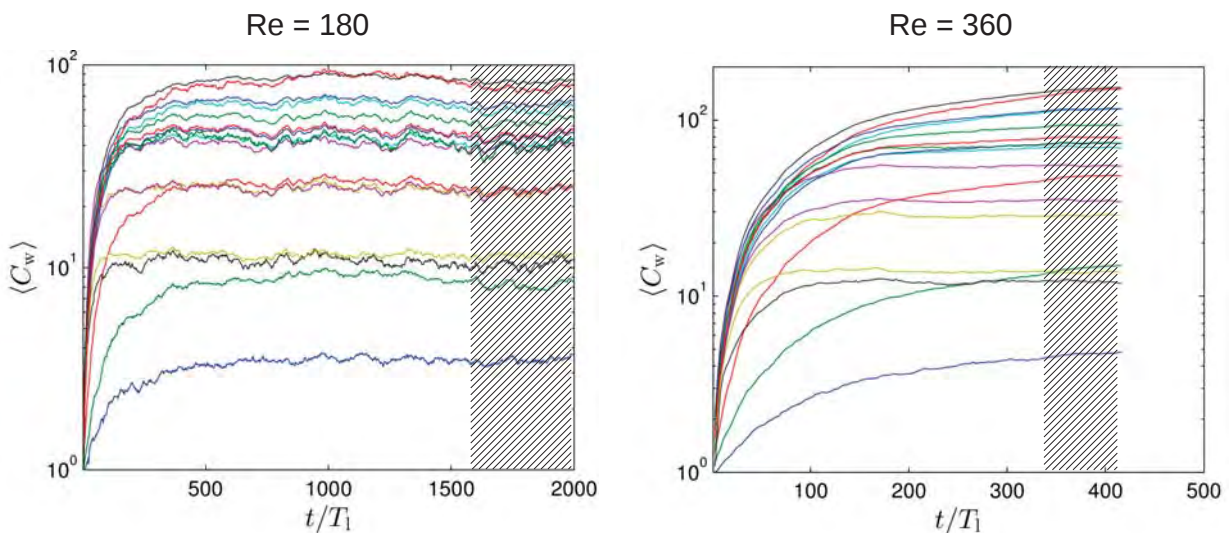
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# Problem setup

- Turbulent channel flow
  - Reynolds number (based on the channel half height and friction velocity): **178 (KMM) and 356**
  - Grid size in wall unit:  $5.8 \times 0.41$ - $5.7 \times 4.4$
- Particle to fluid density ratio:
  - 18, 36, 72, ..., 18432 (i.e. 11 classes  **$18 \times 2^{(0:10)}$** )
- Particle diameter in wall unit:
  - **$Dp^+ = 1/4$**  :  $St^+ = 1/16$ , ..., 64
  - **$Dp^+ = 1/2$**  :  $St^+ = 1/4$ , ..., 256
  - **$Dp^+ = 1$**  :  $St^+ = 1$ , ..., 1024
- Restitution coefficient:
  - **$k = 0.0, 0.25, 0.5, 0.75, 1.0$**
- Volume fraction:
  - **$\Phi = 0$  (one-way-coupled)** :  $Np = 10^5$
  - **$\Phi = \{1/16, 1/4, \dots, 16\} \times 10^{-5}$** :  $Np = 800 - 2 \times 10^5$  (**8X** for higher Re),  $MLR = 4 \times 10^{-5} - 10^{-2}$
- In total
  - **55** classes of particles in the one-way-coupled simulations at each Reynolds
  - **15** classes of particles in the four-way-coupled simulations at each Reynolds



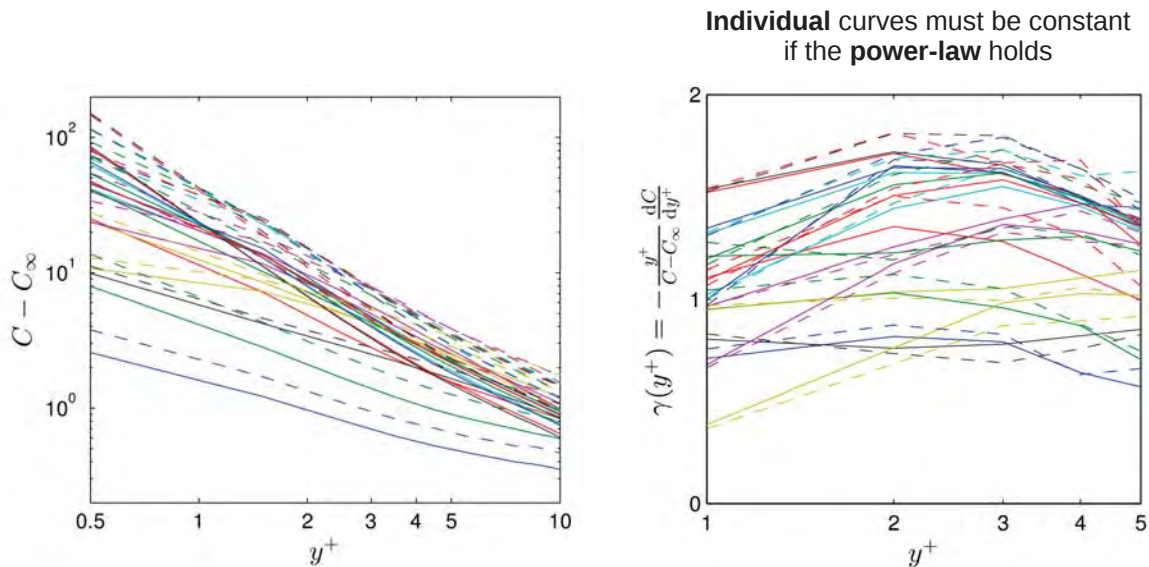
## Time convergence



- Only simulations with collisions will be presented



# Does particle number density follow a power law in the viscous-layer?



- It appears so, however, higher resolution is required before drawing any conclusion



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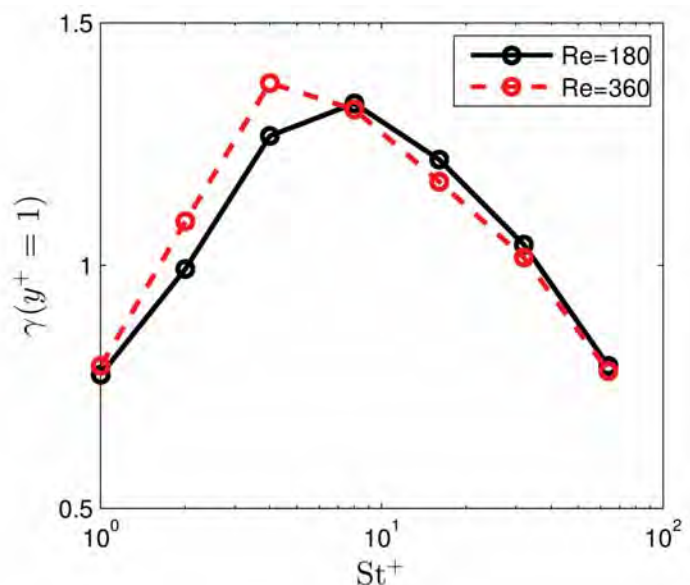
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## Effect of St (number density)

- $Dp^+ = 1/2$ ,  $k = 1$ ,  $\Phi = 10^{-5}$
- Adopting  $\gamma$  for analysis of the near-wall number density allows for a collapse with respect to Re
- Highest concentration is observed at  $St^+ = O(10)$
- Trends are expected to change at lower  $k$



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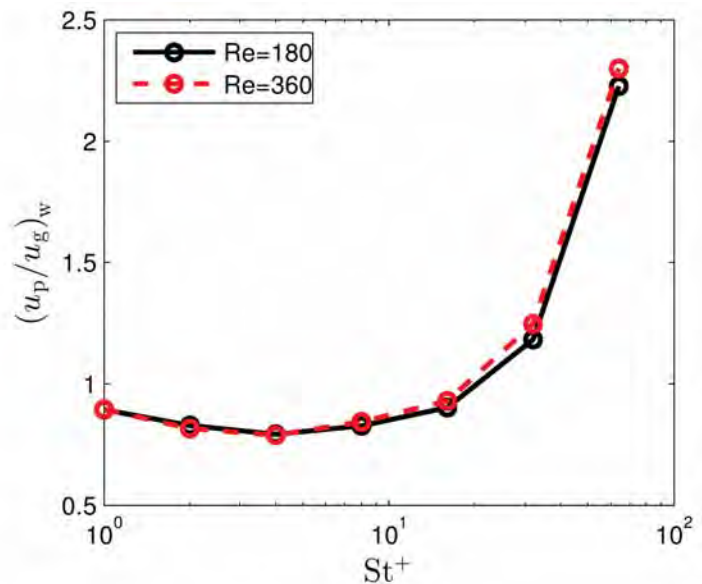
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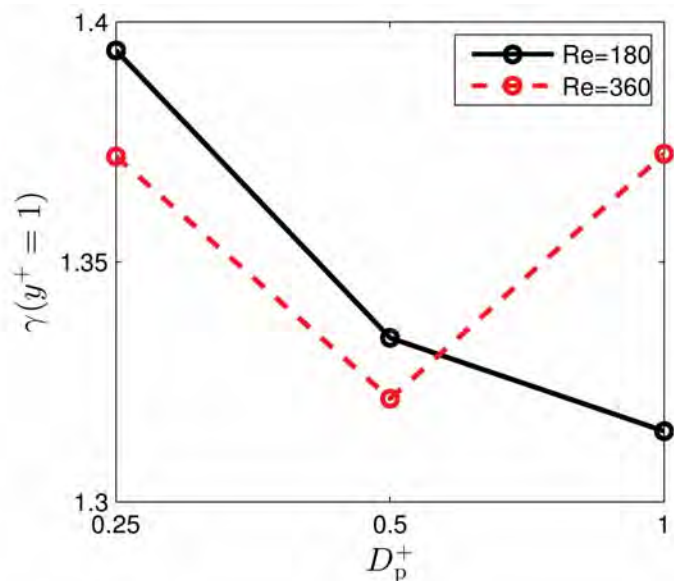
# Effect of Stokes number (velocity)

- $Dp^+ = 1/2$ ,  $k = 1$ ,  $\Phi = 10^{-5}$
- Near-wall velocities in wall-units are independent of  $Re$
- At higher  $St^+$ , particles are faster than the flow in the viscous-layer
- Despite strong turbophoresis, the bulk velocity of particles is similar to the gas at high  $St^+$



# Effect of particle size

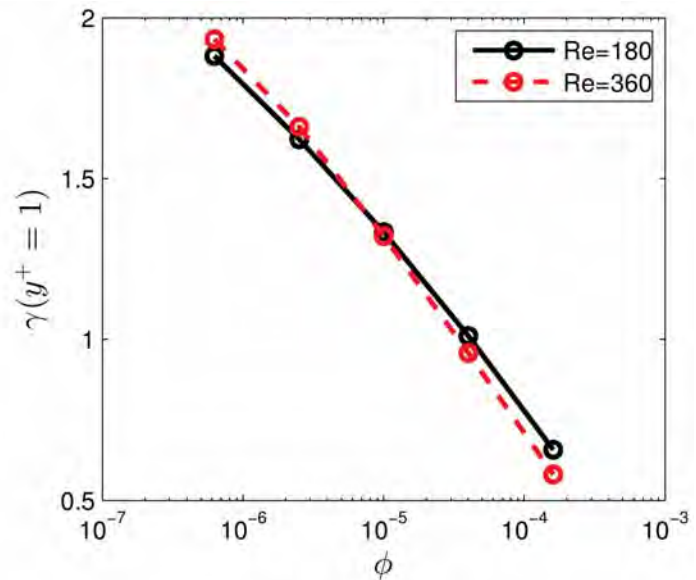
- $St^+ = 8$ ,  $k = 1$ ,  $\Phi = 10^{-5}$
- Density and number of particles are adjusted to keep  $St^+$  and  $\Phi$  constant
- The effect of particle size on  $\gamma$  is an order of magnitude less than  $St^+$
- Particle size has a larger effect when  $\Phi$  is smaller





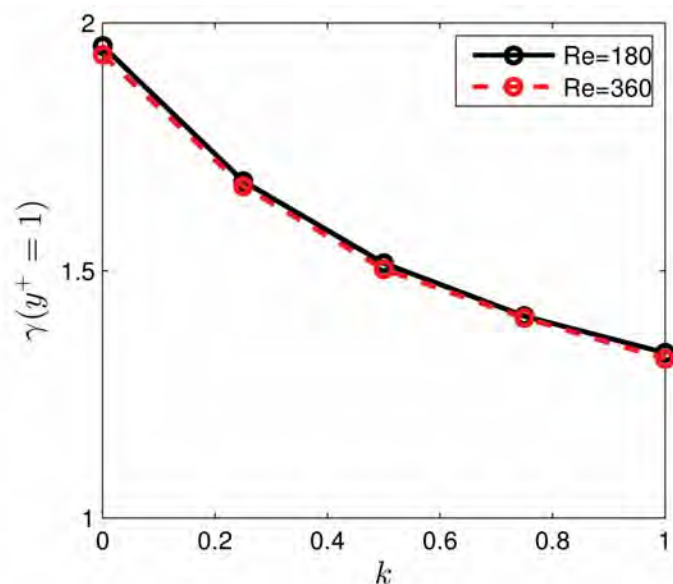
# Effect of volume fraction

- $Dp^+ = 1/2$ ,  $St^+ = 8$ ,  $k = 1$
- $\gamma$  appears to be linearly proportional to  $\log(\Phi)$
- Normalized near wall concentration decreases significantly as  $\Phi$  increases
- The normalized near-wall number density decrease approximately proportional to  $\Phi^{-1/3}$



# Effect of restitution coefficient

- $Dp^+ = 1/2$ ,  $St^+ = 8$ ,  $\Phi = 10^{-5}$
- Same restitution coefficient is employed for the particle-particle as the particle-wall collisions
- The restitution coefficient changes the near-wall number density at most by a factor of two



# Conclusions

- The power-law exponent ( $\gamma$ ) appears to be independent of  $Re$  as oppose to near-wall number density that varies depending on the size of the depletion region (i.e.  $Re$ )
- $\gamma$  is most sensitive to the Stokes number
- The particle size is of minimal importance in the investigated regime
- The volume fraction influence on  $\gamma$  appears to be log-linear
- The restitution coefficient changes near-wall number density at most by a factor two in the investigated regime
- Particles can be faster than the flow in the viscous-layer at higher  $\Phi$  or  $St^+$



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# Open questions

- Is a collision model with a single restitution coefficient an accurate model?
- How does restitution coefficient vary versus  $St^+$ ,  $Dp^+$ , ... ?
- How polydispersity changes the observed trends?
- What is the development length for different  $St^+$ , ...?
- Are there simple universal laws for predicting near-wall behavior of particles?



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A systematic study of  
turbophoresis

Mahdi Esmaily  
Ali Mani

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# Acknowledgment and discussion



PSAAP II



U.S. DEPARTMENT OF  
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A systematic study of  
turbophoresis

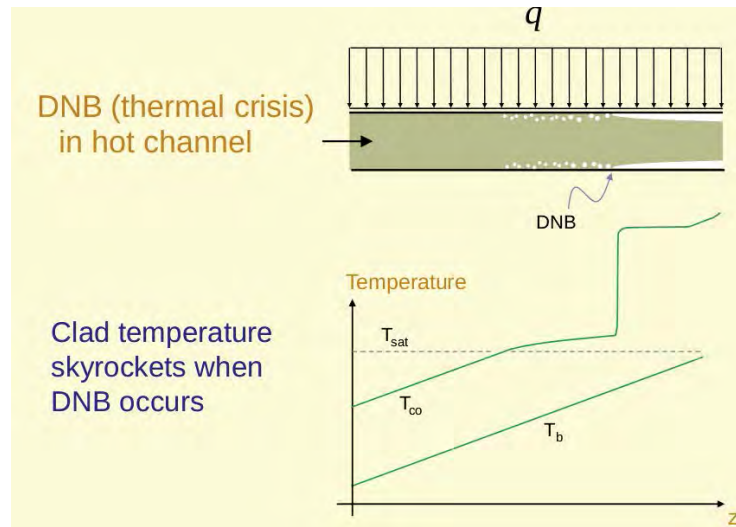
Mahdi Esmaily  
Ali Mani

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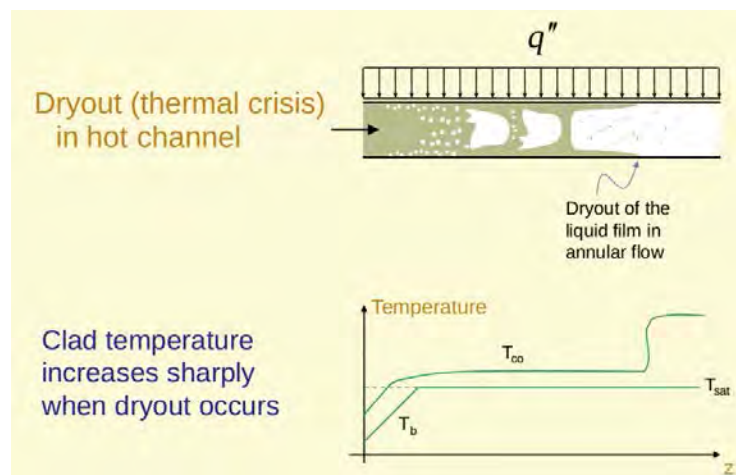
## Departure from Nucleate Boiling/Critical Heat Flux



Buongiorno; MIT Open Courseware

Navigation icons: back, forward, search, etc.

## Departure from Nucleate Boiling/Critical Heat Flux



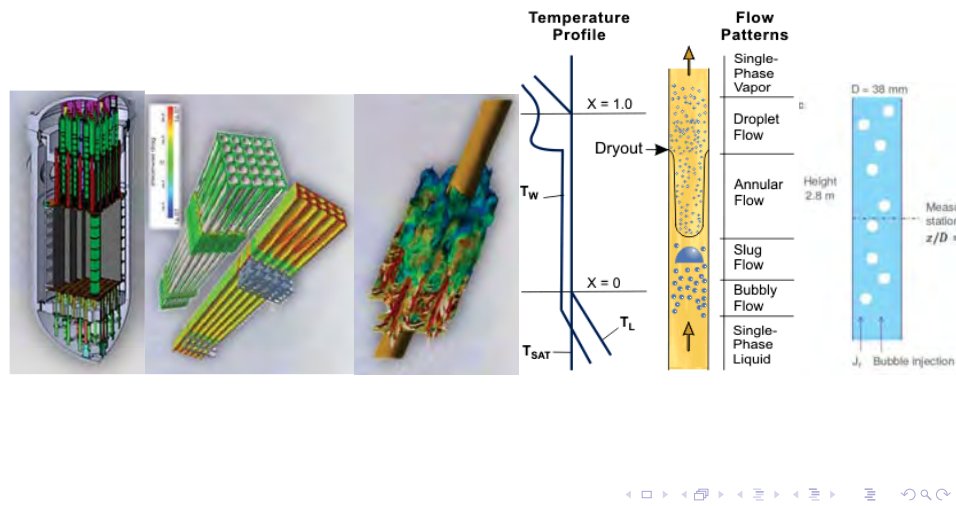
Buongiorno; MIT Open Courseware

Navigation icons: back, forward, search, etc.



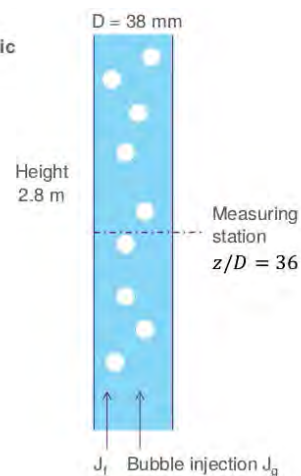
## Complex >>>> Simple

Fuel Assembly >> Rod Bundle >> Heated Pipe >> Adiabatic Bubbly Flow



## Complex >>>> Simple

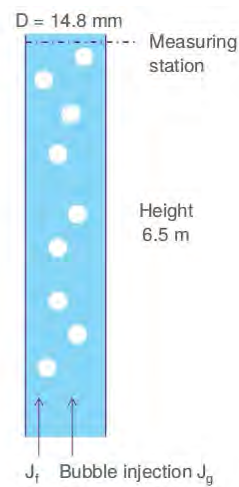
- Upward bubbly flow in a pipe. **Steady, axisymmetric** and **turbulent** flow without phase change  
 $Re \sim 50000$
- **48 flow conditions**
  - liquid superficial velocity  $J_l$
  - gas superficial velocity  $J_g$
- Measuring station  $z/D=36$ . **Radial profiles:**
  - Axial velocity
  - Velocity fluctuations
  - Void fraction



Liu-Bankoff Experiments

## Complex >>>> Simple

- Upward bubbly flow in a pipe. **Steady, axisymmetric** flow without phase change
- Fluid = aqueous solution + glycerine
- **Low Reynolds number:** 875 – 1860
- **9 experiments**
- Measuring station at the top. **Radial profiles:**
  - Liquid velocity and velocity fluctuations
  - Void fraction

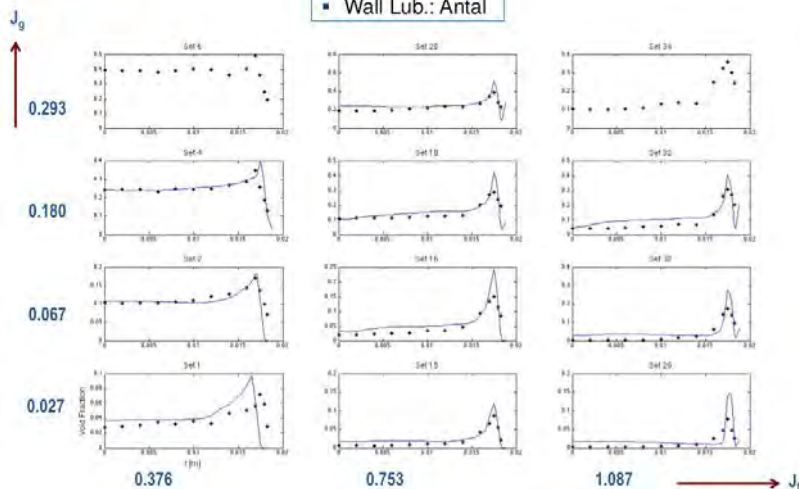


Nakaryakov Experiments

Navigation icons: back, forward, search, etc.

## Void Fraction Predictions 2013 – GEN-I

- Drag: Tomiyama
- Lift:  $C_L = 0.03$
- Wall Lub.: Antal

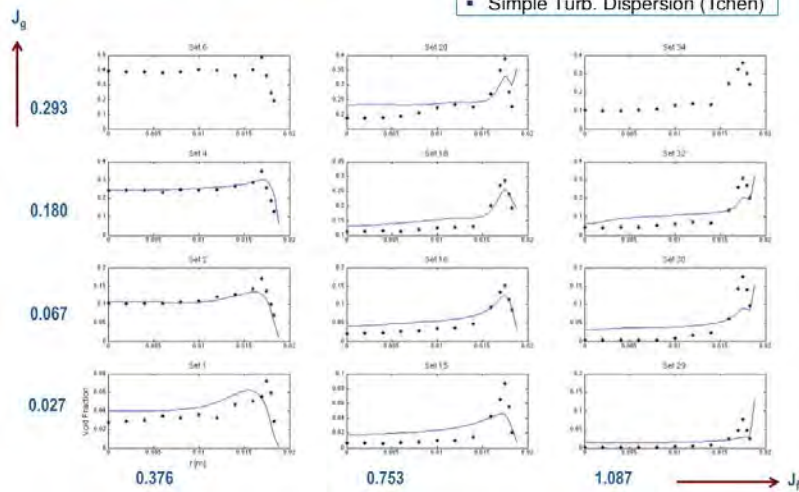


Sugrue-Baglietto

Navigation icons: back, forward, search, etc.

## Void Fraction Predictions Sensitivities example

- Drag: Tomiyama
- Lift:  $CL=0.03$
- Wall Lub.: Antal
- Simple Turb. Dispersion (Tchen)

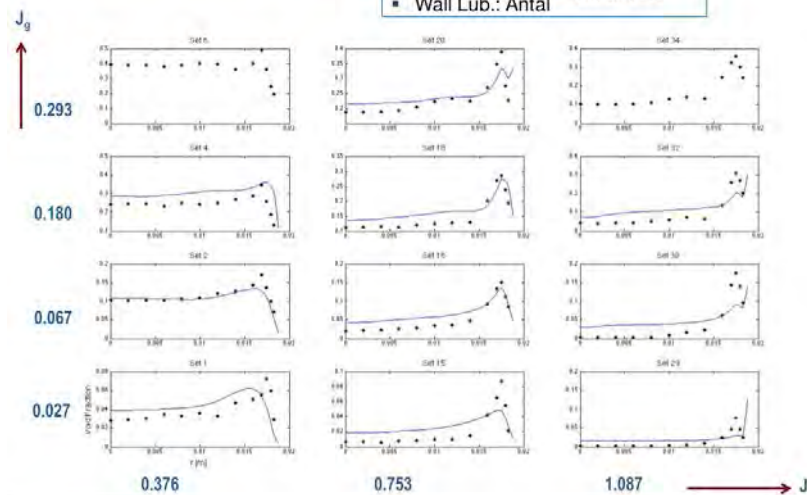


Sugrue-Baglietto

Navigation icons: back, forward, search, etc.

## Void Fraction Predictions Sensitivities example

- Drag: Tomiyama
- Lift:  $CL=0.03$
- Wall Lub.: Antal
- Turb. Dispersion
- Virtual Mass



Sugrue-Baglietto

Navigation icons: back, forward, search, etc.

## Can Bayesian Analysis Be Used to Improve Modeling?

- ▶ Single phase turbulence closures are robust
- ▶ Not the case with multiphase closures
  - ▶ Equation for void fraction can be highly nonlinear and very sensitive to parameter values
  - ▶ Parameterization of different processes can have unintended interactions
- ▶ How can they be made more robust?
  - ▶ Better Analysis of (almost) Direct Numerical Simulations, Experiments

## Summary

- ▶ There is (almost) DNS capability
- ▶ There are multiple parameterizations of multiphase phenomena (drag, lift, turbulent dispersion, ...)
- ▶ Modern UQ, ML, and Bayesian analysis techniques can be used to integrate DNS/Experiments with multiphase phenomenology to make RANS models (ensemble averaged Eulerian Dispersed-Multiphase) more robust
  - ▶ Discrepancy model, Representation error
  - ▶ Hierarchical model

Approach *not unlike* top-down VUQ approach  
that Phil Smilth talked about

# Ensemble-Averaged Dispersed Eulerian Multiphase —Two Fluid—Approach Considered and Thought Sufficient—No Particle-Flow Interaction!

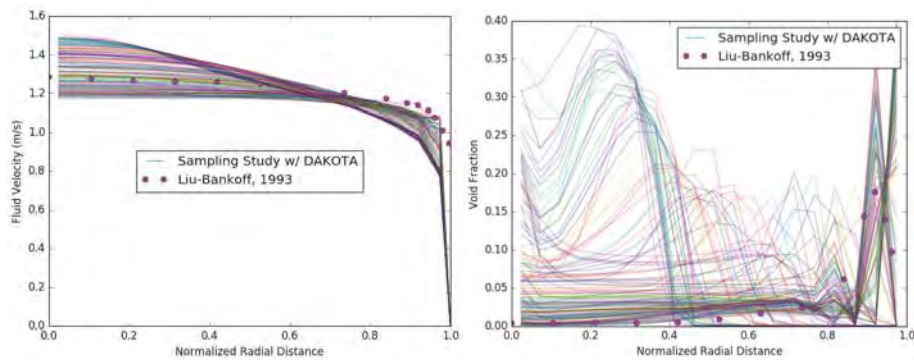
$$\begin{aligned} \frac{\partial \hat{\rho}_k}{\partial t} + \nabla \cdot (\hat{\rho}_k \tilde{\mathbf{v}}_k) &= \Gamma_k \\ \frac{\partial \hat{\rho}_k \tilde{\mathbf{v}}_k}{\partial t} + \nabla \cdot (\hat{\rho}_k \tilde{\mathbf{v}}_k \tilde{\mathbf{v}}_k) &= -\alpha_k \nabla p_k + \nabla \cdot (\alpha_k \boldsymbol{\tau}_k) \\ &\quad + \Gamma_k \tilde{\mathbf{v}}_k + \mathbf{M}_k + \hat{\rho}_k \mathbf{f} \\ \frac{\partial \hat{\rho}_k \tilde{e}_k}{\partial t} + \nabla \cdot (p + \hat{\rho}_k \tilde{e}_k \tilde{\mathbf{v}}_k) &= -\nabla \cdot (\alpha_k \mathbf{q}_k) + \alpha_k Q_k + \\ &\quad \alpha_k \mathbf{T}_k : \nabla \cdot \tilde{\mathbf{v}}_k + e_k \Gamma_k \\ \frac{\partial \alpha_k}{\partial t} + \tilde{\mathbf{v}}_k \cdot \nabla \alpha_k &= \mathcal{V}_k \\ \bar{\rho}_k &= \bar{\rho}_k(p_k, \tilde{T}_k) \end{aligned}$$

$$(\hat{\rho}_k = \alpha_k \bar{\rho}_k, \quad \tilde{\mathbf{v}}_k = \overline{\mathcal{X}_k \rho \mathbf{v}} / \overline{\mathcal{X}_k \rho}, \quad \tilde{e}_k = \overline{\mathcal{X}_k \rho e} / \overline{\mathcal{X}_k \rho})$$

## Sensitivity Analysis

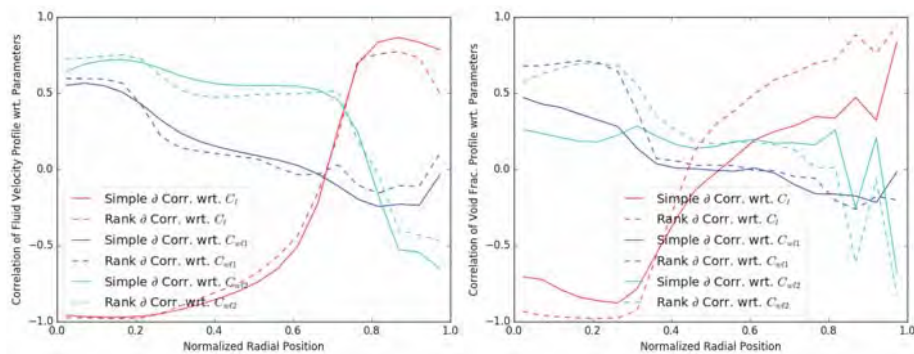


## Non-Intrusive Sensitivity Analysis w/ Starccm+ & Dakota Sampling Study Using Hundreds of Starccm+ Simulations



Void fraction behavior is more sensitive than fluid-velocity behavior

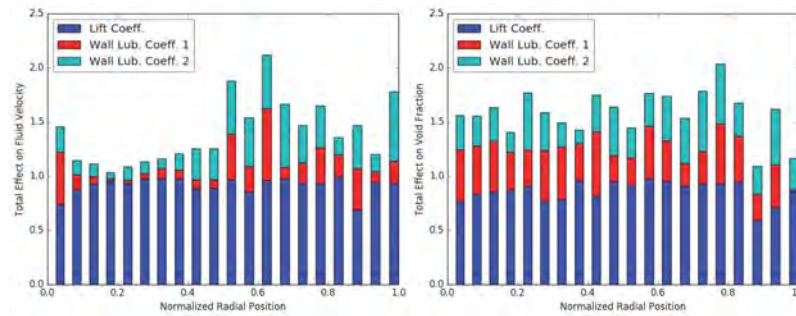
## Simple and Rank Correlations



Larger differences between simple and rank correlations for void fraction indicates a higher degree of nonlinearity in the relationship between the void fraction profile and the multiphase closure parameters

## Polynomial Chaos Expansion Based Sensitivity

Even though wall lubrication is active only near the wall,  
it has an effect on the the velocity and void frac. profiles everywhere

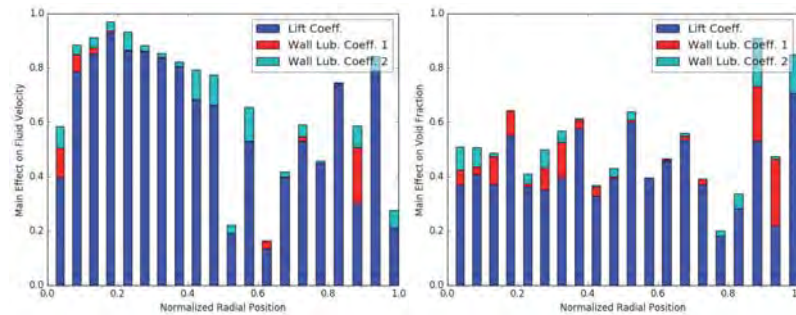


Total Effect

Navigation icons: back, forward, search, etc.

## Polynomial Chaos Expansion Based Sensitivity

Even though wall lubrication is active only near the wall,  
it has an effect on the the velocity and void frac. profiles everywhere

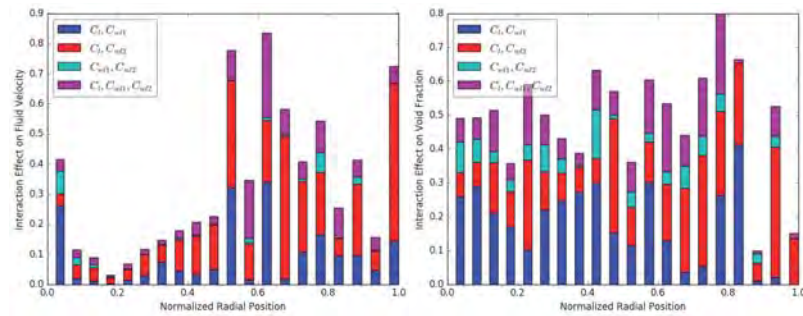


Main Effect

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## Polynomial Chaos Expansion Based Sensitivity

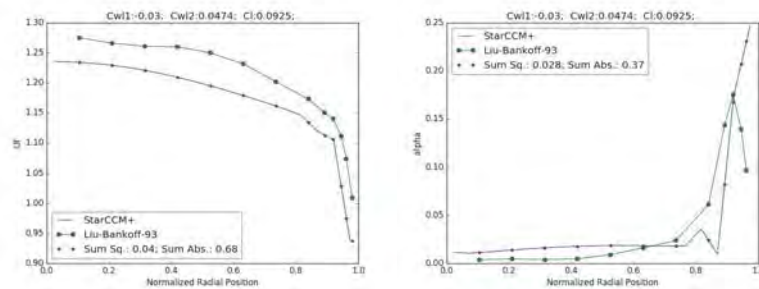
Even though wall lubrication is active only near the wall,  
it has an effect on the the velocity and void frac. profiles everywhere



Interaction Effect

Navigation icons: back, forward, search, etc.

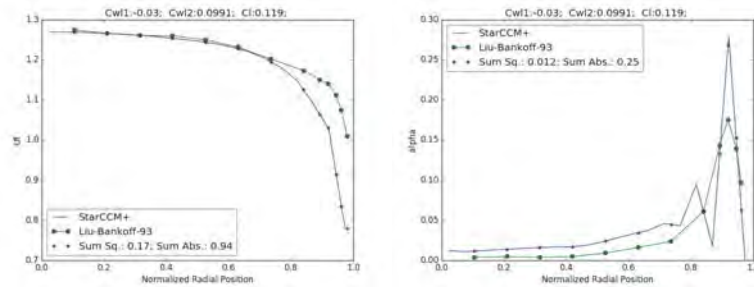
## Starccm+ Parameter Optimization with Dakota



Least squares fit

Navigation icons: back, forward, search, etc.

## Starccm+ Parameter Optimization with Dakota



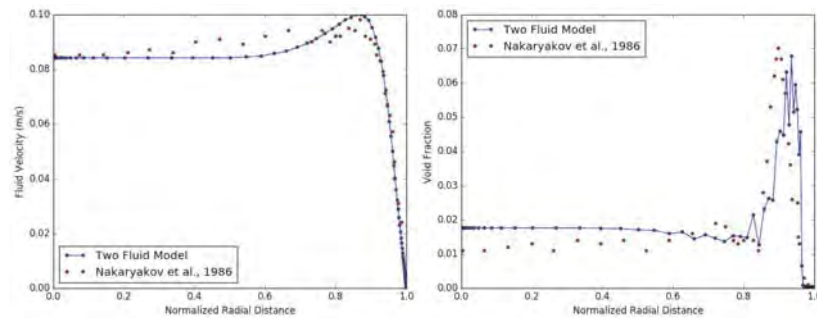
L1 norm produces a better fit of the void fraction profile

Navigation icons: back, forward, search, etc.

## Bayesian Analysis

Navigation icons: back, forward, search, etc.

## Experiments of Nakaryakov et al., 1986

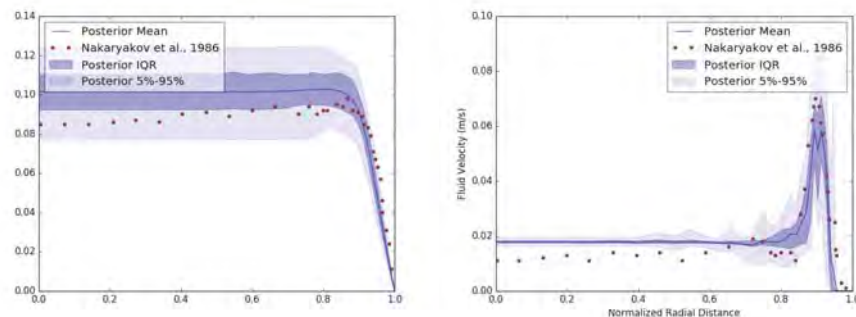


An example model solution using lift, drag, and wall lubrication  
Nakaryakov et al., 1986; Antal, Lahey, Flaherty, 1991

Navigation icons: back, forward, search, etc.

## Bayes. Analysis can Provide Insights into Model Behavior

However, converged statistics may be difficult to obtain with STARCCM+. Here the same parameterizations as in the STARCCM+ studies are used in a Bayesian calibration study



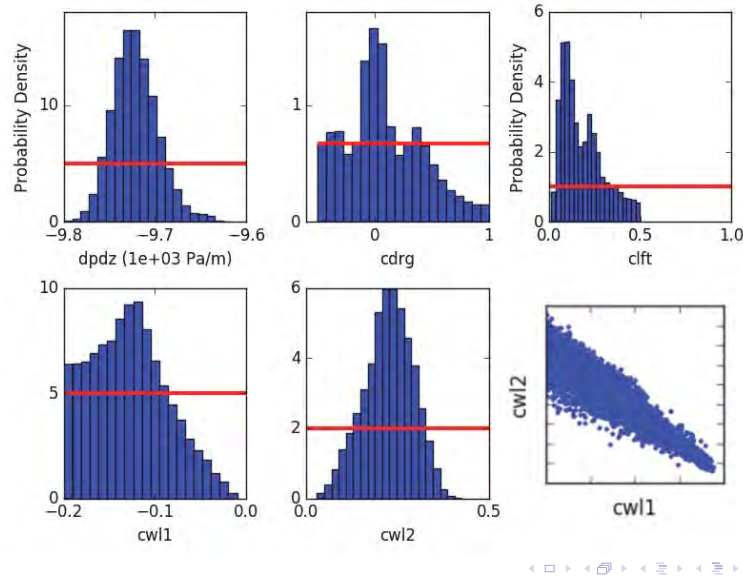
Nakaryakov et al., 1986; Antal, Lahey, Flaherty, 1991

Navigation icons: back, forward, search, etc.



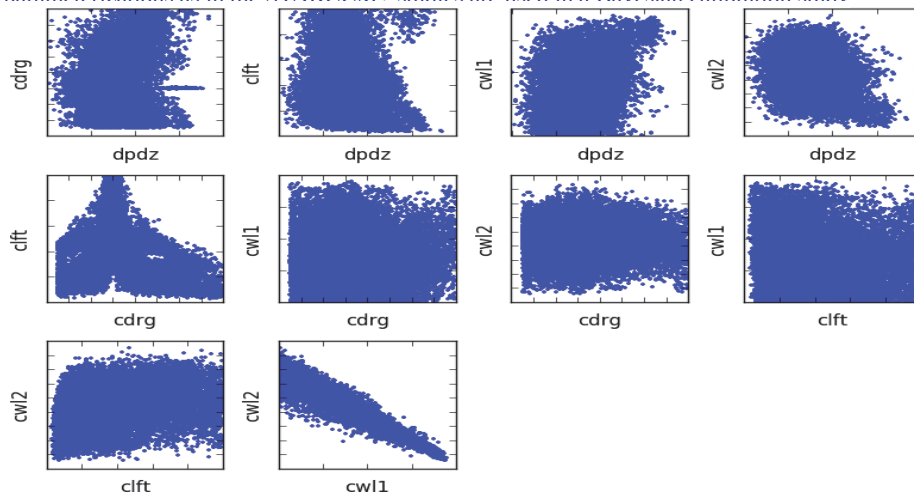
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## Bayes. Analysis can Provide Insights into Model Behavior

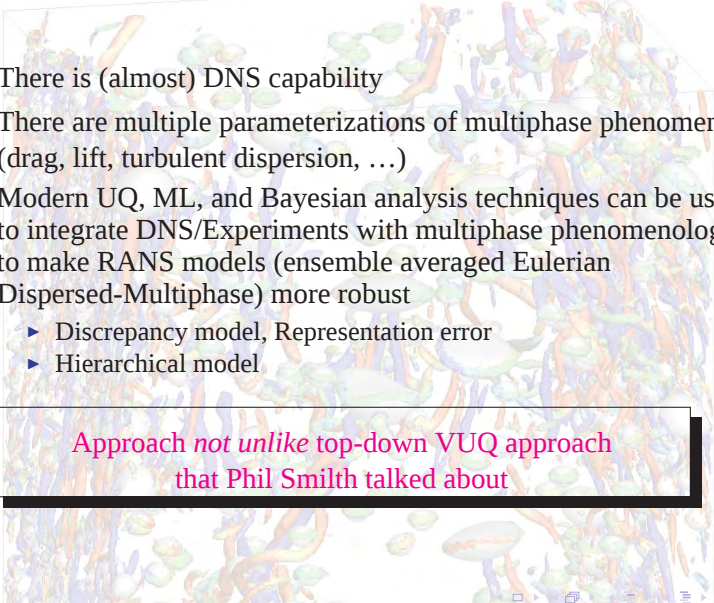
However, converged statistics may be difficult to obtain with STARCCM+. Here the same parameterizations as in the STARCCM+ studies are used in a Bayesian calibration study



Pairwise correlations between parameters

## Looking Forward

### Towards Making Multiphase Modeling Robust

- 
- ▶ There is (almost) DNS capability
  - ▶ There are multiple parameterizations of multiphase phenomena (drag, lift, turbulent dispersion, ...)
  - ▶ Modern UQ, ML, and Bayesian analysis techniques can be used to integrate DNS/Experiments with multiphase phenomenology to make RANS models (ensemble averaged Eulerian Dispersed-Multiphase) more robust
    - ▶ Discrepancy model, Representation error
    - ▶ Hierarchical model

Approach *not unlike* top-down VUQ approach  
that Phil Smilth talked about

# Eulerian Models and Polydispersity Treatment for Dilute Gas-Particles Flows

John Milo Parra-Alvarez (Speaker)

Phil Smith

Sean Smith

## Overview

- Motivation
- Eulerian Moment-Based Methods
- Polydispersity treatment of multiphase flows
  - Size-conditioned moment method
  - Size-reconstructed moment method
- Applications

Motivation

# Industrial Applications

## Chemical Looping Processes



<http://www.netl.doe.gov/research/coal/energy-systems/advanced-combustion/prototype>

## Oxy-Coal combustion

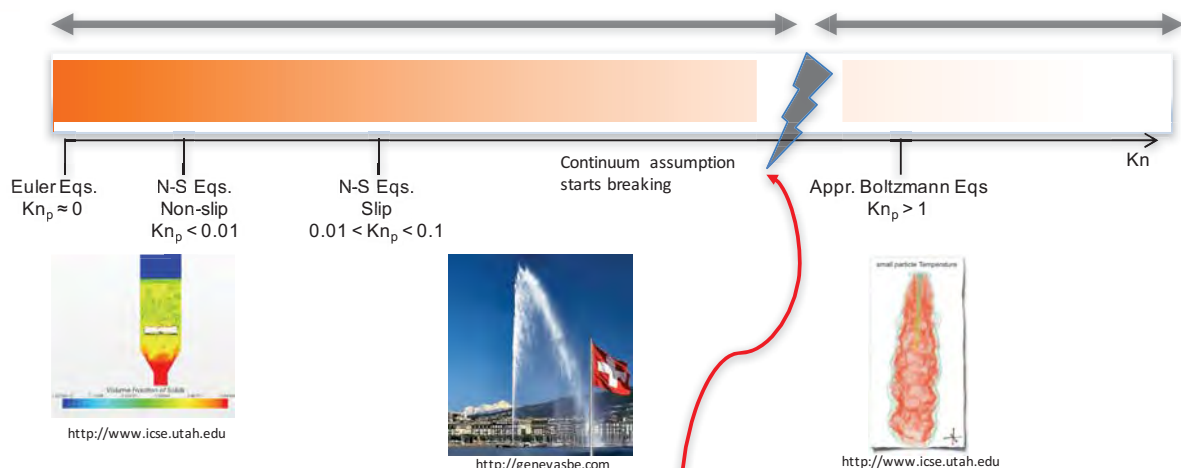


## And many more:

- Catalytic combustion
- Fluidization technologies
- Coal gasification
- Pneumatic transport
- Drying
- plasma arc coating
- Aerosol transport
- ...

Motivation

# Multiscale Phenomena



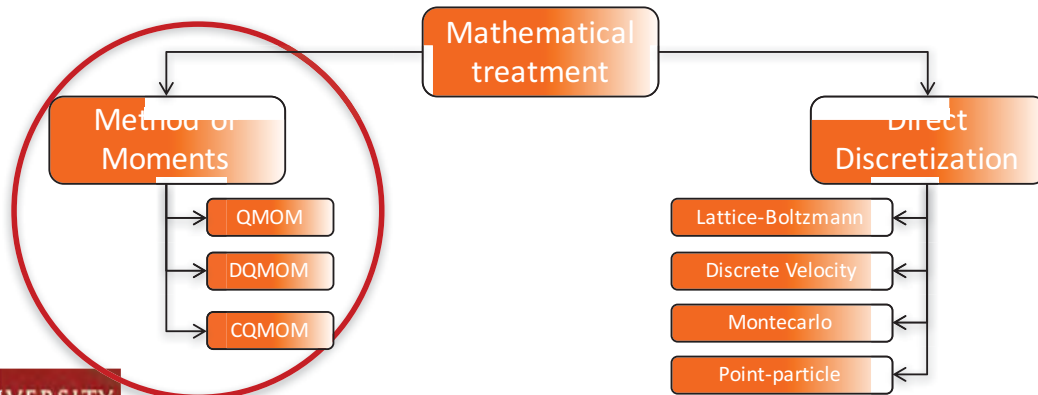
More complex modeling approaches are necessary to describe non-equilibrium phenomena in particulate flows

## The mathematical model

$$\mathfrak{F} = \mathfrak{F}(\mathbf{x}, \mathbf{v}, \theta, \xi)$$

$$\frac{\partial \mathfrak{F}}{\partial t} + \underbrace{\nabla_{\mathbf{x}} \cdot (\mathbf{v} \mathfrak{F})}_{(i)} + \underbrace{\nabla_{\xi} \cdot (\mathbf{G}(\xi, \theta, \mathbf{v}) \mathfrak{F})}_{(ii)} + \underbrace{\nabla_{\theta} \cdot (\mathbf{H}(\xi, \theta, \mathbf{v}) \mathfrak{F})}_{(iii)} + \underbrace{\nabla_{\mathbf{v}} \cdot (\mathbf{F}(\xi, \mathbf{v}) \mathfrak{F})}_{(iv)} = \underbrace{\mathbb{C}[f, f]}_{(v)}$$

(i) Advection in physical space      (ii) Advection in size space      (iii) Advection in scalar space      (iv) Advection in velocity space      (v) Particle-particle interactions



Size-conditioned moment method



## Joint NDF Approximation

$$\frac{\partial \mathfrak{F}}{\partial t} + \nabla_{\mathbf{x}} \cdot (\mathbf{v} \mathfrak{F}) = - \underbrace{\nabla_{\xi} \cdot (\mathbf{G}(\xi, \boldsymbol{\theta}, \mathbf{v}) \mathfrak{F}) + \nabla_{\boldsymbol{\theta}} \cdot (\mathbf{H}(\xi, \boldsymbol{\theta}, \mathbf{v}) \mathfrak{F}) + \nabla_{\mathbf{v}} \cdot (\mathbf{F}(\xi, \mathbf{v}) \mathfrak{F})}_{\mathfrak{S}(\xi, \boldsymbol{\theta}, \mathbf{v})} + \mathbb{C}[f, f]$$

$$\mathfrak{S}(\xi, \boldsymbol{\theta}, \mathbf{v})$$

$$\mathfrak{F}(\mathbf{x}, \mathbf{v}, \boldsymbol{\theta}, \xi) = g(\xi) f(\boldsymbol{\theta} | \xi) \mathfrak{F}(\mathbf{v} | \xi, \boldsymbol{\theta}) \approx \sum_{\alpha=1}^{N_s} w_{\alpha} g(\xi, \xi_{\alpha}) f(\boldsymbol{\theta}, \boldsymbol{\theta}_{\alpha} | \xi_{\alpha}) \mathfrak{F}(\mathbf{v} | \xi_{\alpha}, \boldsymbol{\theta}_{\alpha}) \approx \sum_{\alpha=1}^{N_s} w_{\alpha} g(\xi, \xi_{\alpha}) f(\boldsymbol{\theta}, \boldsymbol{\theta}_{\alpha}) \mathfrak{F}(\mathbf{v} | \xi_{\alpha})$$

$$\begin{aligned} & \sum_{\alpha=1}^{N_s} w_{\alpha} g(\xi, \xi_{\alpha}) f(\boldsymbol{\theta}, \boldsymbol{\theta}_{\alpha}) \left[ \frac{\partial}{\partial t} (\mathfrak{F}(\mathbf{v} | \xi_{\alpha})) + \nabla_{\mathbf{x}} \cdot (\mathbf{v} \mathfrak{F}(\mathbf{v} | \xi_{\alpha})) \right] \\ & + \sum_{\alpha=1}^{N_s} \mathfrak{F}(\mathbf{v} | \xi_{\alpha}) \left[ \frac{\partial}{\partial t} (w_{\alpha} g(\xi, \xi_{\alpha}) f(\boldsymbol{\theta}, \boldsymbol{\theta}_{\alpha})) + \nabla_{\mathbf{x}} \cdot (\mathbf{v} w_{\alpha} g(\xi, \xi_{\alpha}) f(\boldsymbol{\theta}, \boldsymbol{\theta}_{\alpha})) \right] = \mathfrak{S}(\xi, \boldsymbol{\theta}, \mathbf{v}) \end{aligned}$$

$$g(\xi, \xi_{\alpha}) = \delta(\xi - \xi_{\alpha})$$

$$f(\boldsymbol{\theta}, \boldsymbol{\theta}_{\alpha}) = \prod_{k=1}^M \delta(\theta_{m_k} - \theta_{m_k, \alpha})$$

$$\mathfrak{M}^{(l, M, N)} = \int_{\mathbb{R}} \int_{\mathbb{R}^m} \int_{\mathbb{R}^3} \xi^l \boldsymbol{\theta}^{(M)} \mathbf{v}^{(N)} \mathfrak{F}(\xi, \boldsymbol{\theta}, \mathbf{v}; \mathbf{x}, t) d\mathbf{v} d\boldsymbol{\theta} d\xi$$

## System of equations

$$\begin{aligned} & \iiint \xi^l \boldsymbol{\theta}^{(M)} \mathbf{v}^{(N)} \sum_{\alpha=1}^{N_s} w_{\alpha} \delta(\xi - \xi_{\alpha}) \prod_{k=1}^{N_{\theta}} \delta(\theta_{m_k} - \theta_{m_k, \alpha}) \left[ \frac{\partial}{\partial t} (\mathfrak{F}(\mathbf{v} | \xi_{\alpha})) + \nabla_{\mathbf{x}} \cdot (\mathbf{v} \mathfrak{F}(\mathbf{v} | \xi_{\alpha})) \right] d\mathbf{v} d\boldsymbol{\theta} d\xi \\ & + \iiint \xi^l \boldsymbol{\theta}^{(M)} \mathbf{v}^{(N)} \sum_{\alpha=1}^{N_s} \mathfrak{F}(\mathbf{v} | \xi_{\alpha}) \left[ \frac{\partial}{\partial t} (w_{\alpha} g(\xi, \xi_{\alpha}) f(\boldsymbol{\theta}, \boldsymbol{\theta}_{\alpha})) + \nabla_{\mathbf{x}} \cdot (\mathbf{v} w_{\alpha} g(\xi, \xi_{\alpha}) f(\boldsymbol{\theta}, \boldsymbol{\theta}_{\alpha})) \right] d\mathbf{v} d\boldsymbol{\theta} d\xi = \mathfrak{S}(\xi, \boldsymbol{\theta}, \mathbf{v}) \end{aligned}$$

The original NDF can be rewritten in terms of transport equations for primitive variables and conditional moments

$$\left\{ \begin{array}{l} \mathcal{L}[\langle \mathbf{m} \rangle_{\alpha}^N] \\ \mathcal{L}[w_{\alpha} \xi_{\alpha}] \\ \mathcal{L}[w_{\alpha} \boldsymbol{\theta}_{\alpha}] \\ \mathcal{L}[w_{\alpha}] \end{array} \right\} \quad \mathcal{L}[\cdot] \equiv \frac{\partial}{\partial t} + \mathbf{U} \nabla_{\mathbf{x}} - \mathbf{S}$$

Each set of transport equations is weighted by a matrix of coefficients of primitive variables and conditional moments

$$[\mathbf{A}_w] \mathcal{L}[\langle \mathbf{m} \rangle_{\alpha}^N] + [\mathbf{A}_{w\theta}] \mathcal{L}[w_{\alpha} \xi_{\alpha}] + [\mathbf{A}_{w\xi}] \mathcal{L}[w_{\alpha} \boldsymbol{\theta}_{\alpha}] + [\mathbf{A}_m] \mathcal{L}[w_{\alpha}] = \mathbf{0}_{\alpha}$$

## Matrix form, IVNS and final set

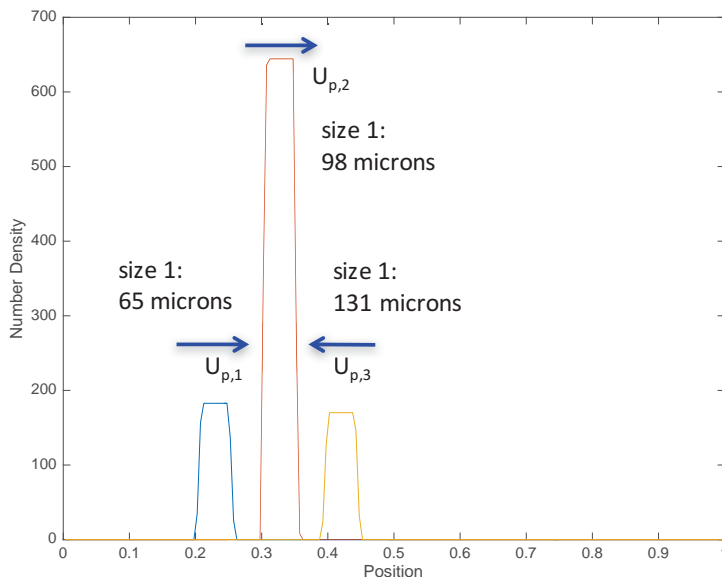
Minimum norm  
solution for  
homogeneous  
linear systems

$$\begin{bmatrix} \mathbf{A}_w & \mathbf{A}_{w\theta} & \mathbf{A}_{w\xi} & \mathbf{A}_m \end{bmatrix} \begin{bmatrix} \mathcal{L}[\langle \mathbf{m} \rangle_\alpha^N] \\ \mathcal{L}[w_\alpha \xi_\alpha] \\ \mathcal{L}[w_\alpha \theta_\alpha] \\ \mathcal{L}[w_\alpha] \end{bmatrix} = \begin{bmatrix} 0_\alpha \\ 0_\alpha \\ 0_\alpha \\ 0_\alpha \end{bmatrix}$$

$$\begin{bmatrix} \mathcal{L}[\langle \mathbf{m} \rangle_\alpha^N] \\ \mathcal{L}[w_\alpha \xi_\alpha] \\ \mathcal{L}[w_\alpha \theta_\alpha] \\ \mathcal{L}[w_\alpha] \end{bmatrix} = \begin{bmatrix} 0_\alpha \\ 0_\alpha \\ 0_\alpha \\ 0_\alpha \end{bmatrix}$$

$$\begin{aligned} \frac{\partial \langle \mathbf{m} \rangle_\alpha^{(N)}}{\partial t} + \nabla_x \langle \mathbf{m} \rangle_\alpha^{(N)+1} &= (N) \int_{\Omega_v} \mathbf{v}^{(N)-1} \mathfrak{F}(\mathbf{v} | \xi_\alpha) \mathbf{F}(\xi, \mathbf{v}) d\mathbf{v} \\ \frac{\partial w_\alpha \xi_\alpha}{\partial t} + \mathbf{U}_\alpha^{(N)} \cdot \nabla_x (w_\alpha \xi_\alpha) &= w_\alpha \mathbf{G}(\xi, \theta, \mathbf{v}) \\ \frac{\partial w_\alpha \theta_\alpha}{\partial t} + \mathbf{U}_\alpha^{(N)} \cdot \nabla_x (w_\alpha \theta_\alpha) &= w_\alpha \mathbf{H}(\xi, \theta, \mathbf{v}) \\ \frac{\partial w_\alpha}{\partial t} + \mathbf{U}_\alpha^{(N)} \cdot \nabla_x (w_\alpha) &= 0 \end{aligned}$$

## Applications ID packets



The drag force has the form:

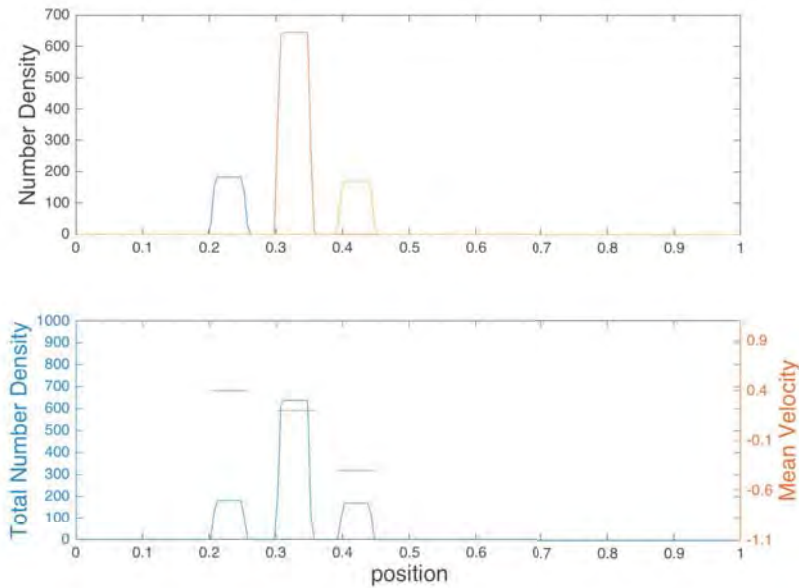
$$\langle A \rangle_{\beta, \alpha} = \frac{m_p}{\tau_p} (|U_f - v_{\beta, \alpha}|)$$

$$\tau_p = \tau_p(\text{Re}_p)$$

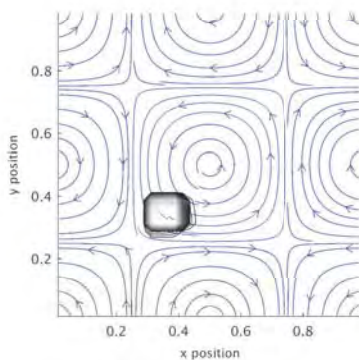
$$\text{Re}_p = \frac{2|U_f - v_{\beta, \alpha}|r}{\mu_f}$$

- Three discrete sizes are considered
- Velocity moments are conditioned on these 3 sizes
- Particles move in a vacuum
- The purpose of this simple example is to capture PTC present in dilute systems

## Applications 1D packets

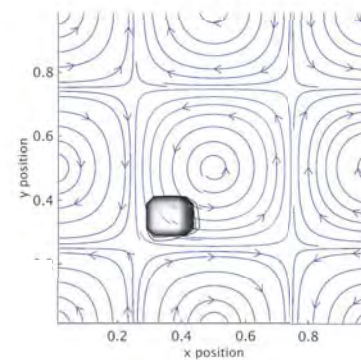
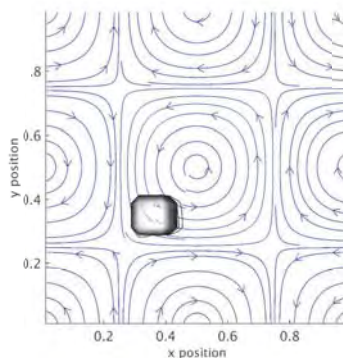


## Applications 2D Taylor vortex



65 Microns

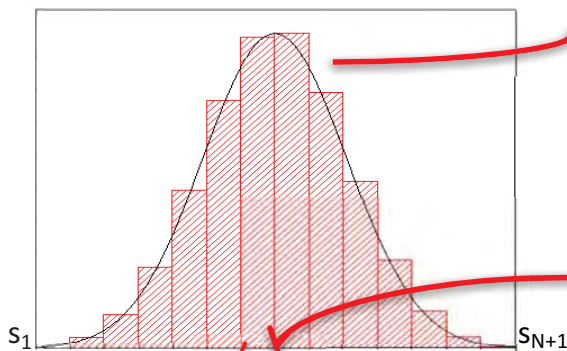
131 Microns



98 Microns

## Size-reconstructed moment method

## Formulation



$$\frac{\partial \mathfrak{F}}{\partial t} + \nabla_{\mathbf{x}} \cdot (\mathbf{v} \mathfrak{F}) = \mathfrak{S}$$

$$\begin{aligned} \mathfrak{F}(\xi, \boldsymbol{\theta}, \mathbf{v}) &= \sum_{k=1}^{N_s+1} \ell_k(\xi) g_k(\xi) f_k(\boldsymbol{\theta}, \mathbf{v} | \xi) \\ &= \sum_{k=1}^{N_s+1} \ell_k(\xi) g_k(\xi) \delta(\boldsymbol{\theta} - \bar{\boldsymbol{\theta}}_k(\xi)) \sum_{\alpha=1}^{N_v} \mathbf{w}_{k,\alpha} \delta(\mathbf{v} - \bar{\mathbf{U}}_{k,\alpha}(\xi)) \end{aligned}$$

$$\ell_k(\xi) = \begin{cases} 1 & \text{if } \xi \in [s_k, s_{k+1}] \\ 0 & \text{otherwise} \end{cases}$$

$$\begin{aligned} \bar{\boldsymbol{\theta}}^{(M)} &= \bar{\theta}_1^{m_1} \bar{\theta}_2^{m_2} \dots \bar{\theta}_M^{m_M} \\ \bar{\mathbf{U}}_{\alpha}^{(N)} &= \bar{U}_{x,\alpha}^{n_1} \bar{U}_{y,\alpha}^{n_2} \bar{U}_{z,\alpha}^{n_3} \end{aligned}$$

$$\frac{\partial \mathfrak{M}_k^{(l,M,N)}}{\partial t} + \nabla_{\mathbf{x}} \cdot \mathfrak{M}_k^{(l,M,N+1)} = \int_{s_k}^{s_{k+1}} \int_{\Omega_{\theta}} \int_{\Omega_{\mathbf{v}}} \xi^l \boldsymbol{\theta}^{(M)} \mathbf{v}^{(N)} \mathfrak{S}_k d\mathbf{v} d\boldsymbol{\theta} d\xi$$

$$\mathfrak{M}_k^{(l,M,N)} = \sum_{\alpha=1}^{N_v} \mathbf{w}_{\alpha} \int_{s_k}^{s_{k+1}} \xi^l g(\xi) \bar{\boldsymbol{\theta}}^{(M)} \bar{\mathbf{U}}_{\alpha}^{(N)} d\xi$$

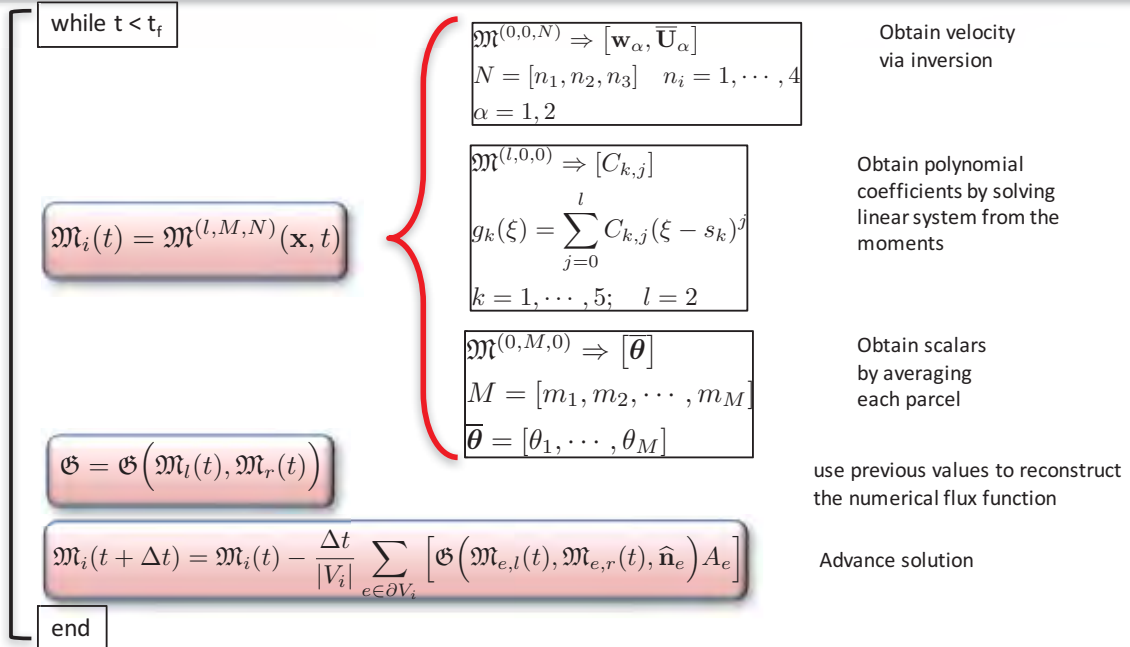
$$\mathfrak{M}^{(l,M,N)} = \sum_{k=1}^{N_s+1} \mathfrak{M}_k^{(l,M,N)}$$

$$g_k(\xi) = \sum_{j=0}^l C_{k,j} (\xi - s_k)^j$$

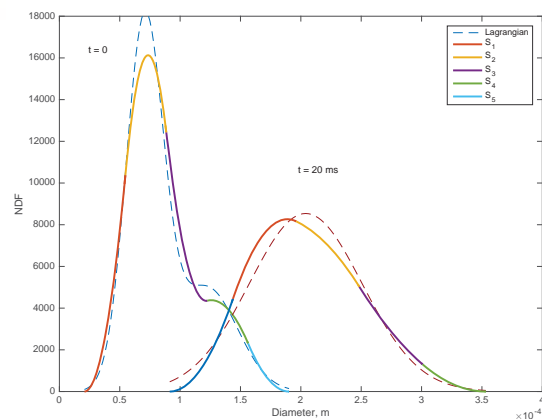
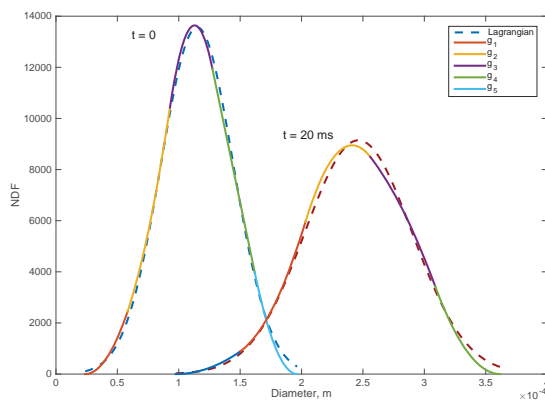
$$[\mathbf{A}] \cdot [\mathbf{C}] = [\mathbf{M}]$$



## Computational loop



## Reconstruction test



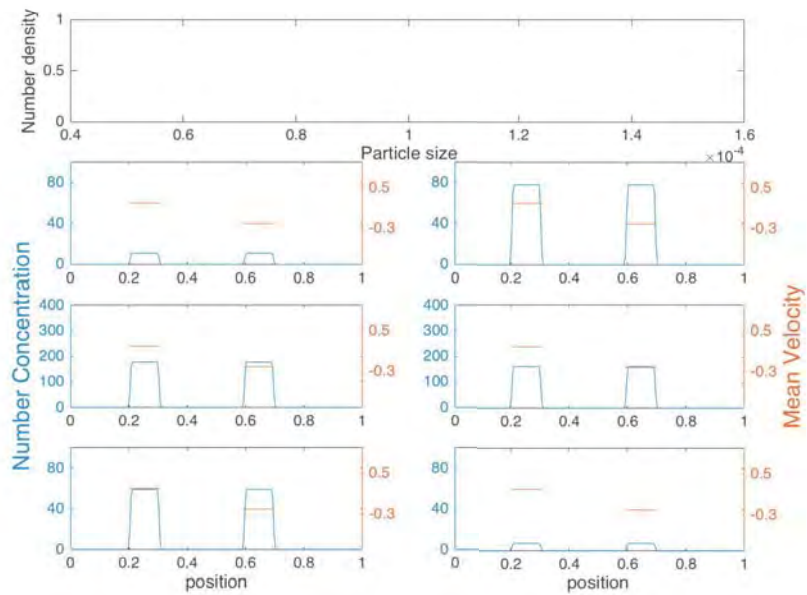
$$\frac{dD_p}{dt} = 0.5 * \sqrt{D_p}$$

- The size distribution is reconstructed at each time step
- 5 parcels are used
- In each parcel a second order polynomial is used

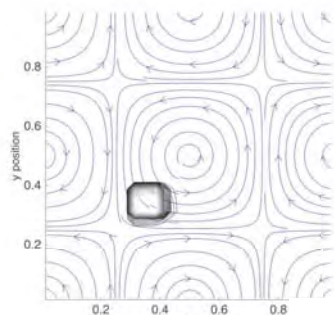
$$[A] \cdot [C] = [M]$$



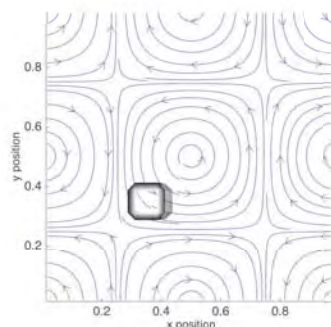
## Applications 1D packets



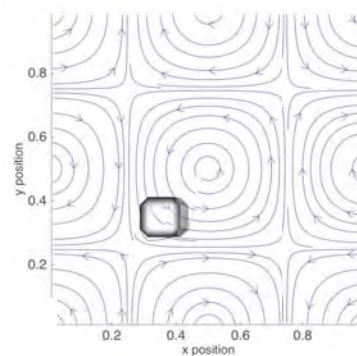
## Applications 2D Taylor vortex



Parcel 1

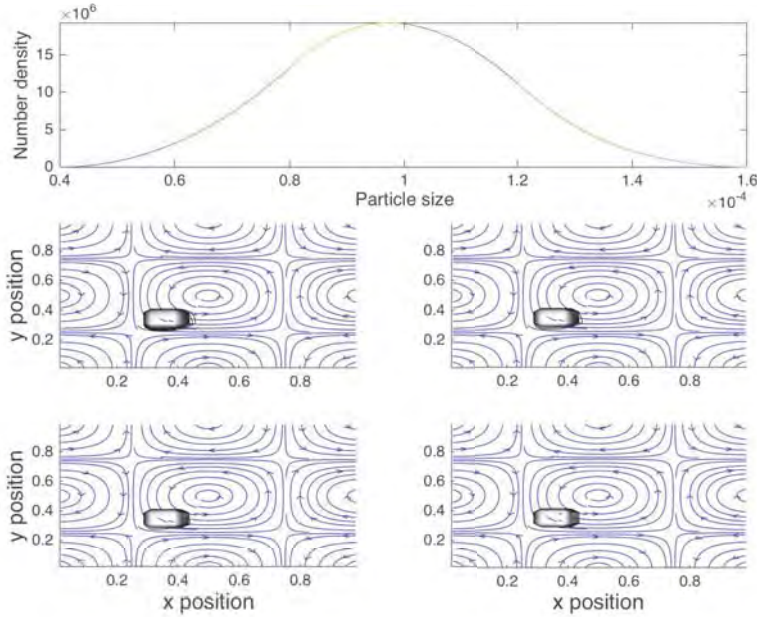


Parcel 6



Parcel 4

## Applications 2D Taylor vortex



## Char oxidation test

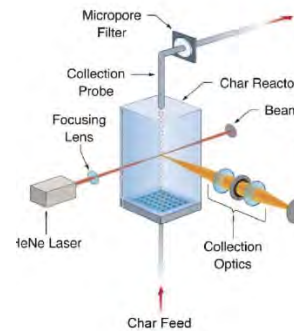
$$\begin{aligned} \frac{\partial \mathfrak{M}_k^{(l,M,T,N)}}{\partial t} + \frac{\partial \mathfrak{M}_k^{(l,M,T,N+1)}}{\partial x} = & \\ & + M \sum_{\alpha=1}^{N_v} w_{\alpha} \int_{s_k}^{s_{k+1}} \xi^l g(\xi) (U_{\alpha}^N(\xi) \bar{m}_p^{M-1}(\xi) \bar{\theta}^T(\xi) G(\xi, \bar{m}_p(\xi), \bar{\theta}(\xi))) d\xi \\ & + T \sum_{\alpha=1}^{N_v} w_{\alpha} \int_{s_k}^{s_{k+1}} \xi^l g(\xi) (U_{\alpha}^N(\xi) \bar{m}_p^M \bar{\theta}^{T-1}(\xi) H(\xi, \bar{m}_p(\xi), \bar{\theta}(\xi))) d\xi \\ & + N \sum_{\alpha=1}^{N_v} w_{\alpha} \int_{s_k}^{s_{k+1}} \xi^l g(\xi) \bar{m}_p^M \bar{\theta}^T U_{\alpha}^{N-1}(\xi) \frac{1}{\bar{m}_p} F_{\alpha}(\xi, \bar{\theta}(\xi), U_{\alpha}(\xi)) d\xi \end{aligned}$$

Numerical  
Splitting

$$\begin{aligned} \frac{\partial \mathfrak{M}_k^{(l,M,T,N)}}{\partial t} + \frac{\partial \mathfrak{M}_k^{(l,M,T,N+1)}}{\partial x} &= 0 \rightarrow \frac{\Delta t}{2} \\ \frac{\partial U_{\alpha}(\xi)}{\partial t} &= \frac{U_g - U_{\alpha}(\xi)}{\tau(\xi, Re)} \rightarrow \Delta t \\ \frac{\partial \bar{m}_p}{\partial t} &= G(\xi, \bar{m}_p, \bar{\theta}) \rightarrow \Delta t \\ \frac{\partial \bar{\theta}}{\partial t} &= H(\xi, \bar{m}_p, \bar{\theta}) \rightarrow \Delta t \\ \frac{\partial \mathfrak{M}_k^{(l,N)}}{\partial t} + \frac{\partial \mathfrak{M}_k^{(l,N+1)}}{\partial x} &= 0 \rightarrow \frac{\Delta t}{2} \end{aligned}$$

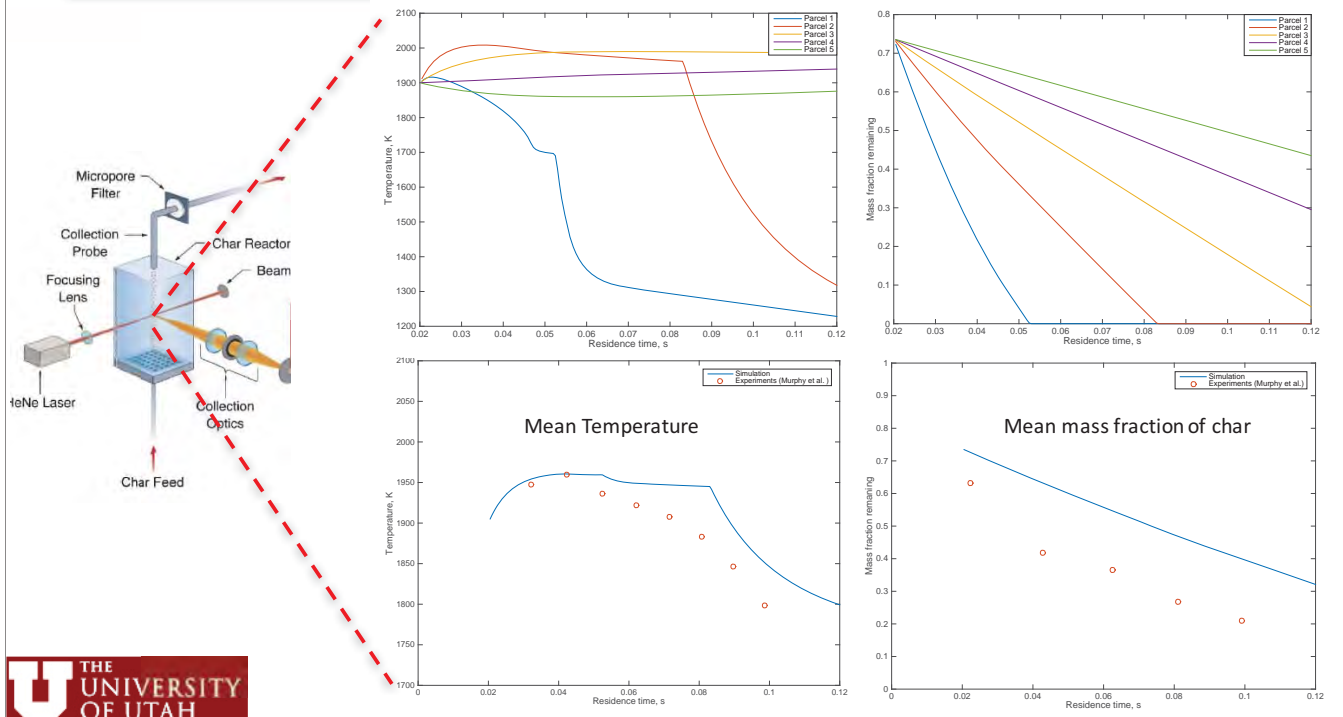
Heat and mass transfer  
from the particle to the gas

Moment equations for a 1D tubular  
reactor describing mass, temperature,  
size and velocity



$$\begin{aligned} G(\xi, \bar{m}_p, \bar{\theta}) &= \pi \xi^2 q M_c \\ H(\xi, \bar{m}_p, \bar{\theta}) &= -\frac{6\epsilon\sigma}{\xi C_p \rho_p} (\theta^4 - T_w^4) \\ &\quad - \frac{12\gamma}{\xi^2 C_p \rho_p} \left[ \frac{\kappa/2}{\exp(\kappa/2) - 1} \right] (\theta - T_g) + q \Delta h \end{aligned}$$

## Char oxidation test



Thank you